UNIT-III

ELECTRICAL AND DIELECTRIC PROPERTIES OF MATERIALS

1. Explain the failures of Classical free electron theory.

Failures of classical free electron theory:

Classical free electron theory fails to explain the temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

i)Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T.

i.e. $\sigma_{exp} \ \alpha \ \frac{1}{T} \rightarrow (1)$

According to the assumptions of classical free electron theory

or $\sigma \ \alpha \ \frac{1}{\sqrt{T}}$ →(2)

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

ii) Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$\sigma = \frac{ne^2\tau}{m}$ i.e., σ α n , where n is the electron concentration,

Consider copper and aluminum. Their electrical conductivities are $5.88\times10^7/\Omega m$ and $3.65x10^7/\Omega m$. The electron concentrations for copper and aluminum are $8.45\times10^{28}/m^3$ and $18.06x10^{28}/m^3$. Hence the classical free electron theory fails to explain the dependence of σ on electron concentration.

Experimental results:

Metals	Electron concentration(n)	conductivity (σ)
Copper	$8.45\times10^{28}/m^3$	$5.88x10^7/\Omega m$
Aluminium	$18.06x10^{28}/m^3$	$3.65x10^7/ \Omega m$

2. Explain the assumptions of quantum free electron theory. Also discuss the merits of quantum free electron theory.

Assumptions of quantum free electron theory:

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

Merits of Quantum free electron theory:

Quantum free electron theory has successfully explained following observed experimental facts where as the classical free electron theory failed.

i) Temperature depends on electrical conductivity:

Experimentally electrical conductivity $\sigma_{expt}\ \alpha \frac{1}{T}$

According to quantum free electron theory electrical conductivity is given by

$$\sigma_{QFT} = \frac{ne^2\lambda}{m^* v_F} \quad \text{------------------ (1)}$$

Where m^* is called effective mass of an electron.

According to quantum free electron theory E_F and V_F are independent of temperature. The dependence of λ & T is as follows

Conduction electrons are scattered by the vibrating ions of the lattice. As the temperature increases the vibrational cross sectional areas (πr^2) increases and hence mean free path decreases.

The mean free path of the electrons is given by

$$\therefore \lambda \alpha \frac{1}{\pi r^2}$$

Where 'r' is the amplitude of vibrations of lattice ions

Considering the facts the energy of vibrating body is proportional to the square of amplitude and the energy of ions is due to the thermal energy.

The thermal energy is proportional to the temperature 'T'.

We can write $r^2 \alpha T$

$\therefore \lambda \alpha 1/T$ substituting for λ in eqn (1)

Hence $\sigma_{QFT} \ \alpha \ \frac{1}{T}$

Thus $\sigma_{QFT} \ \alpha \frac{1}{T}$ is correctly explained by quantum free electron theory.

ii) Electrical conductivity and electron concentration:

Aluminium and gallium which have three free electrons per atom have lower electrical conductivity than that of copper and silver, which have only one free electron per atom.

As per quantum free electron the electrical conductivity is

$$\sigma_{QFT} = \frac{ne^2\lambda}{m^* v_F}$$

i.e $\sigma \ \alpha \ (n)$ and $\sigma \ \alpha \ (\lambda/v_F)$

The value of (n) for aluminium is 2.13 times higher than that of copper. But the value of (λ/v_f) for copper is about 3.73 times higher than that of aluminium. Thus the conductivity of copper exceeds that of aluminium.

3. Define Fermi energy and Fermi factor. Explain the variation of Fermi factor with temperature.

***Fermi energy* and Fermi level:**

The energy of electrons corresponding to the highest occupied energy level at absolute $0°K$ is called Fermi energy and the energy level is called Fermi level.

Fermi factor:

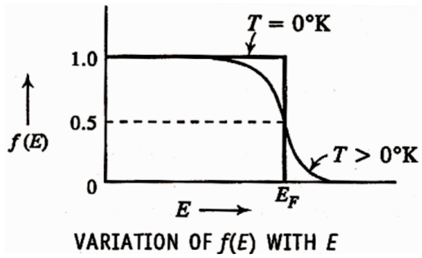
Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability f(E) that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

Dependence of Fermi factor with temperature and energy:

The dependence of Fermi factor on temperature and energy is as shown in the figure.



i) Probability of occupation for E<E$_F$ at T=0K:

When T=0K and E<E$_F$

$$f(E) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

The probability of occupation of energy state by the conduction electrons below the fermi level is 100%

ii) Probability of occupation for E>E$_F$ at T=0K:

When T=0K and E>E$_F$

$$f(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state by the conduction electrons above the fermi level is 0%

iii) The probability of occupation at ordinary temperature(for E≈E$_F$ at T>0K)

for E= E$_f$, $e^{(E-E_f)/kT} = e^0 = 1$

$$\therefore f(E) = \frac{1}{e^{(E-E_f)/kT}+1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

4. What is Fermi Energy? Derive an Expression for Fermi Energy at Absolute Zero temperature (T=0K)

The energy of electrons corresponding to the highest occupied energy level at absolute $0°K$ is called Fermi energy

Expression for Fermi Energy at Absolute Zero temperature

According to Fermi-Dirac statistics the distribution of electrons among the various allowed energy levels is given by

N(E) dE = g(E) f(E) dE .... (1)

The number of free electrons distributed /unit volume of the material up to Fermi level is given by

$$n = \int_0^{E_{F0}} (E)dE = \int_0^{E_{F0}} g(E)\, f(E) dE$$

But for the energy levels from E=0 to E= E_{F_0}

f (E) = 1 at T= 0K

$$n = \int_0^{E_{F0}} (E)\, dE$$

g(E) is given by $g(E)= \frac{8\pi\sqrt{2}m^{3/2}}{h^3} E^{1/2} dE$

where 'm' is the mass of electron, 'h' is the planck's constant

$$n = \frac{8\pi\sqrt{2}m^{3/2}}{h^3}\int_0^{E_{F0}} E^{1/2}\, dE = \frac{8\pi\sqrt{2}m^{3/2}}{h^3}\left[\frac{2}{3}E^{\frac{3}{2}}\right]_0^{E_{F0}}$$

$$n = \frac{8\pi\sqrt{2}m^{3/2}}{h^3} \times \frac{2}{3}(E_{F_0})^{3/2} = \left[\frac{8\pi\; 2^{3/2} 2m^{3/2}}{h^3}\right]\left[\frac{\pi}{3}\right](E_{F_0})^{3/2}$$

$$(E_{F_0})^{3/2} = \frac{h^3}{(8m)^{3/2}}\left(\frac{3n}{\pi}\right)$$

$$E_{F_0} = \left(\frac{h^2}{8m}\right)\left(\frac{3n}{\pi}\right)^{2/3}$$

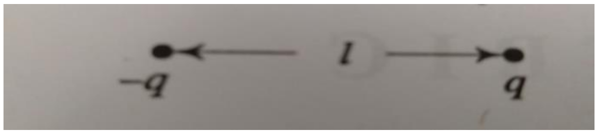
$$E_{F_0} = \left(\frac{h^2}{8m}\right)\left(\frac{3}{\pi}\right)^{2/3} n^{2/3} = 5.85\times10^{-38}\, n^{2/3}\ J$$

Note:

<u>Dielectric materials</u>: Dielectric materials are nothing but insulators which have the ability to get electrically polarized with the application of external electric field.

Examples: Glass, rubber, plastic, mica.

Electric dipole:



Two equal and opposite charges separated by a distance is called dipole.

The product of one of the charge and distance of separation between them is called dipole moment (μ).

$$\mu = q\,l$$

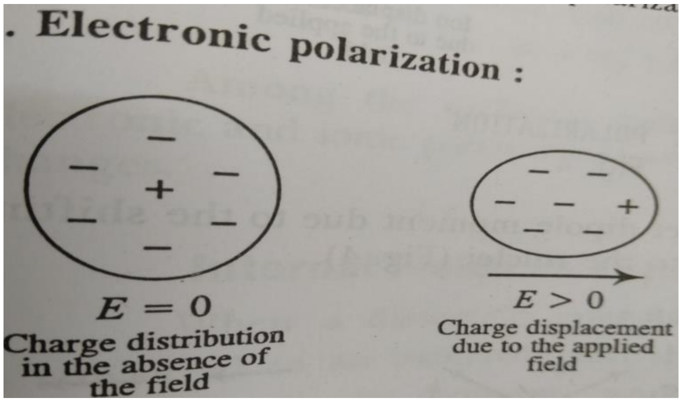
5. What is polarization? Explain various types of polarization mechanism.

Polarization: The displacement of charges in the atoms or molecules of a dielectric under the action of an applied electric field loads to the development of dipole moment is called polarization.

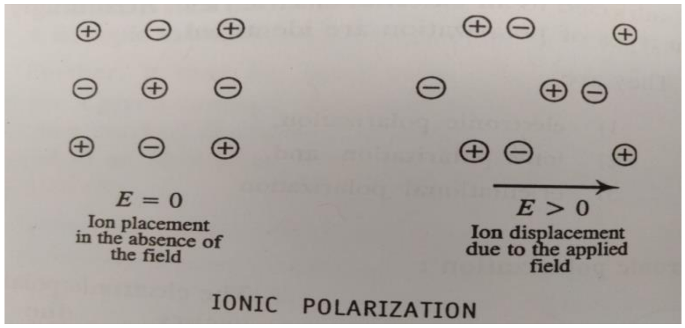
TYPES OF POLARIZATION MECHANISMS:

Electronic Polarization: The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material with the application of external electric field, which leads to development of dipole moment. The electronic polarizability is given by,

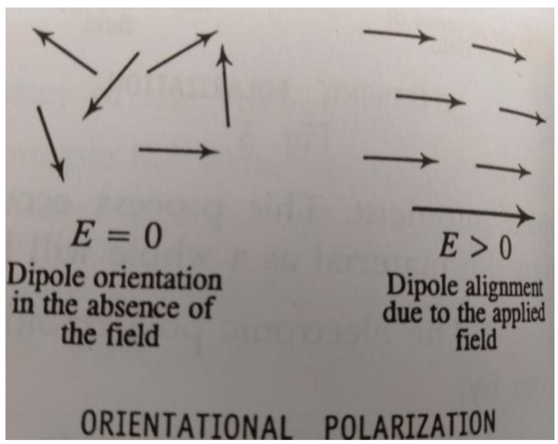
$$\alpha_e = \epsilon_0 (\epsilon_r - 1) / N .$$



- **Ionic Polarization:** This is kind of polarization occurs in some dielectric materials which have ionic bonds like Nacl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement. The distance of separation between the ion pair depends on the location of atoms in the lattice results in the development of dipole moment



- **Oriental Polarization:** This type of polarization occurs in polar dielectric material. (Permanent dipoles).



The orientation of molecules is random in the absence of electric field; therefore net dipole moment in the material is zero. But under the influence of an applied electric field each dipole reorient along the field direction. Thus the material develops an electrical polarization. This type of polarization is temperature dependent. In polar dielectrics the orientation polarizability α_0 is given by $\alpha_0 = \mu^2 / 3KT$

6. Define the term internal field in case of solid dielectrics. Explain polar and non polar dielectrics with examples

Internal fields in solids:

"The internal field is the electric field that acts at the site of any atom of a solid subjected to an external external electric field and is the resultant of the applied field and the field due to all the surrounding dipoles".

In one dimension the internal field is given by

$$E_i = E_{applied} + E_{dipoles}$$

$$E_i = E + \left(\frac{}{\epsilon_0}\right)P$$

where ν→ internal field constant, p→ polarization

For, 3D the internal field is called Lorentz field and is given by

$$\mathbf{E_{Lorentz} = E + \frac{P}{3\epsilon_0}}$$

Types of dielectric materials:

Polar dielectrics: In some dielectric materials, the effective centers of the negative and positive charges in the molecules do not coincide with each other and exhibit permanent dipoles in the absence of electric field are called polar dielectrics. **Example**: H_2O.

Non polar dielectrics: In some dielectric materials, the effective centers of the negative and positive charges in the molecules do not coincide with each other and do not exhibit permanent dipoles in the absence of electric field are called non polar dielectrics. **Examples**: paper, wood, glass.

7. Derive Clausius – Mossotti Equation.

Consider a solid dielectric material with dielectric constant Er. If 'n' is the number of atoms/unit volume and 'μ' is the dipole moment of the atoms in the material.

Therefore, the dipole moment / unit volume = N μ ………(1)

The field experienced by the atom is an internal field Ei. α_e is the electronic polarizability of the atoms.

Then dipole moment / unit volume = $N\alpha_e E_i$

The dipole moment $\mu = \alpha_e E_i$ ………(2)

Dipole moment / unit volume is nothing but polarization (P)

$$P = N\alpha_e E_i$$

$$E_i = \frac{P}{N\alpha e} \quad \text{……….(3)}$$

we have $P = \epsilon 0 (\epsilon r - 1) E$

$$E = \frac{P}{\epsilon 0 (\epsilon r - 1)} \quad \text{………(4)}$$

The expression for internal field for 3D material is given by $E_i = E + \frac{P}{3\epsilon 0}$

Substituting (3) and (4) in the above equation

$$\frac{P}{N\alpha e} = \frac{P}{\epsilon 0(\epsilon r - 1)} + \frac{P}{3\epsilon 0}$$

$$\frac{1}{N\alpha e} = \frac{1}{\epsilon 0(\epsilon r - 1)} + \frac{1}{3\epsilon 0} = \frac{1}{\epsilon 0}\left[\frac{1}{(\epsilon r - 1)} + \frac{1}{3}\right]$$

$$\boldsymbol{\frac{\epsilon r - 1}{\epsilon r + 2} = \frac{N\alpha e}{3\epsilon 0}}$$

This is called Clausius Mossotti equation.