

Electrical properties of materials.Quantum Free electron theory :-

After the failure of CFET in 1928 Sommerfeld proposed a theory for electrical properties of metals based on quantum mechanics. His theory is known as QFET. Sommerfeld retained some of the assumptions of CFET and included quantum mechanical concepts and Fermi-Dirac Statistics to the free electrons in the metal.

Assumptions of Quantum Free electron theory :-

- 1) The free electrons are partially free inside the metal and are bound to the metal. They ^{are possible} ~~for the electrical conduction.~~
- 2) The free electrons obey the laws of quantum mechanics.
- 3) The energy levels of free electrons are quantized.
- 4) The free electrons obey the Pauli's exclusion principle.
- 5) The distribution of ^{energy} among the free electrons according to Fermi-Dirac Statistics.
- 6) The potential is constant inside the metal.
- 7) The attraction between the electrons and Ion Core, The repulsion between electrons are ignored.

C.B	over lap
V.B	

The Band representation of metals ^{or solid} is used to explain electric conduction.

The energy levels are close together is called band.

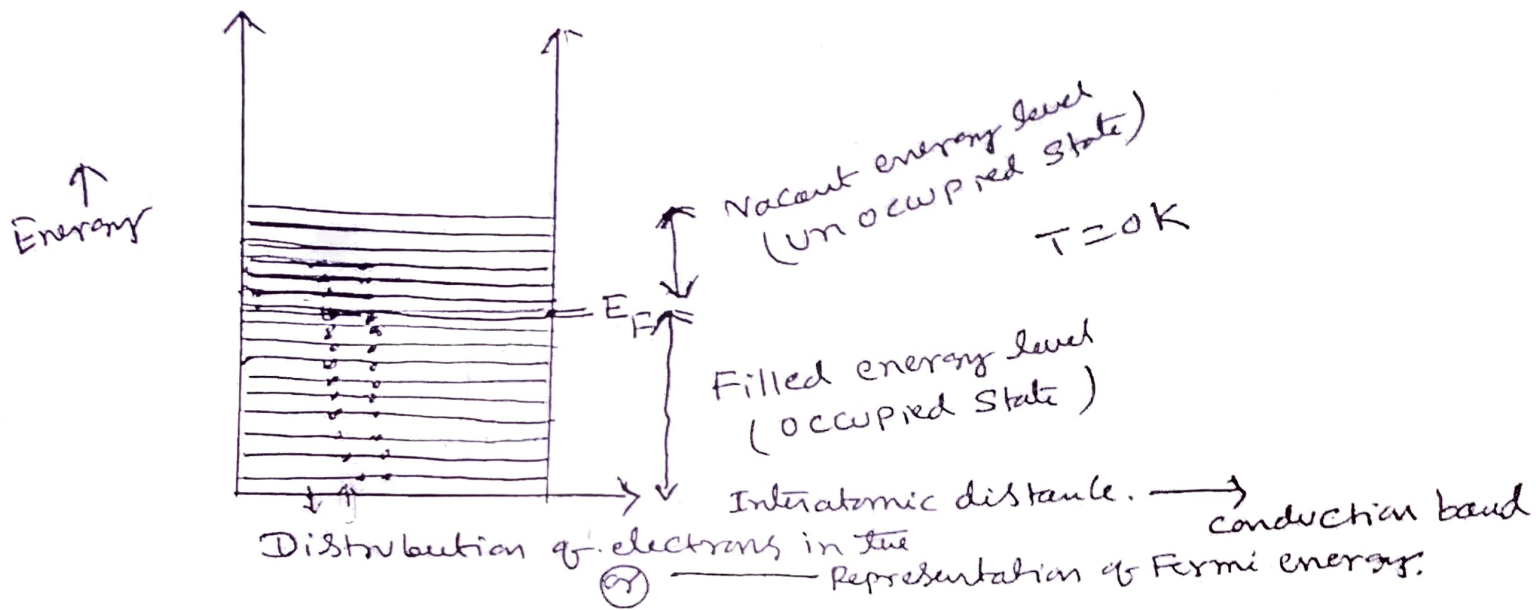
In a solid there will be different energy bands, each band containing a large number of energy levels.

The band occupied by the valence electrons is called V.B. The band occupied by conduction electron is known as C.B.

The C.B is empty for insulators and only partially filled for metals. The gap between the C.B and the V.B is called forbidden gap or band gap.

In a metal the V.B and C.B are overlapped, there is no gap between V.B and C.B. Therefore large number of free electrons are available for conduction in the conduction band at room temperature.

Fermi Energy and Fermi level :-



In a metal The conduction band is partially filled with an electrons at 0K as shown in figure. The energy levels are filled by the free electrons in accordance with the Pauli's exclusion principle.

According to this principle, each energy levels can accommodate only two electrons, one with Spin up and the other with Spin down. The filling up of energy levels takes place with one pair of electrons, one with Spin up and the other with Spin down occupy the lowest level, another pair the next higher level and so on until all the free electrons in the metal are accommodated as shown in figure.

We see that all the energy levels below certain levels marked E_F will be completely filled and the above it will remain completely vacant as shown in figure.

Here we define Fermi energy and Fermi level as follows.

The energy corresponding to the highest filled level at 0K in a metal is called Fermi energy.

OR
The Fermi energy is the maximum energy level up to which a free electrons are occupied at 0K in a metal.

The highest filled level at 0K in a metal is known as Fermi level

(OR)
The energy level which divides the filled and empty levels is called Fermi level.

It is found that for a large majority of metals, the Fermi energy E_F is less than 10 eV.

What is the importance of Fermi energy?
Electrical conduction takes place only when electrons acquire energy beyond the Fermi energy.

Fermi factor :-

The distribution function is called Fermi factor. It describes how energy is distributed among the free electrons.

According to Fermi, The free electrons cannot occupy all the states, They can occupy certain energy states.

Therefore The probability of an electron occupying an energy level E is given by the following relationship, termed as Fermi factor denoted by $f(E)$. It is given by the expression.

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}}$$

where $f(E) \rightarrow$ probability of finding an electron in the energy state E

$E \rightarrow$ Energy level occupied by an electron in eV

$E_F \rightarrow$ Fermi level or characteristic energy

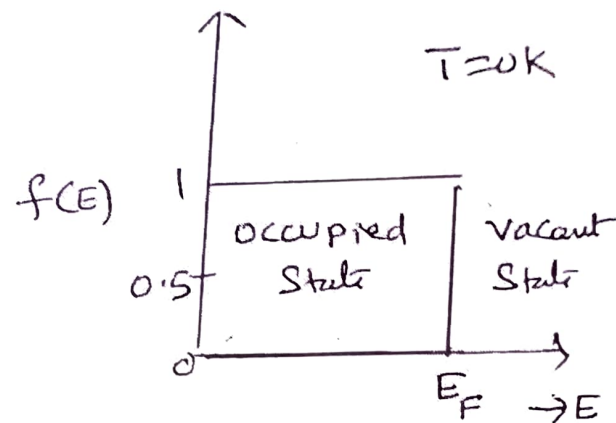
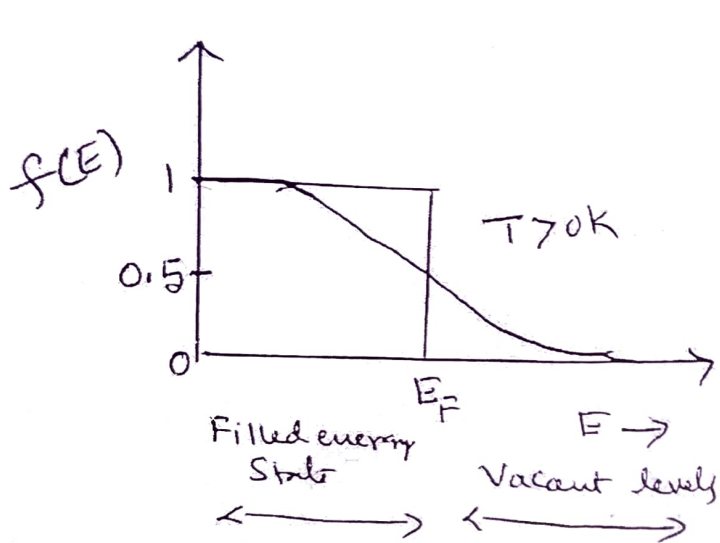
$K \rightarrow$ Boltzmann's constant in eV/K

$T \rightarrow$ Temperature in K

Fermi factor depends on the distribution of energy levels and the number of electrons available.

Variation of Fermi factor with Temperature :-

The variation of the Fermi factor with temperature is shown in figure



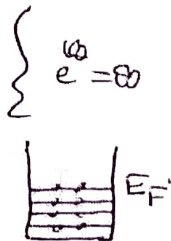
The different cases of probability of occupation are given below.

Case (i) Probability of occupation for $E < E_F$ at $T = 0K$

For $E - E_F$ [For any energy level below Fermi level]

The term $E - E_F = -ve$ quantity $\therefore \frac{E - E_F}{KT} = \frac{-\Delta E}{K \cdot 0} = -\infty$

$$\therefore f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + \frac{1}{e^{\infty}}} = \frac{1}{1 + \frac{1}{0}} = \frac{1}{1 + 0} = 1$$



Therefore $f(E) = 1$ it means that all the energy level below E_F is completely filled by electrons, i.e. there is 100% probability of the occupation of electrons in these energy levels.

Case (ii) Probability of occupation for $E > E_F$ at $T = 0K$

For $E - E_F$ [For any energy level above E_F]

The term $E - E_F = +ve$ quantity $\therefore \frac{E - E_F}{KT} = \frac{\Delta E}{K \cdot 0} = \infty$

$$\therefore f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}} = \frac{1}{1 + e^{\infty}} = \frac{1}{1 + \infty} = \frac{1}{\infty} = 0$$



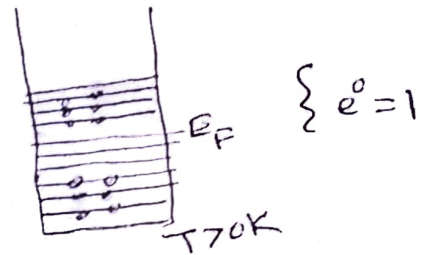
$\therefore f(E) = 0$ it means that all the energy level above E_F is completely vacant, i.e. there is no possibility for the occupation of electrons in the energy level above Fermi level.

15

Thus the function is discontinuous and reduces to zero
 Case (iii) Probability of Occupation at $T > 0K$ at $E = E_F$

if $E = E_F$ Then $\frac{E - E_F}{kT} = \frac{0}{kT} = 0$

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}} = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2}$$

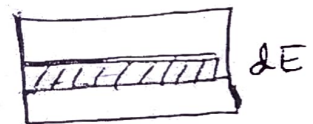


$f(E) = \frac{1}{2}$ means that the probability of occupation of electrons is 50% at any temperature above 0K, i.e. 50% of energy levels are occupied by electrons.

It is observed that at a temperature higher than 0K, there is some energy state above Fermi level E_F will be filled and there is some state below Fermi level is empty.

Therefore as the temperature increases, the electrons near the Fermi level gain the energy and are excited into higher states which results in a decrease in the Fermi factor.

Density of States $D(E)$:- (No derivation)



Energy range in an energy band

In a small energy range dE there are many discrete energy levels. These energy levels are not uniformly distributed. Hence we have to consider the density of energy states.

It is defined as The number of energy states per unit energy range (dE). In other words the number of energy states available in a certain energy range. It is denoted by $D(E)$ and it is given by the expression

$$D(E)dE = \frac{4\pi}{h^3} (2m)^{3/2} \cdot E^{1/2} dE$$

This is the expression for density of energy states.

where $D(E)dE$ is the number of available states in the energy interval dE $\odot$ The no. of energy states with small energy dE

m is the mass of the electron

h is the Planck's constant

E is the any energy state.

Fermi distribution function provides the probability of the electron occupying an energy level, but it is impossible to know the actual number of electrons. The number of electrons in the energy range dE is given by

$$N(E)dE = D(E)dE \cdot f(E)$$

where $D(E)dE$ is the number of available states

$f(E)$ is the probability of occupation of electrons in the given energy.

Expression for Fermi energy at $T=0K$ $\odot$

The number of electrons in a small energy dE is given by

$$N(E)dE = D(E)dE \cdot f(E)$$

where $D(E)dE \rightarrow$

$f(E) \rightarrow$

Let ' n ' be the number of free electrons per unit volume present in all energy states, is given by

$$\text{i.e. } n = \int_0^{\infty} N(E)dE$$

$$\therefore n = \int_0^{\infty} D(E)dE \cdot f(E)$$

The number of electrons can be determined by integrating the above equation from $E=0$ to $E=E_F$ because electrons are filled up to Fermi level E_F

At $T=0K$ Then $f(E)=1$ $\left\{ \begin{array}{l} \text{For energy levels between} \\ E=0 \text{ and } E=E_F \end{array} \right.$

17

$$n = \int_0^{E_F} D(E) dE \quad \text{--- (1)}$$

$$= \frac{4\pi}{h^3} (2m)^{3/2} \int_0^{E_F} E^{1/2} dE$$

$$= \frac{4\pi}{h^3} (2m)^{3/2} \left[\frac{E^{3/2}}{3/2} \right]_0^{E_F}$$

$$= \frac{4\pi}{h^3} (2m)^{3/2} \left[\frac{2}{3} E_F^{3/2} \right]$$

$$n = \frac{8\pi}{3h^3} (2mE_F)^{3/2} = \frac{8\pi}{3h^3} (2m)^{3/2} E_F^{3/2}$$

$$E_F^{3/2} = \frac{3n}{8\pi} \cdot \frac{h^3}{(2m)^{3/2}}$$

$$E_F = \left(\frac{3n}{8\pi} \right)^{2/3} \left[\frac{h^3}{(2m)^{3/2}} \right]^{2/3} = \left(\frac{3n}{8\pi} \right)^{2/3} \cdot \frac{h^2}{2m}$$

$$E_F = \frac{h^2}{8m} \cdot \left(\frac{3n}{\pi} \right)^{2/3}$$

This is the expression for the Fermi energy of electrons in metals at $T=0K$.
 Re arranging we get (ii) This is used to determine the highest energy.

$$E_F = \frac{h^2}{8m} \cdot \left(\frac{3}{\pi} \right)^{2/3} \times n^{2/3}$$

$$E_F = B \times n^{2/3}$$

$$\text{But } B = \frac{h^2}{8m} \cdot \left(\frac{3}{\pi} \right)^{2/3} \text{ is a constant}$$

$$B = 5.85 \times 10^{-38} \text{ J}$$

$$E_F = 5.85 \times 10^{-38} \times (n)^{2/3}$$

18

Fermi Temperature (T_F) :-

The Temperature at which the average thermal energy of free electron in the metal ($\sim kT$) becomes equal to the Fermi energy at $T=0K$ is called Fermi Temperature

i.e. $E_F = kT$

At $T = T_F$ $E_F = kT_F$

② The product of k and T is equal to Fermi energy i.e. average energy of free electrons.

$$\therefore T_F = \frac{E_F}{k}$$

it is defined as the ratio of Fermi energy to Boltzmann's Constant

T_F is only a Theoretical Concept

Reason: For metals: E_F a few eV ~ 3 eV (Say)
 $3 \times 1.6 \times 10^{-19} \text{ J} \sim 4.8 \times 10^{-19} \text{ J}$

$$T_F = \frac{E_F}{k} = \frac{4.8 \times 10^{-19}}{1.38 \times 10^{-23} \text{ J/K}} = 34800 \text{ K}$$

This is quite high Temperature to be realized in practice. Because at this Temperature, the metal will not be in Solid State.

Fermi Velocity (V_F) :-

The Velocity of electrons at Fermi level is called Fermi velocity and hence the K.E is given by

$$E = \frac{1}{2} mv^2$$

At Fermi level $E = E_F$ and $v = v_F$

$$E_F = \frac{1}{2} m v_F^2 \quad \therefore v_F = \sqrt{\frac{2E_F}{m}}$$

19 based on

Expression for Electrical Conductivity in terms of QFET

According to the quantum theory, The current is carried out only by few electrons which are close to the Fermi Surface having Fermi velocity v_F . The approach from these assumptions also leads to the same equation for electrical conductivity but this equation is more accurate.

According to CFET, The expression for σ is given by

$$\sigma = \frac{n e^2 \tau}{m}$$

According to QFET The expression σ is given by

(Substituting τ_F we get)

$$\lambda_F = v_F \tau_F$$

$$\tau_F = \frac{\lambda}{v_F}$$

$$\sigma = \frac{n e^2 \lambda_F}{m^* v_F}$$

where $\tau \rightarrow$ Relaxation Time at Fermi Surface.

Where $n \rightarrow$ The density of Free electrons

$m^* \rightarrow$ effective mass of electron in a given metal and varies from Solid to Solid.

$\lambda_F \rightarrow$ Mean Free path

$v_F \rightarrow$ Fermi velocity

Merits of quantum Free electron Theory :-

The QFET has been successful in explaining the important experimental results related to electrical conductivity and other related properties of the following three cases.

[1] Specific heat at constant volume (C_V) :-

Based on CFET

$$[C_V]_{CFET} = \frac{3}{2} R \text{ which is Temp independent}$$

on the other hand experimentally

$$[C_V]_{EXPT} = \text{Constant} \cdot R \cdot T \text{ which is Temp dependent}$$

whereas From quantum Free electron theory

$$[C_v]_{QFET} = \frac{2k}{E_F} \times RT = 10^{-4} RT$$

which agrees with the experimental result.

Thus the QFET is successful in explaining the experimental behaviour of specific heat at constant volume with respect to the Temperature.

[2] Temperature dependence of electrical conductivity :-

Acc QFET σ of metal is given by

$$\sigma = \frac{ne^2 \lambda_F}{m^* v_F}$$

Here $\sigma \propto \lambda$ λ depends on Temperature

$$\therefore \lambda \propto \frac{1}{T}$$

$\sigma \propto \frac{1}{T}$ This relation explains the experimental result

So based on Classical Free electron Theory $[\sigma]_{CFET} \propto \frac{1}{\sqrt{T}}$

whereas the experimental relation $[\sigma]_{EXPT} \propto \frac{1}{T}$

and based on quantum Free electron Theory $[\sigma]_{QFET} \propto \frac{1}{T}$

Thus the QFET is once again successful in explaining the experimental observations

[3] Dependence of Electrical conductivity on electron Concentration :-

According to QFET $\sigma = \frac{ne^2 \lambda_F}{m^* v_F}$

From the above equation The electrical conductivity depends on both electron concentration and ratio $\left(\frac{\lambda_F}{v_F}\right)$

This relation satisfactorily explains the experimental facts.

21

Differences between Classical and Quantum Free electron theory

C. F. E. T

- 1) Electron energy is distributed according to Maxwell-Boltzmann Statistics
- 2) Electron can have continuous energy values
- 3) Many electrons can have the same energy
- 4) Free electrons are completely free
- 5) In the absence of field electrons move with random direction
- 6) When electric field is applied electrons acquire a drift velocity in the opposite direction of the field

Q. F. E. T

- Electron energy is distributed according to Fermi-Dirac Statistics
- Electron energies are discrete and quantized
- No two electrons can have the same energy.
- Free electrons are not completely free
- Electrons occupy energy levels according to Pauli's exclusion principle
- When electric field is applied electrons acquire additional energy.

- ① Estimate the probability of occupancy for a state whose energy is (a) 0.1 eV below the Fermi energy (b) 0.1 eV above the Fermi energy (c) equal to the Fermi energy. Assume the temperature to be 300 K.

① a) $E < E_F$ $\frac{E - E_F}{kT} = \frac{-0.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} = -1.45$

$f(E) = \frac{1}{1 + e^{-1.45}} = 0.81 \text{ or } 81\%$

① b) $E > E_F$ $\frac{E - E_F}{kT} = 1.45$

$f(E) = \frac{1}{1 + e^{1.45}} = 0.19 \text{ or } 19\%$

① c) $E = E_F$ $f(E) = \frac{1}{1 + e^0} = \frac{1}{2} \text{ or } 50\%$

22

- ② The occupied level of the electron at 1262 K is above 0.5 eV of the Fermi level of a material. What is the probability of occupation of electron at this temperature?

$$T = 1262 \text{ K}$$

$$E - E_F = 0.5 \text{ eV} = 0.5 \times 1.6 \times 10^{-19} \text{ J}$$

$$E - E_F = 8 \times 10^{-20} \text{ J}$$

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}} = \frac{1}{1 + e^{\frac{8 \times 10^{-20}}{1.38 \times 10^{-23} \times 1262}}} = 0.01$$

- ③ Calculate the temperature at which there is 1% probability that a state with an energy 0.5 eV above Fermi energy is occupied.

The difference between energy level to Fermi level = $E - E_F = 0.5 \text{ eV}$

$$= 0.5 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 8.0 \times 10^{-20} \text{ J}$$

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

$$0.01 = \frac{1}{1 + e^{\frac{8 \times 10^{-20}}{1.38 \times 10^{-23} \times T}}} = \frac{1}{1 + e^{\frac{5797.10}{T}}}$$

$$\frac{1}{0.01} = 1 + e^{\frac{5797.10}{T}}$$

$$\frac{1}{0.01} - 1 = e^{\frac{5797.10}{T}}$$

$$e^{\frac{5797.10}{T}} = 99$$

$$\frac{5797.10}{T} \times \ln e = \ln 99$$

$$T = \frac{5797.10}{\ln 99} = \frac{5797.10}{4.5951} = 1261.5 \text{ K}$$

[4] Calculate the probability of an electron occupying an energy level of 0.02 eV above the Fermi level at 200 K and 400 K in a metal.

$$E - E_F = 0.02 \text{ eV} = 0.02 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-21} \text{ J}$$

$$T = 200 \text{ K} \quad f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}} = \frac{1}{1 + e^{\frac{3.2 \times 10^{-21}}{1.38 \times 10^{-23} \times 200}}} = \frac{1}{1 + e^{1.1594}} = 0.2387$$

$$T = 400 \text{ K} \quad f(E) = \frac{1}{1 + e^{\frac{3.2 \times 10^{-21}}{1.38 \times 10^{-23} \times 400}}} = \frac{1}{1 + e^{0.5797}} = 0.3589$$

[5] Evaluate the Fermi function for an energy KT above the Fermi energy.

The Fermi function is given by,

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}} = \frac{1}{1 + e^1} = \frac{1}{1 + 2.78} = 0.269$$

(Given that $E - E_F = KT$)

[6] Show that the occupation probability at $E = E_F + \Delta E$ is equal to the non-occupation probability at $E = E_F - \Delta E$.

We know that the occupation probability for the energy state E is given by the Fermi factor

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}}$$

∴ The occupation probability for the energy state $E = E_F + \Delta E$ will be

$$f(E_F + \Delta E) = \frac{1}{1 + e^{\frac{E_F + \Delta E - E_F}{KT}}} = \frac{1}{1 + e^{\frac{\Delta E}{KT}}} = \frac{1}{1 + x} \quad \text{--- (1) where } x = e^{\frac{\Delta E}{KT}}$$

We know from the Theory of probability that
Probability of absence = 1 - Probability of presence

$$\begin{aligned} \therefore \text{Non-occupation probability} &= 1 - \text{occupation probability} \\ &= 1 - \frac{1}{1 + e^{\frac{E - E_F}{KT}}} \end{aligned}$$

∴ The non occupation probability for the energy state $E = E_F - \Delta E$ will be

$$f(E_F - \Delta E) = 1 - \frac{1}{1 + e^{\frac{E_F - \Delta E - E_F}{KT}}} = 1 - \frac{1}{1 + e^{-\frac{\Delta E}{KT}}}$$

$$\begin{aligned}
 &= 1 - \frac{1}{1 + \frac{1}{e^{\frac{\Delta E}{kT}}}} = 1 - \frac{1}{1 + \frac{1}{x}} \\
 &= 1 - \frac{1}{\frac{x+1}{x}} = 1 - \frac{x}{x+1} = \frac{x+1-x}{x+1} = \frac{1}{x+1} \quad (2)
 \end{aligned}$$

Equation (1) and (2) are equal Thus the given problem is proved

(7) Show that the sum of the probability of occupancy of an energy state at ΔE below Fermi level and that at ΔE above Fermi level is unity.

Prove that

$$\begin{array}{l}
 \text{Probability of an electron} \\
 \text{occupying below the Fermi} \\
 \text{level}
 \end{array}
 +
 \begin{array}{l}
 \text{probability of an electron} \\
 \text{occupying above the} \\
 \text{Fermi level}
 \end{array}
 = 1$$

P.T $f(E_F - \Delta E) + f(E_F + \Delta E) = 1$

The occupation probability for the energy state E is given by

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

The probability of occupancy of an energy state ΔE below the Fermi level i.e. $E = E_F - \Delta E$

$$\therefore f(E_F - \Delta E) = \frac{1}{1 + e^{\frac{E_F - \Delta E - E_F}{kT}}} = \frac{1}{1 + e^{\frac{-\Delta E}{kT}}} = \frac{1}{1 + \frac{1}{x}} = \frac{x}{x+1}$$

The probability of an occupancy of an energy state ΔE above the Fermi level i.e. $E = E_F + \Delta E$

$$\therefore f(E_F + \Delta E) = \frac{1}{1 + e^{\frac{E_F + \Delta E - E_F}{kT}}} = \frac{1}{1 + e^{\frac{\Delta E}{kT}}} = \frac{1}{1+x}$$

Substituting in the above equation

$$\frac{x}{1+x} + \frac{1}{1+x} = \frac{1+x}{1+x} = 1$$

$$1 = 1$$

$\therefore$ Sum of the probabilities = 1

- ⑧ The concentration of electrons in copper is $8.4905 \times 10^{28} \text{ m}^{-3}$
Find its Fermi energy at 0K

$$E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{2/3} = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31}} \times \left(\frac{3 \times 8.4905 \times 10^{28}}{\pi} \right)^{2/3}$$

$$E_F = \frac{1.1297 \times 10^{-18} \text{ J}}{1.6 \times 10^{-19}} = 7.06 \text{ eV}$$

- ⑨ Find the electron density for a metal with a Fermi energy of 3eV
Fermi energy at 0K $E_F = 3 \text{ eV} = 3 \times 1.6 \times 10^{-19} \text{ J} = 4.8 \times 10^{-19}$

$$E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{2/3} = \frac{(6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31}} \times \left(\frac{3}{\pi} \right)^{2/3} n^{2/3}$$

$$4.8 \times 10^{-19} = 5.8437 \times 10^{-38} \times n^{2/3}$$

Re arranging the above equation we get

$$n = \left(\frac{4.8 \times 10^{-19}}{5.8437 \times 10^{-38}} \right)^{3/2} = 2.36 \times 10^{28} \text{ m}^{-3}$$

- ⑩ The Fermi energy of Copper is 7eV. Find the charge carrier density and Fermi velocity

$$E_F = 7 \text{ eV} = 7 \times 1.6 \times 10^{-19} \text{ J} = 1.12 \times 10^{-18} \text{ J}$$

$$E_F = \frac{h^2}{8m} \times \left(\frac{3}{\pi} \right)^{2/3} n^{2/3}$$

$$1.12 \times 10^{-18} = 5.85 \times 10^{-38} \times n^{2/3}$$

$$n = \left(\frac{1.12 \times 10^{-18}}{5.85 \times 10^{-38}} \right)^{3/2} = 8.39 \times 10^{28} \text{ m}^{-3}$$

$$v_F = \sqrt{\frac{2E_F}{m}}$$

$$= \sqrt{\frac{2 \times 1.12 \times 10^{-18} \text{ J}}{9.11 \times 10^{-31}}}$$

$$= 1.568 \times 10^6 \text{ m/s}$$

- ⑪ Fermi Temperature of a metal is 24600K. Find the Fermi velocity of an electron

$$T_F = 24600 \text{ K}$$

$$m = 9.11 \times 10^{-31}$$

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2kT_F}{m}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 24600}{9.11 \times 10^{-31}}}$$

$$v_F = 0.8633 \times 10^6 \text{ m/sec}$$

26

- (12) Calculate Fermi energy and Fermi Temperature in a metal. Fermi velocity of an electron in the metal is $0.86 \times 10^6 \text{ m/s}$

We know that the Fermi energy

$$E_F = \frac{1}{2} m v_F^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times 0.86 \times 10^6 = \frac{3.368 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ J} = 2.1 \text{ eV}$$

We also know that the Fermi temperature

$$T_F = \frac{E_F}{k} = \frac{3.368 \times 10^{-19}}{1.38 \times 10^{-23}} = 2.43 \times 10^4 \text{ K}$$

- (13) Find the Fermi velocity and mean free path for conduction electrons in silver. Given that its Fermi energy is 5.5 eV and relaxation time for electrons is $3.83 \times 10^{-14} \text{ sec}$

$$v_F = \sqrt{\frac{2 E_F}{m}} = 1.39 \times 10^6 \text{ m/s}$$

$$\lambda = v_F \times \tau = 5.32 \times 10^{-4} \text{ m}$$