An Autonomous Institute
Affiliated to VTU, Belagavi,
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DEPARTMENT OF MEDICAL ELECTRONICS ENGINEERING

SUBJECT CODE : 18ML42

SUBJECT NAME: COMMUNICATION SYSTEMS

LECTURE PRESENTATION UNIT– 4

DIGITAL MODULATION

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Unit: Structure

DIGITAL MODULATION:

✓**Sampling theorem for low pass and band pass signal, statement and proof**

✓ **PAM, Natural Sampling, Flat-Top**

✓**sampling, Quantization of Signals, Quantization error.**

✓**PCM, Electrical representations of Binary digits, The PCM Systems**

✓**DPCM, Delta Modulation, ADM, ASK, FSK**

TEXT BOOK

“Communication Systems”, Simon Haykins & Moher, 5th Edition, John Willey, India Pvt. Ltd, 2010, ISBN 978 – 81 – 265 – 2151 – 7.

Sampling theorem

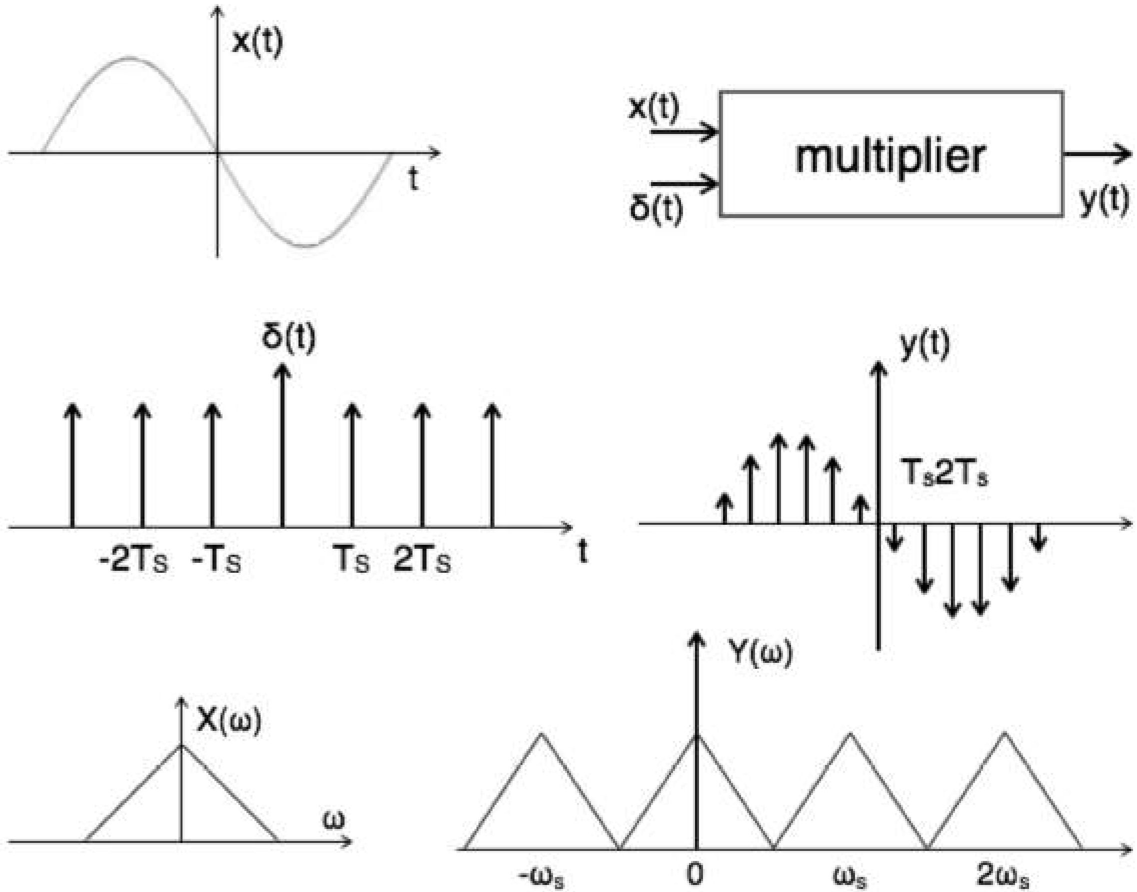
Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

$$f_s \geq 2 f_m.$$

Proof: Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega|>\omega_m$.

Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:

Continued



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

$$\text{Sampled signal } y(t) = x(t).\,\delta(t) \ \ldots\ldots (1)$$

The trigonometric Fourier series representation of δ (t) is given by

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t) \ \ldots\ldots (2)$$

Where $a_0 = \frac{1}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$

$$a_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\cos n\omega_s\, dt = \frac{2}{T_2}\delta(0)\cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\sin n\omega_s t\, dt = \frac{2}{T_s}\delta(0)\sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore\ \delta(t) - \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t + 0)$$

$$\therefore \delta(t) - \frac{1}{T_s} + \Sigma_{n=1}^{\infty}\left(\frac{2}{T_s}\cos n\omega_s t + 0\right)$$

Substitute δ(t) in equation 1.

$$\rightarrow y(t) = x(t).\,\delta(t)$$

$$- x(t)\left[\frac{1}{T_s} + \Sigma_{n=1}^{\infty}\left(\frac{2}{T_s}\cos n\omega_s t\right)\right]$$

$$- \frac{1}{T_s}\left[x(t) + 2\Sigma_{n=1}^{\infty}(\cos n\omega_s t)x(t)\right]$$

$$y(t) = \frac{1}{T_s}\left[x(t) + 2\cos\omega_s t.\,x(t) + 2\cos 2\omega_s t.\,x(t) + 2\cos 3\omega_s t.\,x(t) \ldots \ldots\right]$$

Take Fourier transform on both sides.

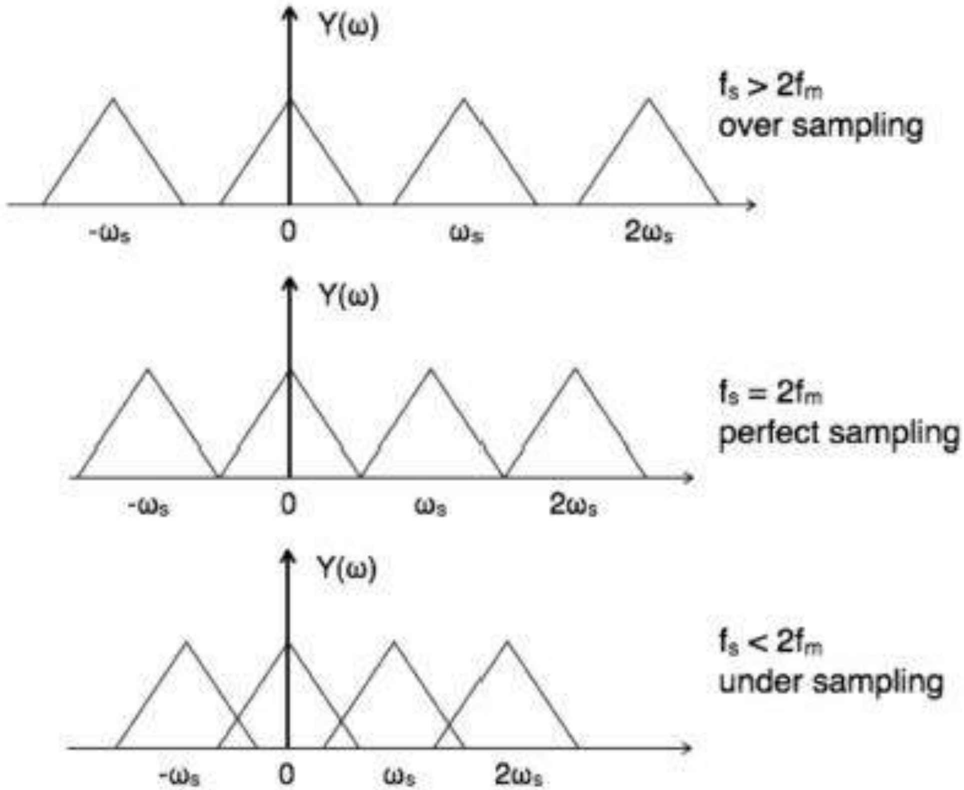
Take Fourier transform on both sides.

$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s) + X(\omega + 2\omega_s) + \dots]$$

$$\therefore\ Y(\omega) = \frac{1}{T_s}\Sigma_{n-- \infty}^{\infty} X(\omega - n\omega_s) \qquad where\ n = 0, \pm 1, \pm 2, \dots$$

To reconstruct x(t), you must recover input signal spectrum X(ω) from sampled signal spectrum Y(ω), which is possible when there is no overlapping between the cycles of Y(ω).

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



Aliasing Effect

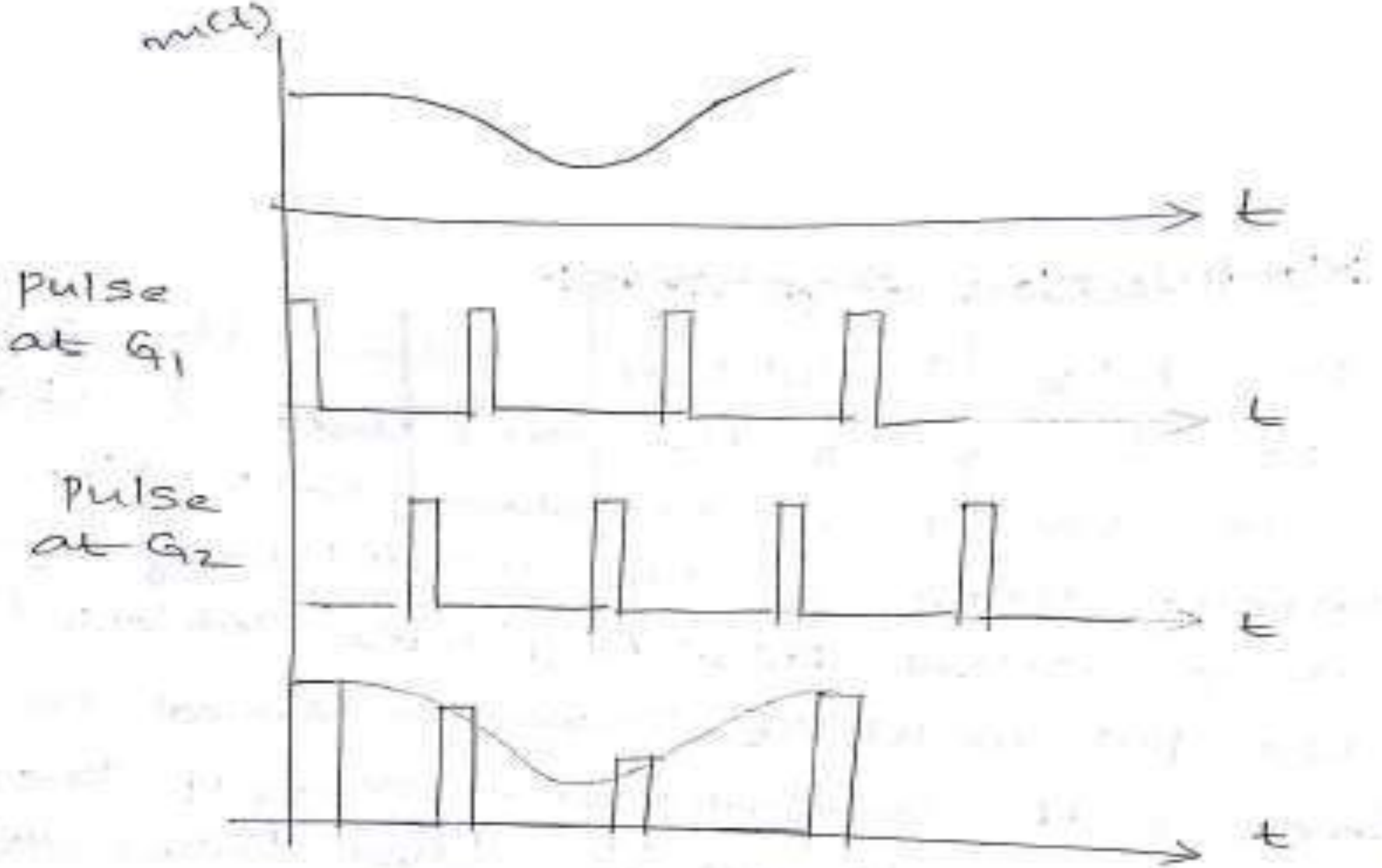
The overlapped region in case of under sampling represents aliasing effect, which can be removed by

- considering $f_s > 2f_m$
- By using anti aliasing filters.

PAM GENERATION

- During this period, the capacitor is quickly charged to a voltage equal to instantaneous sample value of incoming signal x(t)
- Now the sampling switch is opened and capacitor holds the charge.
- The discharge switch is then closed by a pulse applied to gate G2 of second transistor.
- Due to this the capacitor is discharged to zero volts. The discharge switch is then opened and the capacitor has no voltage.
- Hence the output of sample and hold circuit consists of a sequence of flat top samples.

PAM GENERATION



Transmission bandwidth of PAM

In PAM signal the pulse duration τ is assumed to be very small compared to time period T_s i.e $\tau < T_s$

If the maximum frequency in the modulating signal $x(t)$ is f_m then sampling frequency f_s is given by $f_s <= 2f_m$ Or $1/T_s <= 2f_m$ or $T_s <= 1/2f_m$

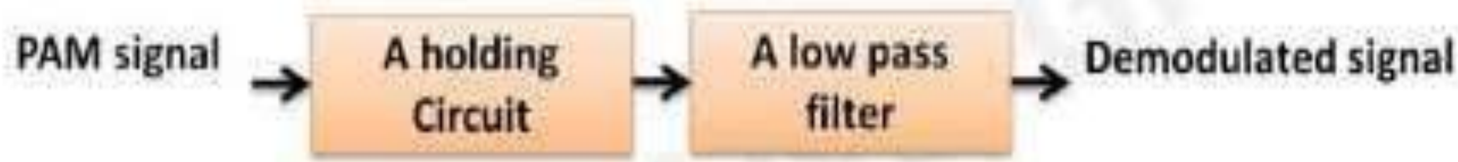
Therefore, $\tau < T_s <= 1/2f_m$

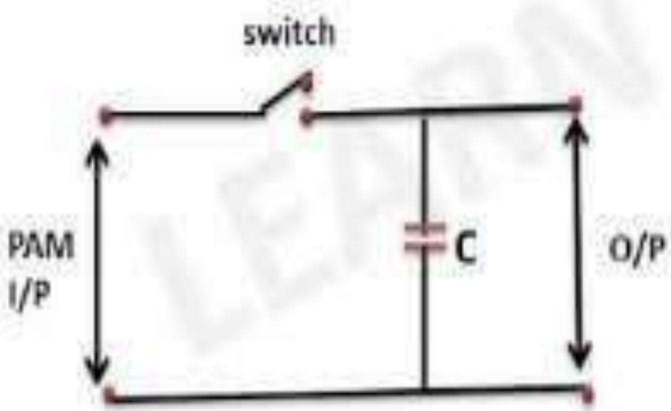
If ON and OFF time of PAM pulse is equal then maximum frequency of PAM pulse will be $f_{max} = 1/\tau + \tau = 1/2\tau$

Therefore, transmission bandwidth $>= 1/2\tau >= 1/[2(1/2f_m) >= f_m$

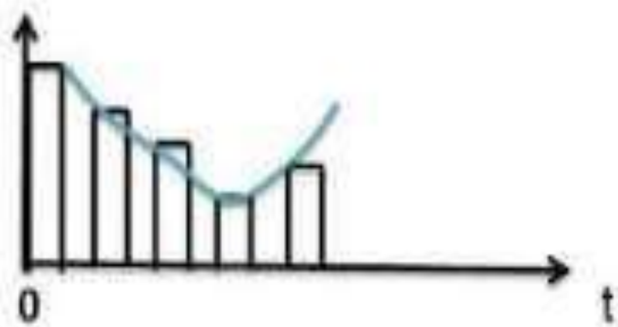
Demodulation of PAM

Demodulation is the reverse process of modulation in which modulating signal is recovered back from the modulated signal.





zero order holding circuit



O/P of holding circuit

Demodulation of PAM

- For PAM signals, demodulation is done using a holding circuit.
- The received PAM signal is first passed through a holding circuit and then through a lowpass filer.
- Switch S is closed during the arrival of the pulse and is opened at the end of the pulse.
- Capacitor C is charged to pulse amplitude value and holds this value during the interval between two pulses.
- Holding circuit output is then passed through a low pass filter to extract the original signal.

Advantages, Disadvantages of PAM

Advantages:

- It is the simple process for modulation and demodulation
- Transmitter and receiver circuits are simple and easy to construct.

Disadvantages:

- Bandwidth requirement is high
- Interference of noise is maximum
- Power requirement is high

Applications:

- Used in microcontrollers for generating control signals
- Used as electronic driver for LED lighting

SAMPLING

It is the process of converting a continuous time signal into a discrete time signal

During sampling, sufficient number of samples of the signal must be taken so that original signal is correctly represented in its samples and possible for reconstruction.

Number of samples to be taken depends on maximum signal frequency present in the signal.

- Different types of samples are,
- Ideal
- Natural
- Flat top

SAMPLING

Sampling theorem:

A continuous time signal may be completely represented in its samples and recovered back if the sampling frequency $fs>2fm$

Nyqyist rate and Nyquist interval:

When sampling rate becomes exactly equal to 2fm samples per second, it is called Nyquist rate

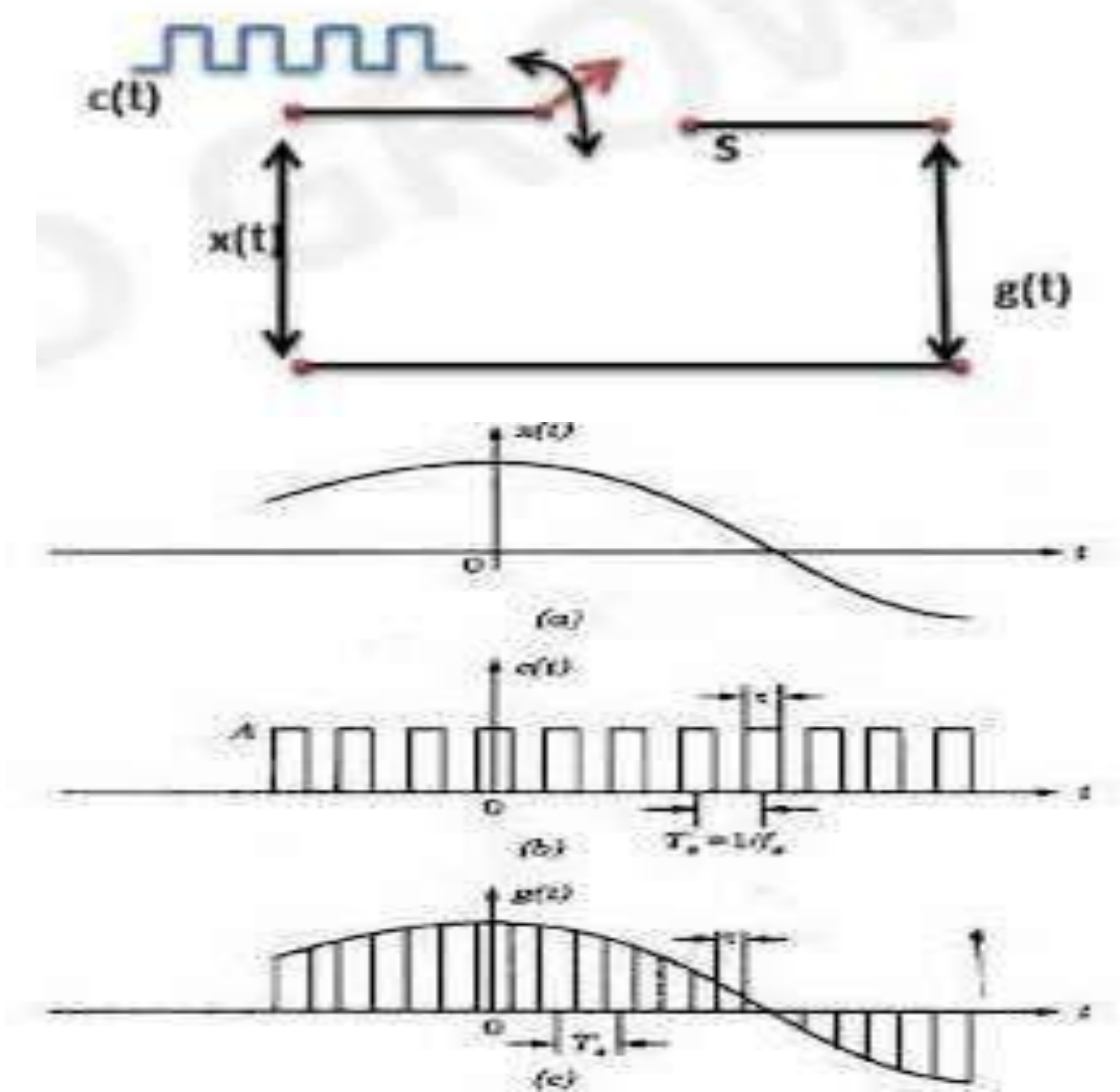
$$fs=2fm \text{ Hz}$$

Maximum sampling interval is called Nyquist interval.

$$Ts = 1/fs=1/2fm \text{ sec}$$

NATURAL SAMPLING

In natural sampling, pulse has a finite width



NATURAL SAMPLING

Let an analog continuous time signal x(t) sampled at a rate fs Hz and sampling function c(t) which is a train of periodic pulse of width τ and frequency fs Hz

Case i: When c(t) is high

Switch S is closed and output g(t) is exactly equal to input

$$g(t) = x(t)$$

NATURAL SAMPLING

Case ii: When c(t) is low

Switch s is open

$$g(t) = 0$$

The time domain representation of naturally sampled signal is given by,

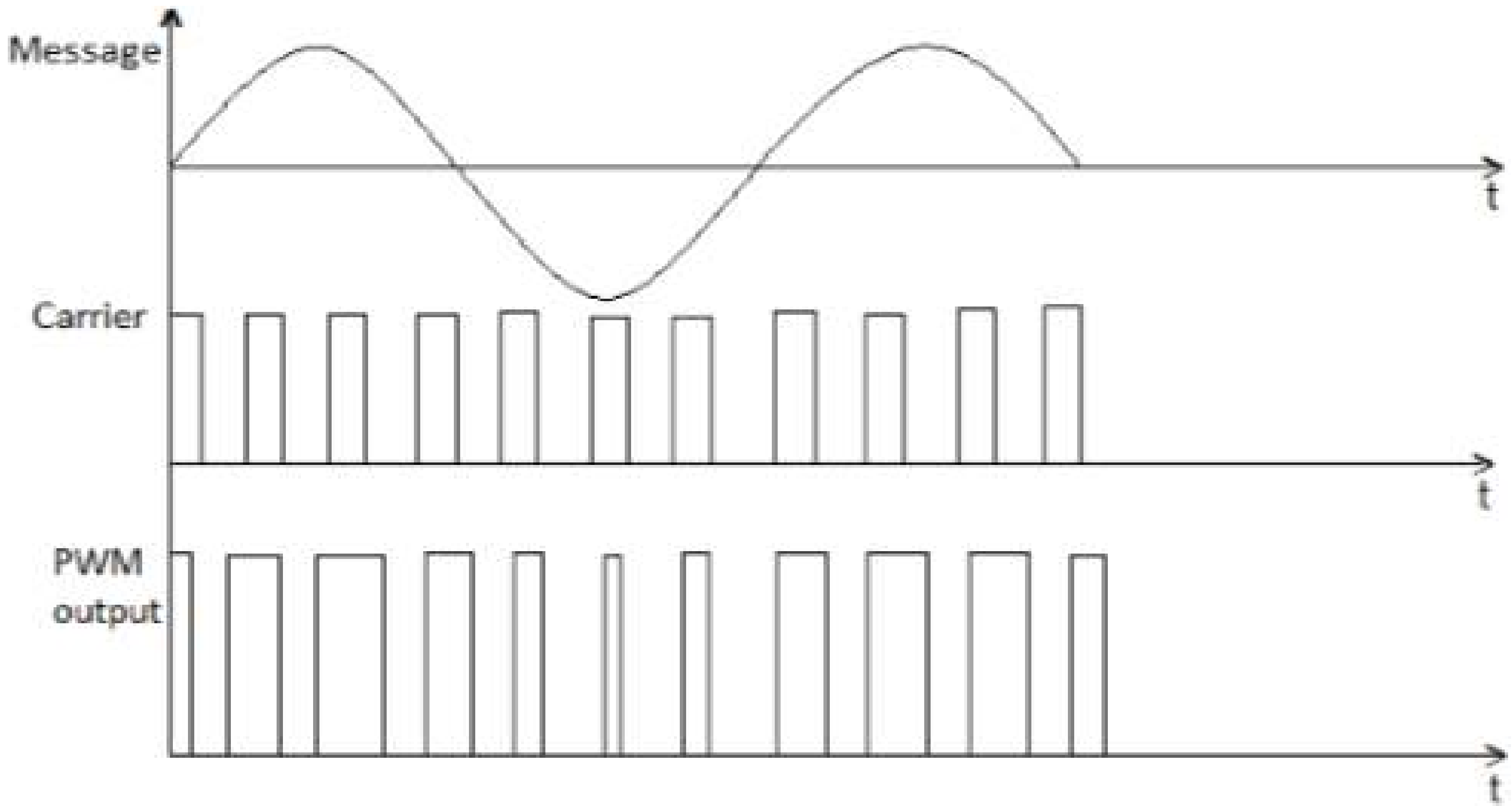
$$g(t) = x(t)\sum_{n=-\alpha}^{\alpha} \frac{\tau A}{Ts} Sinc(fn.\tau)e^{j2\pi fsnt}]$$

The spectrum of naturally sampled signal is given by,

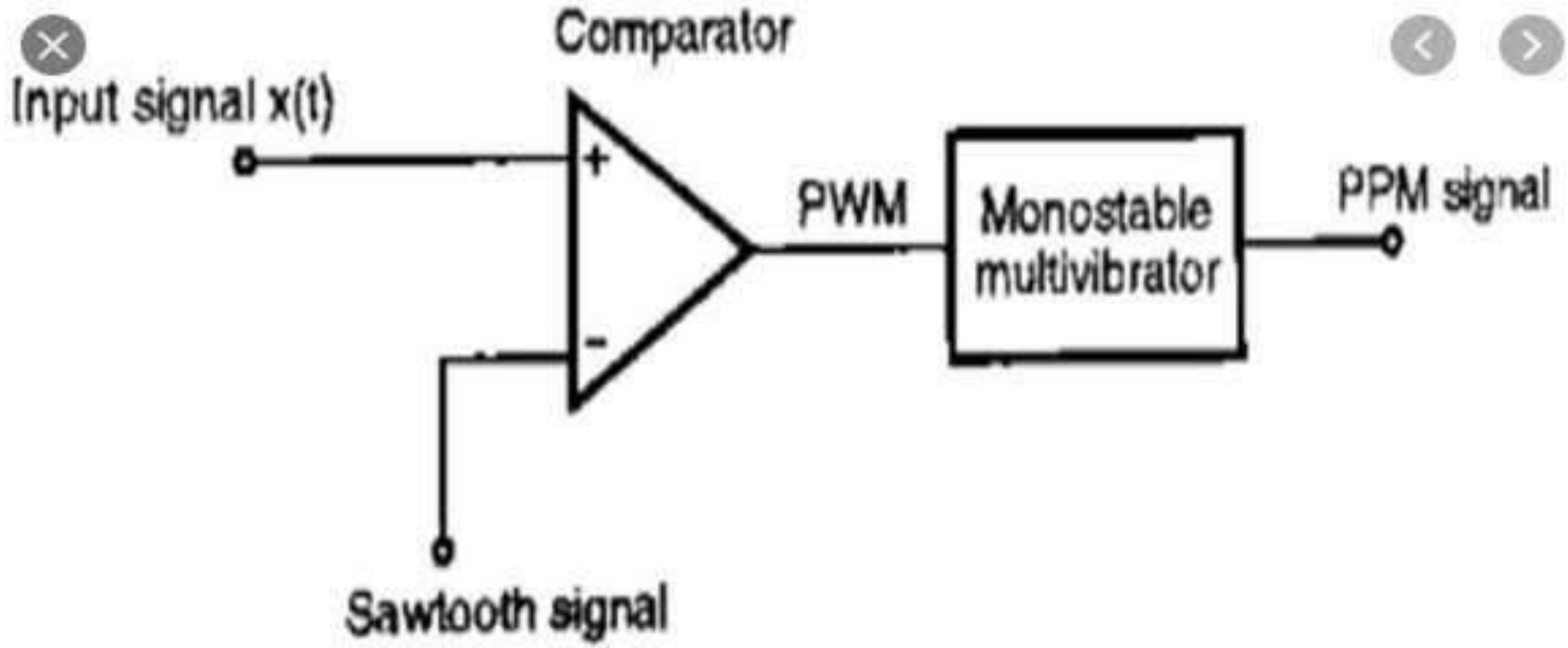
$$G(f) = \frac{\tau A}{Ts}\sum_{n=-\alpha}^{\alpha} Sinc(nfs\tau)X(f - nfs)$$

Pulse Width Modulation(PWM)

In PWM, the width of pulses of carrier pulse train is varied in proportion with amplitude of modulating signal.



PWM GENERATION



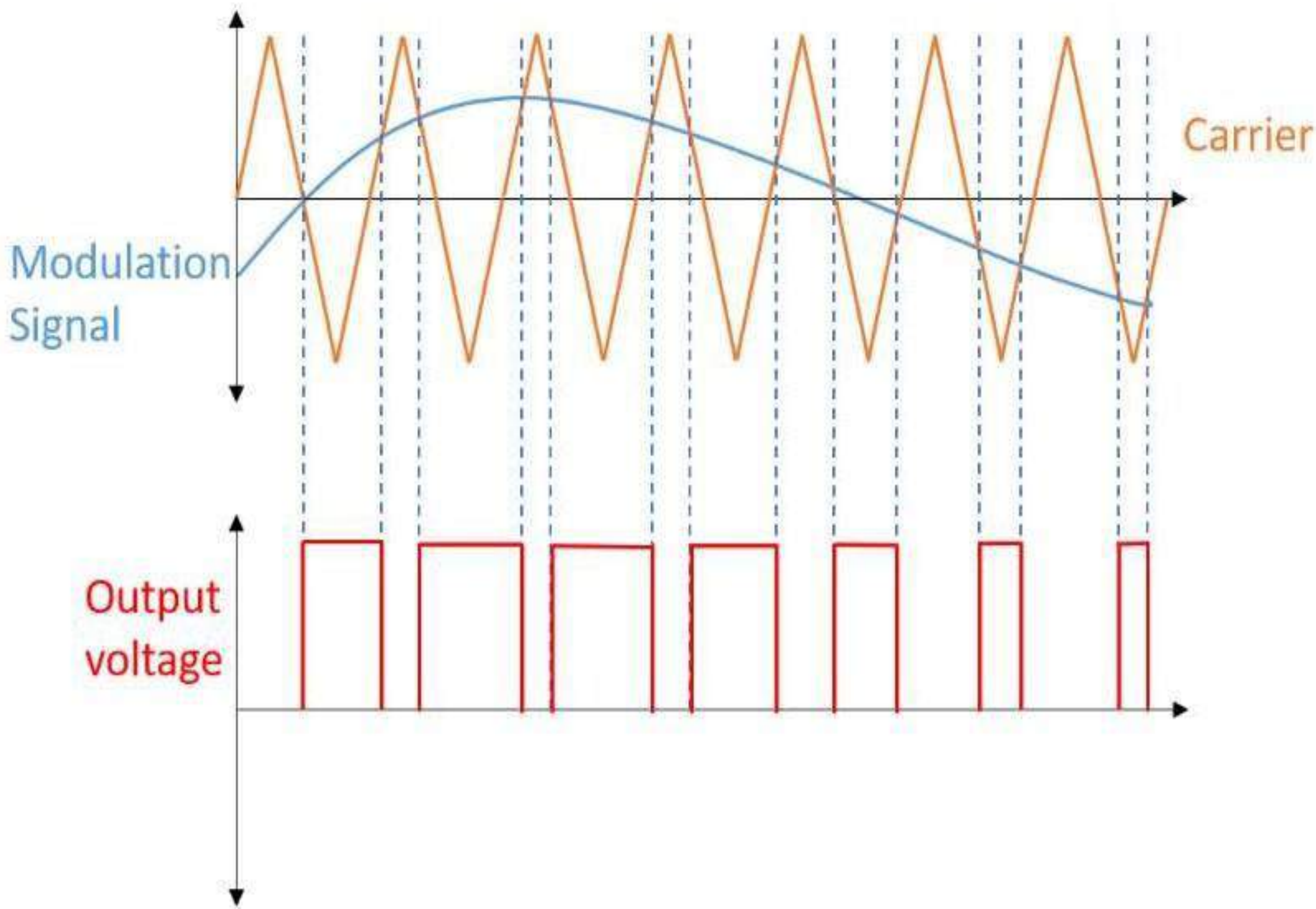
A sawtooth generator generates a sawtooth signal of frequency fs.

This is applied to inverting terminal of comparator.

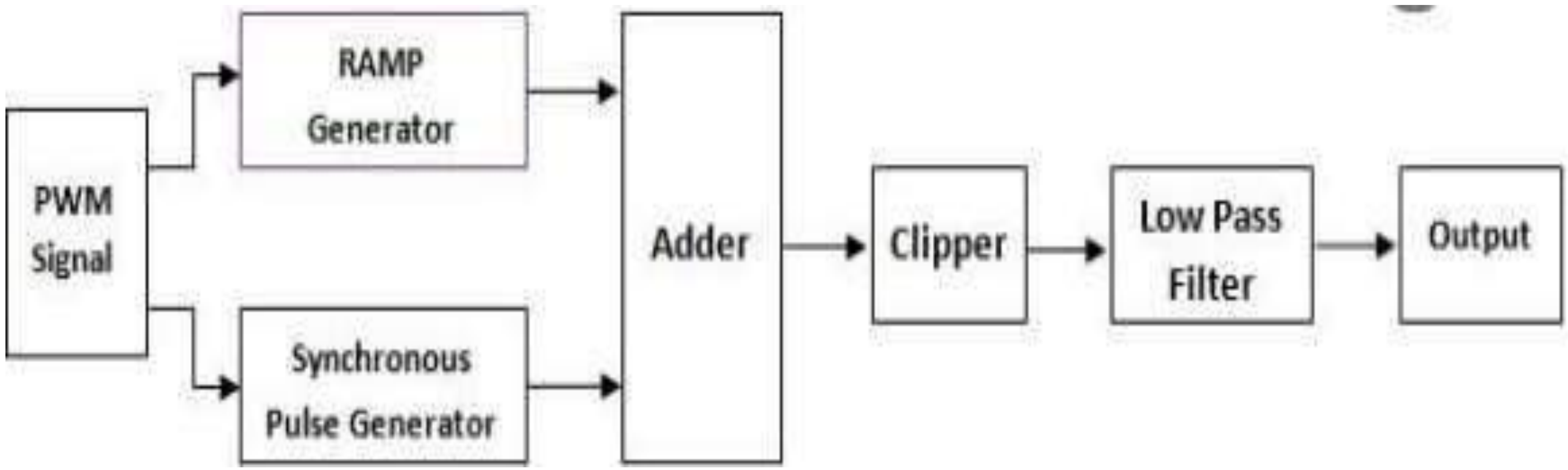
PWM GENERATION

- Modulating signal x(t) is applied to non-inverting terminal of comparator.
- Comparator output remains high as long as instantaneous amplitude of x(t) is higher than sawtooth signal.
- This gives the PWM output at the output of comparator.
- The leading edges of PWM waveform coincide with falling edges of ramp signal
- Therefore, leading edges of PWM signal are always generated at fixed time intervals
- Occurrence of falling edge of PWM signal is dependent on instantaneous amplitude of x(t)

PWM GENERATION



DETECTION OF PWM

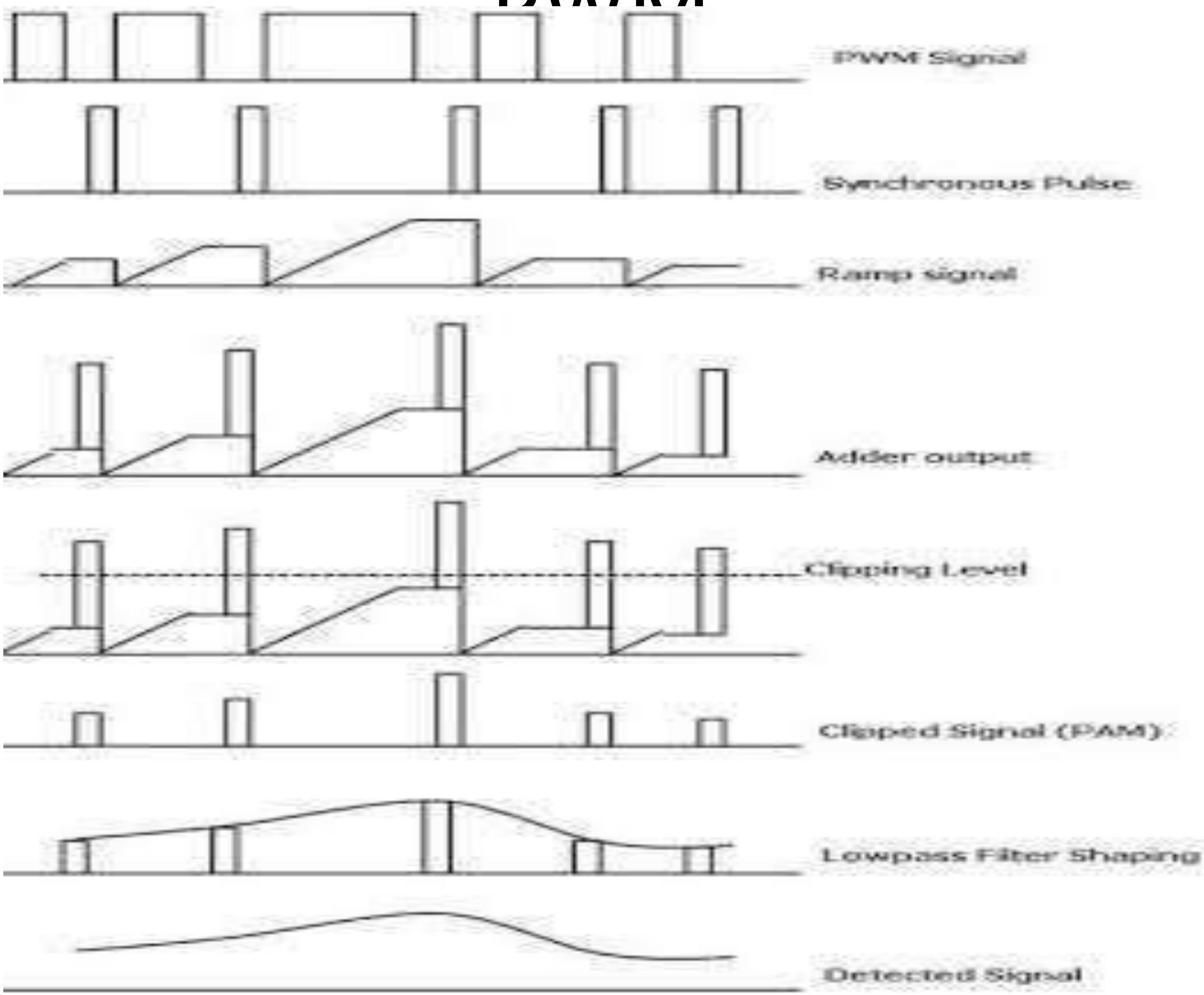


- The PWM signal received at the input of detector circuit will contain noise
- This signal is applied to a pulse generator which regenerates the PMW signal.
- Some of the noise is removed and the pulses are squared up.

DETECTION OF PWM

- The regenerated pulses are applied to a reference pulse generator.
- It produces a train of constant amplitude and constant width pulses.
- These pulses are synchronized to the leading edges of regenerated PWM pulses but delayed by fixed intervals.
- The regenerated PWM pulses are also applied to a ramp generator whose o/p is a constant slope ramp for the duration of the pulse.
- At the end of the pulse a sample and hold circuit retains the final ramp voltage until it is reset at the end of the pulse.
- The constant amplitude pulses at the o/p of the reference generator are then added to ramp signal.
- O/P of the adder is then clipped off at a threshold level to generate a PAM signal.
- A low pass filter is used to recover the original modulating signal back from PAM signal.

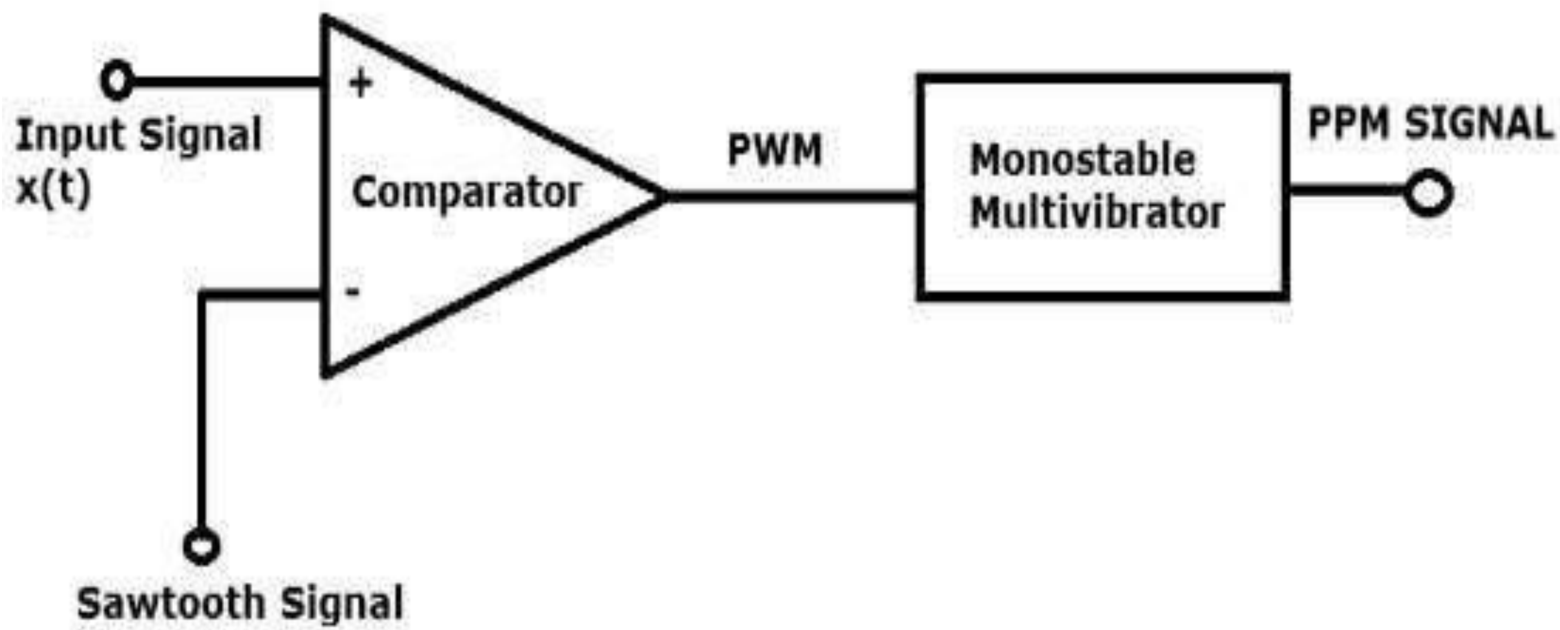
DETECTION OF PWM



PULSE POSITION MODULATION(PPM)

Modulation technique in which position of pulses of carrier pulse train is varied in accordance with amplitude of modulating signal.

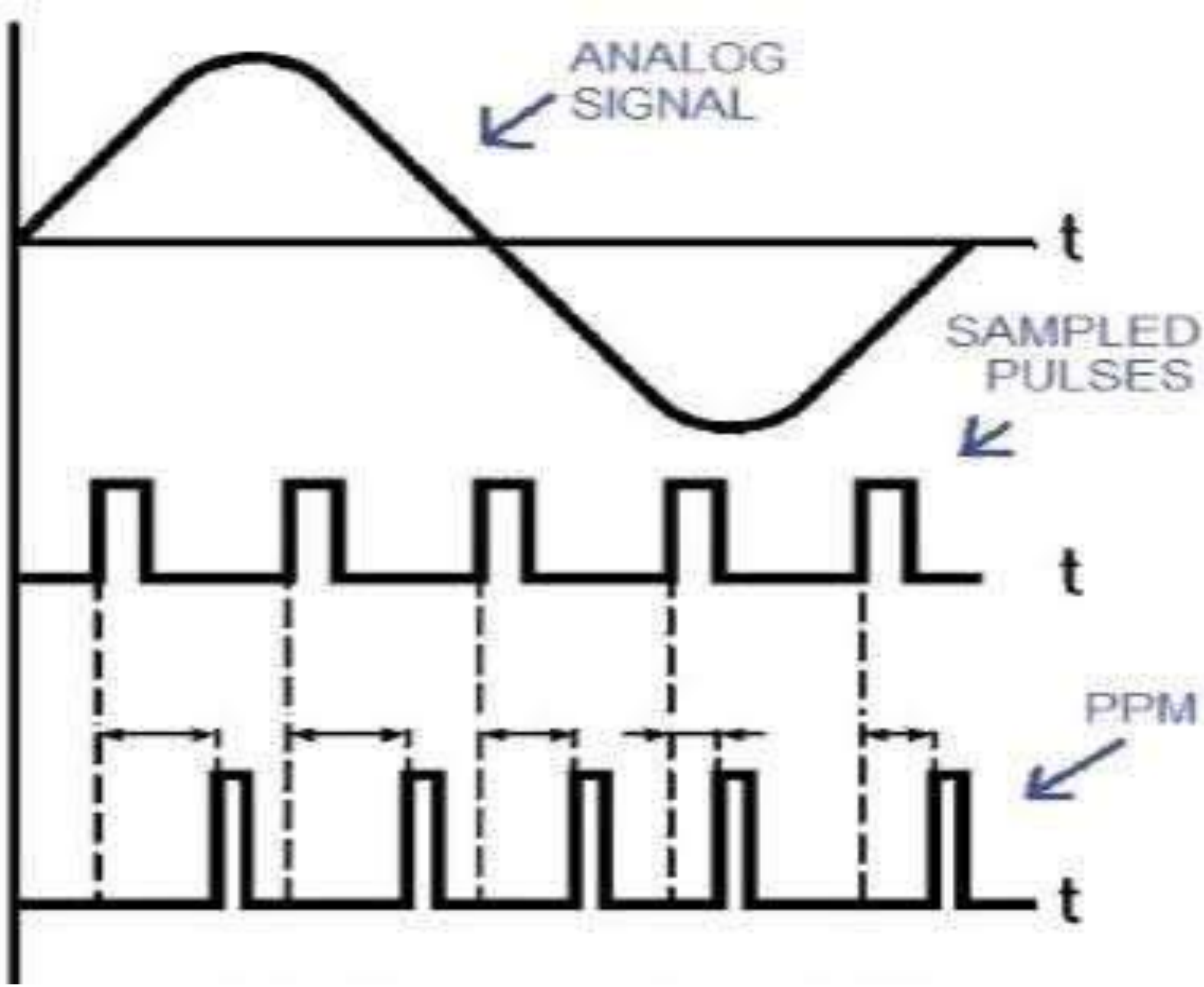
Generation:



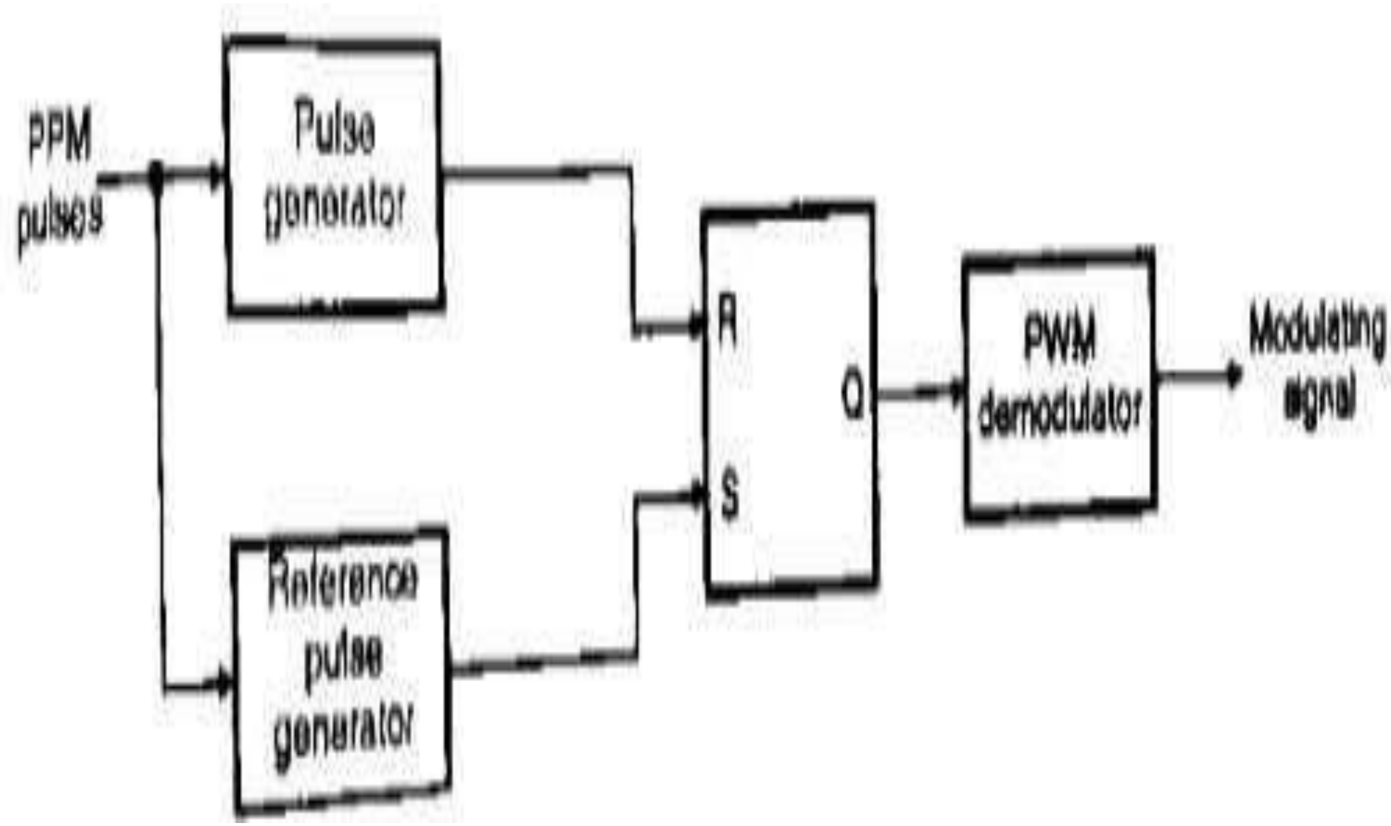
PPM GENERATION

- The block diagram is similar to PWM except monostable multivibrator.
- PWM pulses obtained at the output of comparator are applied to a monostable multivibrator.
- monostable multivibrator is a negative edge triggered circuit. At each trailing edge of PWM signal the monostable output goes high.
- PPM output remains high for a fixed duration from trailing edge of PWM signal.

PPM GENERATION



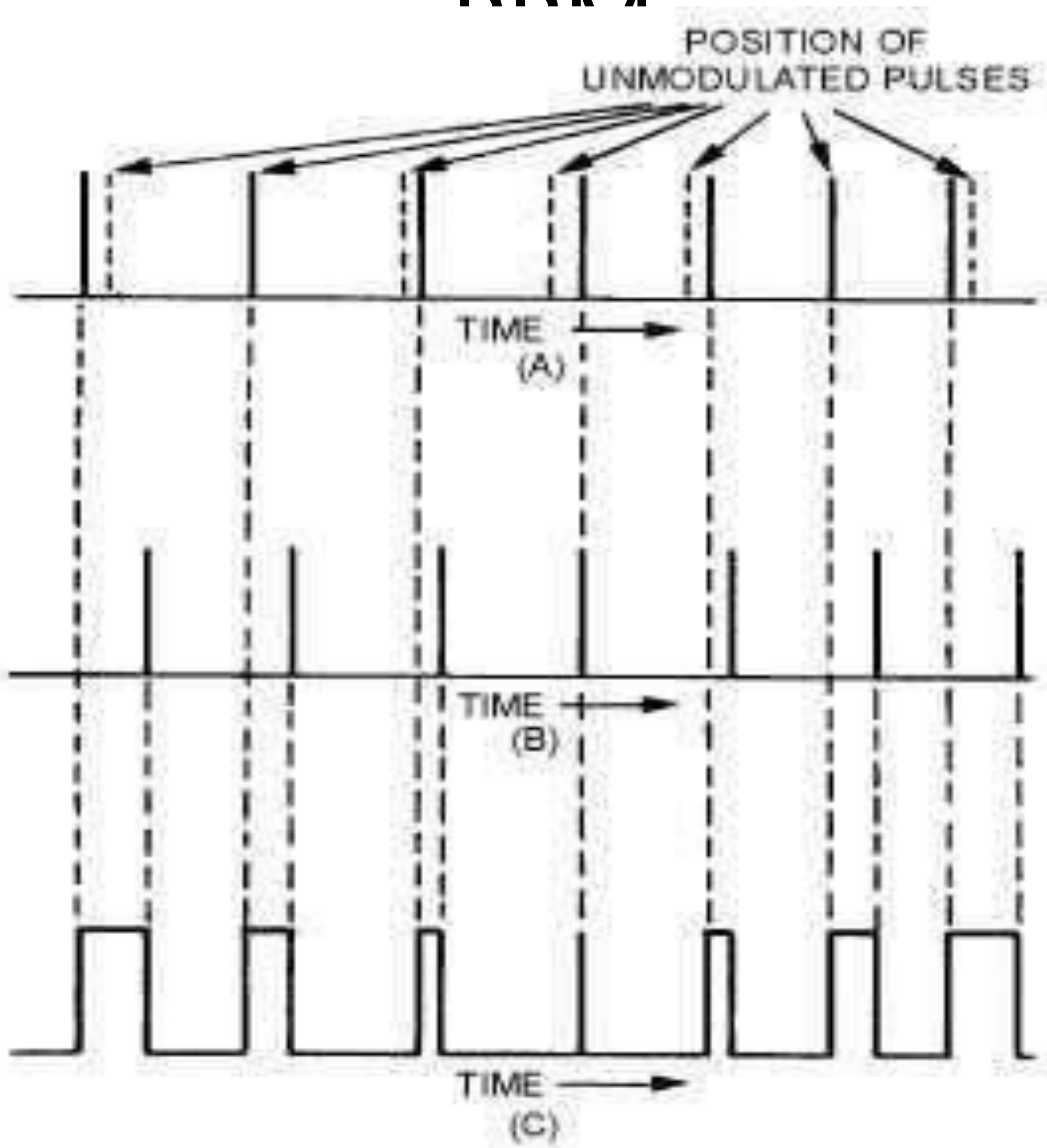
DETECTION OF PPM



DETECTION OF PPM

- The circuit consists of S-R flipflop which is set or gives high output when reference pulses arrive.
- Reference pulses are generated by a reference pulse generator.
- Flip-flop circuit is reset and gives low output at the leading edge of PPM signal.
- The process repeats and we get PWM pulses at the output of flip-flop.
- PWM pulses are then demodulated in a PWM demodulator to get original modulating signal.

DETECTION OF PPM



Radio receiver measurements

The important characteristics of superheterodyne radio receiver are,

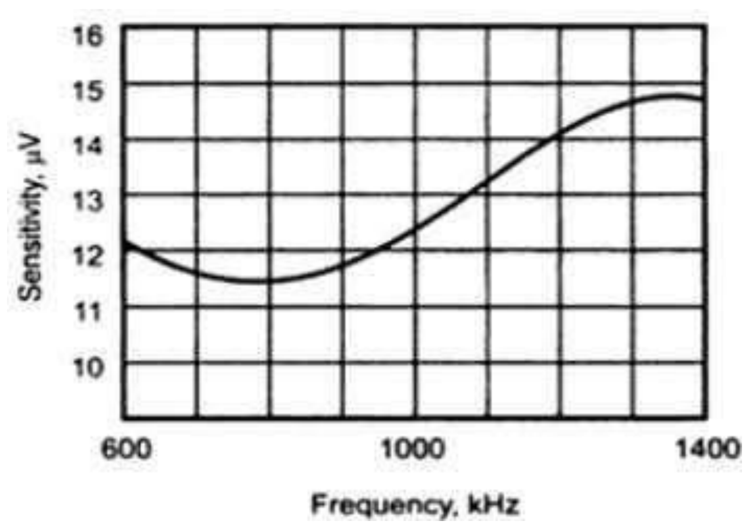
- Sensitivity
- Selectivity
- Fidelity

Sensitivity:

- It is defined as the ability of receiver to amplify weak signals
- It is defined in terms of voltage which must be applied at the receiver input terminals to provide a standard output power at the receiver output.

Radio receiver measurements

- Sensitivity is expressed in milli volts
- For practical receivers sensitivity is expressed in terms of signal power required to produce minimum acceptable output with minimum acceptable noise.

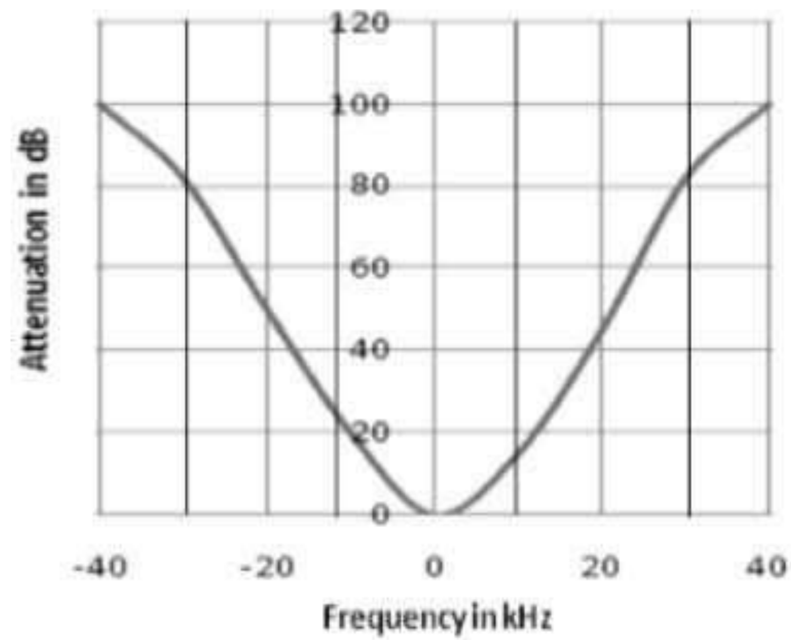


- Sensitivity of superheterodyne radio receiver depends on
- Gain of RF amplifier
- Gain of IF amplifier
- Noise figure of RX

Radio receiver measurements

Selectivity:

It is defined as the ability of receiver to reject unwanted signals.



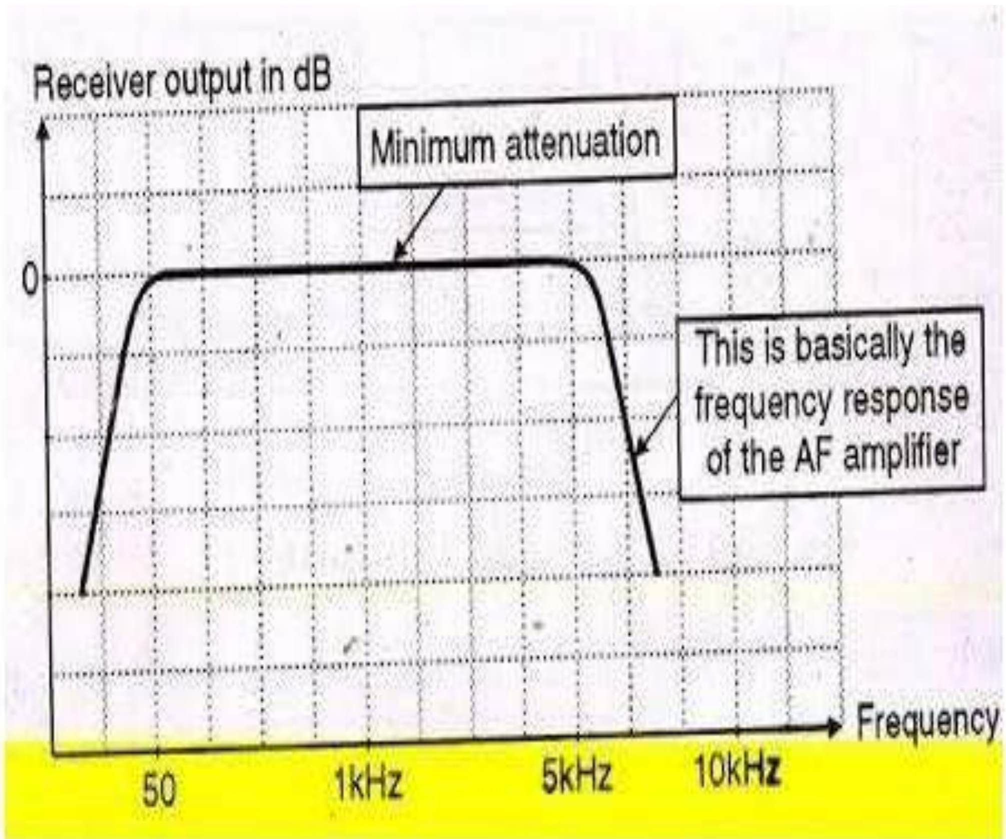
Selectivity depends on

- Receiving frequency
- Response of IF section

Radio receiver measurements

Fidelity:

It is the ability of a receiver to reproduce all the modulating frequencies equally.



INFORMATION & CHANNEL CAPACITY

Information:

Information is defined as a sequence of letters, alphabets, symbols which carries a message with specific meaning.

Source of Information:

The sources of information can be divided into 2 types.

- Analog Information sources
- Digital information sources

Analog information sources produce continuous amplitude continuous time electrical waveforms.

Discrete information sources produces messages consisting of discrete letters or symbols.

Information content of a message

- The information content of a message is represented by the probability or uncertainty of the event associated with the message.
- The probability of occurrence of a message is inversely related to amount of information.
- Therefore, a message with least probability of occurrence will have maximum amount of information.
- The relation between information content of message and its probability of occurrence is given by,

$$I_k = \log (1/P_k)$$

- The unit of information is bit.
- $I_k = \log_2(1/P_k)$ bits, $I_k = \log_{10}(1/P_k)$ Decits, $I_k = \log_e(1/P_k)$ nats.

Entropy (Average information content)

Entropy is defined as the average amount of information conveyed by a message. It is denoted by H.

$$H(X) = \sum_{K=1}^{M} Pk \log \frac{1}{Pk}$$

Properties of Entropy:

1. Entropy is always non negative i.e H(x) ≥ 0.
2. Entropy is zero when probability of all symbols is zero except probability one symbol is one.
3. Entropy is maximum when probability occurrence of all symbols is equal

i.e H(x) = $log_2 M$

Entropy of symbols in long independent sequences

- In a statistically independent sequence, the occurrence of a particular symbol during a time interval is independent of occurrence of symbols at other time interval.
- If P1, P2, P3, ……. P_M are the probabilities of occurrences of M symbols, then the total information content of the message consisting N symbols is given by,

$$I\ total = \sum_{k=1}^{M} N.Pk \log\left(\frac{1}{Pk}\right) bits$$

- To obtain entropy or average information per symbol, the total information content is divided by number of symbols in a message.

Therefore, $H(X) = \frac{Itotal}{N} = \sum_{K=1}^{M} Pk \log \frac{1}{Pk}\ bits/symbol$

Entropy of symbols in long dependent sequences

- In statistically dependent sequences, occurrence of one message alters the occurrence of other message.
- Due to this type of dependency, amount of information coming from a source is gradually decreased.
- To determine the entropy and information rate of symbols for long statistically dependent sequences a special model is developed which is **Markoff statistical model.**

Markoff statistical model for information sources

A random process in which probability of future values depends on probability of previous events is called Markoff process.

The sequence generated from such process is called Markoff sequence.

Entropy of Markoff sources:

Entropy of Markoff sources is defined as average entropy of each state.

$$H = \sum_{i=1}^{n} Pi\, Hi$$

$$H = \sum_{i=1}^{n} Pi\, [\sum_{j=1}^{n} Pij \log \frac{1}{Pij}$$

Information rate of Markoff sources

The information rate of Markoff sources is given by,

$$R = rH$$

Where, r = Rate at which symbols are generated

H= Entropy of Markoff sources

Information rate is measured in bits/sec

$\frac{Y}{X}$

Different types of Entropies

Marginal Entropies:

$$H(X) = -\sum_{i=1}^{M} P(xi)\log 2\, P(xi)$$

$$H(Y) = -\sum_{i=1}^{M} P(yi)\log 2\, P(yi)$$

Joint entropy: $$H(X,Y) = -\sum_{i=1}^{M}\sum_{j=1}^{N} P(xi,yj)\log 2P(xi,yj)$$

Conditional entropy: $$H\left(\frac{Y}{X}\right) = -\sum_{i=1}^{M}\sum_{j=1}^{N} P(xi,yj) log2\left(P\left(\frac{yj}{xi}\right)\right)$$

$$H\left(\frac{X}{Y}\right) = -\sum_{i=1}^{M}\sum_{j=1}^{N} P(xi,yj) log2\left(P\left(\frac{yj}{xi}\right)\right)$$

Relation between Entropies:

H(X,Y)=H(X)+H(Y/X) = H(Y)+H(X/Y)

Mutual Information

I(X; Y) of a channel is equal to difference between initial uncertainty and final uncertainty.

I(X;Y) = Initial uncertainty – final uncertainty.

$I(X;Y) = H(X) - H(X/Y)$ bits/symbol

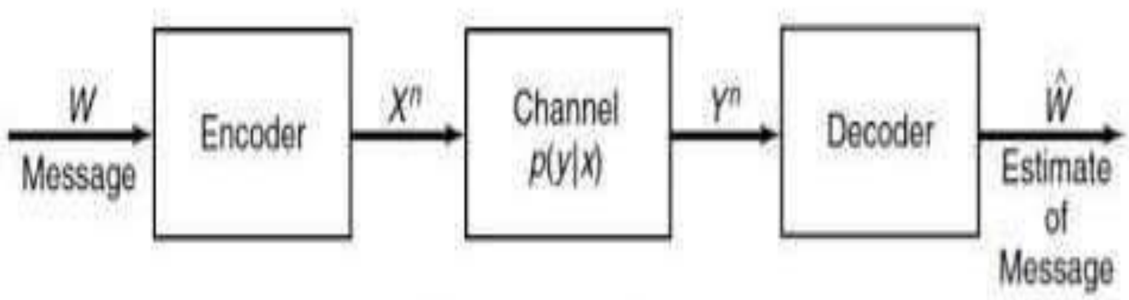
Where, H(X) - entropy of the source andH(X/Y) - Conditional Entropy.

Properties of mutual information:

1. $I(X;Y) = I(Y;X)$
2. $I(X;Y)>=0$
3. $I(X;Y) = H(X) - H(X/Y)$
4. $I(X;Y)) = H(X)+H(Y)-H(X,Y)$.

Discrete communication channel

The communication channel in which both input and output is a sequence of symbols is called a discrete communication channel or coding channel.

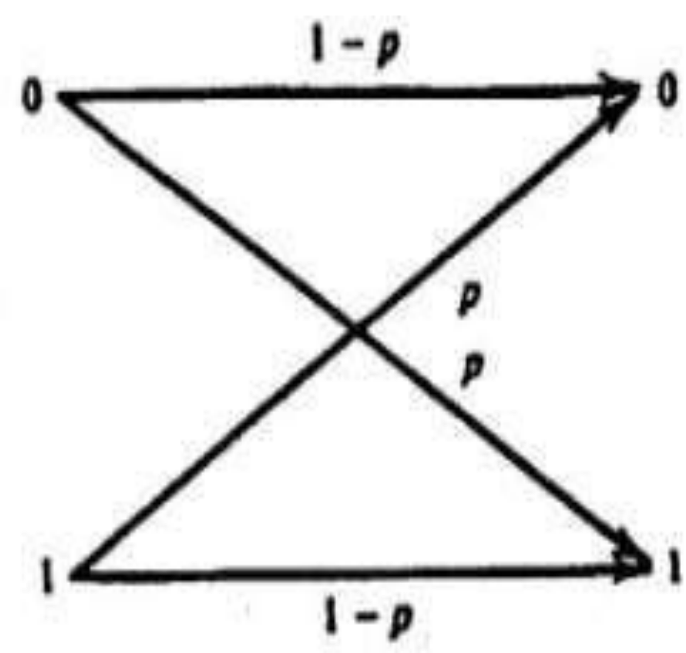


A discrete4 channel is characterized by a set of transition probability Pij which depends on the parameters of modulator, transmission medium or channel, noise and demodulator.

Discrete communication channel

The input to the discrete channel is any of the M symbols of an alphabet provided and output is the symbol belonging to same alphabet.

Model of discrete channel is shown below:



Rate of information over a discrete channel

- In discrete channels, the average rate of information transmission is assumed to be the difference between input data rate and error rate.
- The average rate of information transmission over a discrete channel is defined as the amount of information transmitted over the channel minus information lost.
- It is denoted by Dt and is given by,

$$Dt \cong \left[H(X) - H\left(\frac{X}{Y}\right) \right] rs$$

Capacity of discrete memoryless channel

The maximum allowable rate of information that can be transmitted over a discrete channel is called capacity of memoryless channel.

When channel matches with the source, maximum rate of transmission takes place.

Therefore, channel capacity,

$$C = \max [I(X,Y)] = \text{Max} [\, H(X) - H(X/Y) \,]$$

M-Ary discrete memoryless channel

The channel which transmit and receive one of the 'm' possible symbols depending on the present input and independent of previous input is called M-Ary discrete memoryless channel.

The relation between conditional entropy and joint entropy can be written as,

$$H(X,Y) = H(X/Y) + H(Y) = H(Y/X) + H(X)$$

Capacity of Gaussian channel- Shannon Hartley Theorem

Shannon- Hartley theorem states that the capacity of Gaussian channel having bandwidth 'W' is given as,

$$C = W\ log2\left[1 + \frac{S}{N}\right] bits/sec$$

Where, W = Channel bandwidth

S= Average signal power

N= Average noise power

Shannon- Fano algorithm

Shannon Fano coding is source encoding technique which is used to remove the redundancy (repeated information). The following steps are involved

1. For a given list of symbols, develop a corresponding list of probabilities or frequency counts so that each symbol's relative frequency of occurrence is known.
2. Sort the lists of symbols according to frequency, with the most frequently occurring symbols at the left and the least common at the right.

Shannon- Fano algorithm

3. Divide the list into two parts, with the total frequency counts of the left part being as close to the total of the right as possible.
4. The left part of the list is assigned the binary digit 0, and the right part is assigned the digit 1. This means that the codes for the symbols in the first part will all start with 0, and the codes in the second part will all start with 1.
5. Recursively apply the steps 3 and 4 to each of the two halves, subdividing groups and adding bits to the codes until each symbol has become a corresponding code leaf on the tree.