An Autonomous Institute
Affiliated to VTU, Belagavi,
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DEPARTMENT OF MEDICAL ELECTRONICS ENGINEERING

SUBJECT CODE : 18ML42

SUBJECT NAME: COMMUNICATION SYSTEMS

LECTURE PRESENTATION UNIT– 3

NOISE IN ANALOG MODULATION

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Unit: Structure

Module 3

- **Signal to noise Ratio :AM**
- **Receiver Model, DSBSC Receiver, SSB Receiver, FM Receiver Model,**
- **Noise in FM Reception, FM Threshold effect, Pre-Emphasis and De-Emphasis in FM.**

TEXT BOOK

"Communication Systems", Simon Haykins & Moher, 5th Edition, John Willey, India Pvt. Ltd, 2010, ISBN 978 – 81 – 265 – 2151 – 7.

COURSE OUTCOMES

After studying this course, students will be able to:

- Understand and derive SNR for AM & FM .

NOISE- External noise

- Equipment / Man-made Noise is generated by any equipment that operates with electricity
- Atmospheric Noise is often caused by lightning
- Space Noise is strongest from the sun and, at a much lesser degree, from other stars

NOISE- Internal noise

- Shot Noise is due to random variations in current flow in active devices.
- Partition Noise occurs only in devices where a single current separates into two or more paths, e.g. bipolar transistor.
- Excess Noise is believed to be caused by variations in carrier density in components.
- Transit-Time Noise occurs only at high f.

NOISE- Short *Noise*

- Arises due to discrete nature of current flow in Diodes and Transistors.
- Photons in an optical device or electrons in an electronic circuit.
- If we assume the electrons in these components are emitted at random interval, τk where $-\infty < k < \infty$.
- If this random emission are observed for a long period.
- The total current flow through the component can be expressed as

$$X(t) = \sum_{k=-\infty}^{\infty} h(t - \tau_k)$$

- Where X (t) is stationary and is called Shot Noise.

NOISE- *Thermal Noise*

- Thermal Noise is produced by the random motion of electrons in a conductor due to heat. Noise power, $P_N = kTB$

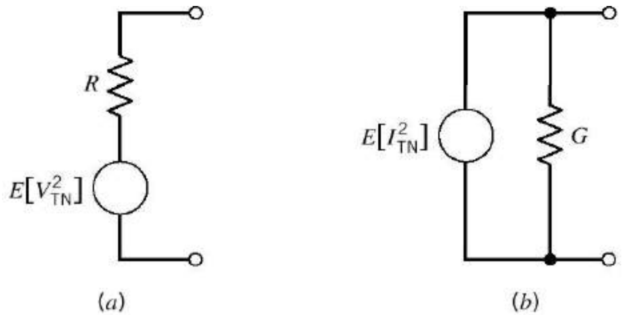
where T = absolute temperature in °K

k = Boltzmann's constant, 1.38×10^{-23} J/K

B = noise power bandwidth in Hz

Noise voltage,

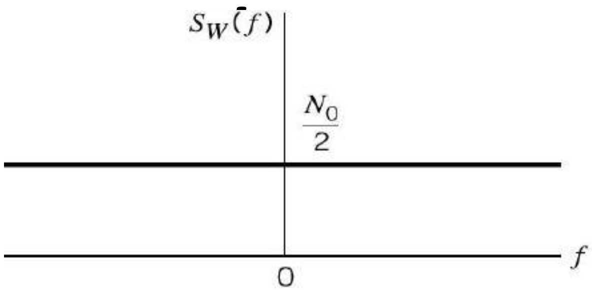
$$V_N = \sqrt{4kTBR}$$



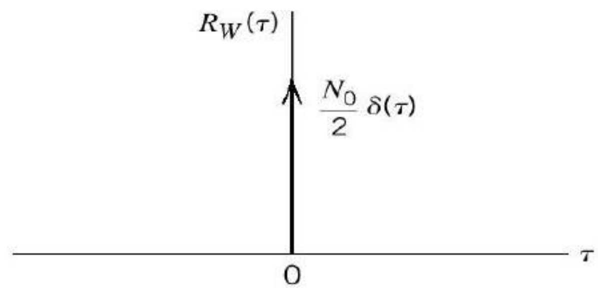
$$E\left[V_{TN}^2\right] = 4kTR\Delta f \qquad \text{volts}^2$$

$$E\left[I_{TN}^2\right] = \frac{1}{R^2} E\left[V_{TN}^2\right] = 4kT\frac{1}{R}\Delta f = 4kTG\Delta f \qquad \text{amps}^2$$

NOISE-



(a)



(b)

$$S_W(f) = \frac{N_0}{2}$$

$$N_0 = kT_e$$

T_e : equivalent noise temperature of the receiver

$$R_W(\tau) = \frac{N_0}{2}\delta(\tau)$$

NOISE

A system with transfer function H(ω) and input signal power density spectrum Si(ω) has a mean square valve of the output signal

$$\overline{v_0^2} = \frac{K}{2\pi}\int_{\infty}^{-\infty} S_{i(\omega)} |H(\omega)|^2 \, d\omega$$

- BW is low, noise have constant spectral density

$$\overline{v_0^2} = \frac{K}{2\pi}\int_{\infty}^{-\infty} |H(\omega)|^2 \, d\omega$$

i.e $S_{i(\omega)}$ is a constant say k

$$\overline{v_0^2} = \frac{K}{\pi}\int_{0}^{-\infty} |H(\omega)|^2 \, d\omega$$

- Equivalent Noise Bandwidth

$$w_0 = \frac{1}{|H(\omega)|^2}\int_{0}^{\infty} |H(\omega)|^2 \, d\omega$$

Hence

$$\overline{v_0^2} = \frac{K}{\pi}|H(\omega_0)|^2 W_0$$

RMS of noise signal ≡ideal BP system with constant gain

Az Documents

NOISE EQUIVALENT BANDWIDTH

Properties of NBN

1. $n_1(t)$ and $n_Q(t)$ of $n(t)$ has Zero mean.
2. If $n(t)$ is Gaussian then $n_I(t)$ and $n_Q(t)$ are jointly Gaussian.
3. If $n(t)$ is stationary then $n_1(t)$ and $n_Q(t)$ are jointly stationary.
4. Both $n_1(t)$ and $n_Q(t)$ have the same psd related to the psd of $n(t)$.
5. $n_1(t)$ and $n_Q(t)$ have the same variance as that of $n(t)$.
6. The cross spectral density of $n_1(t)$ and $n_Q(t)$ of $n(t)$ is purely imaginary.
7. If $n(t)$ is Gaussian and the psd of $n(t)$ is symmetric about fc then $n_1(t)$ and $n_Q(t)$ are statistically independent.

NOISE FIGURE

NOISE FIGURE

- Noise Figure is a figure of merit that indicates how much a component, or a stage degrades the SNR of a system:

  $$NF = (S/N)_i / (S/N)_o$$

  where $(S/N)_i$ = input SNR (not in dB)

  and $(S/N)_o$ = output SNR (not in dB)

  $NF(dB)=10 \log NF = (S/N)_i (dB) - (S/N)_o (dB)$

NOISE IN ANALOG COMMUNICATIONS

In this chapter, we revisit the analog modulation methods of Unit 1 and 2 in light of the noise-related concepts introduced.

There may be many sources of noise in a communication system, but often the major sources are the communication devices themselves or interference encountered during the course of transmission. There are several ways that noise can affect the desired signal, but one of the most common ways is as an additive distortion. That is, the received signal is modeled as1

$$r(t) = s(t) + w(t)$$

where is the s(t) transmitted signal and w(t) is the additive noise. If we knew the noise exactly, then we could subtract it from and recover the transmitted signal exactly. Unfortunately, this is rarely the case. Much of communication system design is related to processing the r(t) received signal in a manner that minimizes the effect of additive noise.

This chapter will focus on the detection of analog signals in the presence of additive noise. The material in this chapter teaches us the following lessons.

- ▶ *Lesson 1: Minimizing the effects of noise is a prime concern in analog communications, and consequently the ratio of signal power to noise power is an important metric for assessing analog communication quality.*

- ▶ *Lesson 2: Amplitude modulation may be detected either coherently requiring the use of a synchronized oscillator or non-coherently by means of a simple envelope detector. However, there is a performance penalty to be paid for non-coherent detection.*

- ▶ *Lesson 3: Frequency modulation is nonlinear and the output noise spectrum is parabolic when the input noise spectrum is flat. Frequency modulation has the advantage that it allows us to trade bandwidth for improved performance.*

- ▶ *Lesson 4: Pre- and de-emphasis filtering is a method of reducing the output noise of an FM demodulator without distorting the signal. This technique may be used to significantly improve the performance of frequency modulation systems.*

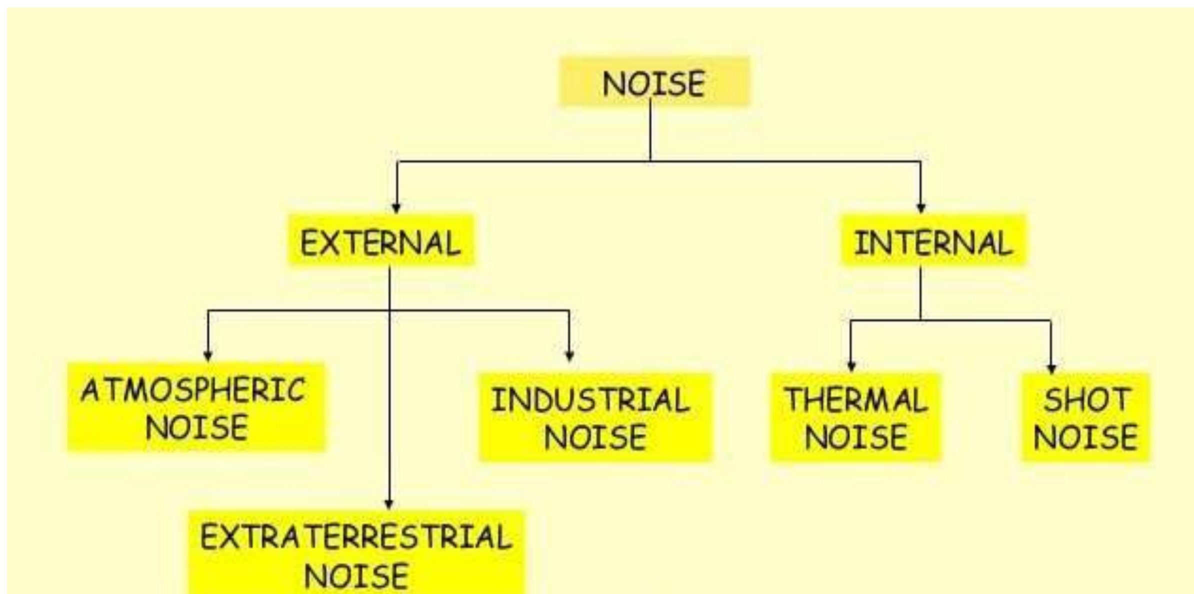
NOISE IN COMMUNICATION SYSTEMS

Noise: It is an unwanted signal which tends to interfere with the modulating signal.

Types of noise:

Noise is basically divided into,

1. External Noise
2. Internal Noise



Classification of Noise

1.External Noise:

- **Atmospheric Noise**: Radio noise caused by natural atmospheric processes, primarily lightening discharges in thunder storms.
- **Extraterrestrial Noise**: Radio disturbances from sources other than those related to the Earth.

  Cosmic Noise: Random noise that originates outside the Earth's atmosphere.

  Solar Noise: Noise that originates from the Sun is called Solar noise.

Classification of Noise

- **Industrial Noise**: Noise generated by automobile ignition, aircrafts, electric motors, Switch gears, welding etc.

2. **Internal Noise**:

- **Shot Noise**: Random motion of electrons in the semiconductor devices generates shot noise.
- **Thermal or Johnson's Noise**: Random motion of electrons in the resistor is called Thermal noise.

$$V_n = KT_0BR$$

Where, K= Boltzmann constant, R= Resistance

T0= Absolute temperature B= Bandwidth

Noise Temperature and Noise Figure

Noise temperature(Te): It is a means for specifying noise in terms of an equivalent temperature. It is expressed as ,

$$T_e = (F_n - 1)\, T_0$$

Where, F_n = Noise Figure, T_0 = Absolute temperature

Noise figure(F_n): It is the ratio of output and input noise of an amplifier or network. It is expressed as, $F_n = \frac{KT_0BG + \Delta N}{KT_0BG}$

Where, Δ = Noise added by the network or amplifier.

G = gain of an network or amplifier

Noise Temperature and Noise Figure

Noise Figure of Cascade Amplifier or Network:

Noise Figure of an cascade network or amplifier is expressed as,

$$F_n = F1 + \frac{(F2-1)}{G1} + \frac{F3-1}{G1G2} + \frac{F4-1}{G1G2G3} + \cdots \ldots + \frac{Fn-1}{[G1G2G3 \ldots \ldots G(n-1)]}$$

Where, F1= Noise figure of 1^{st} stage

G1= Gain of 1^{st} stage

F2= Noise figure of 2^{nd} stage

G2 = Gain of 2^{nd} stage

Fn = Noise figure of nth stage

Gn = Gain of nth stage

Noise equivalent Bandwidth

When white noise (flat spectrum of frequencies like white light) is passed through a filter having a frequency response , some of the noise power is rejected by the filter and some is passed through to the output.

The noise equivalent bandwidth is defined in the following picture,

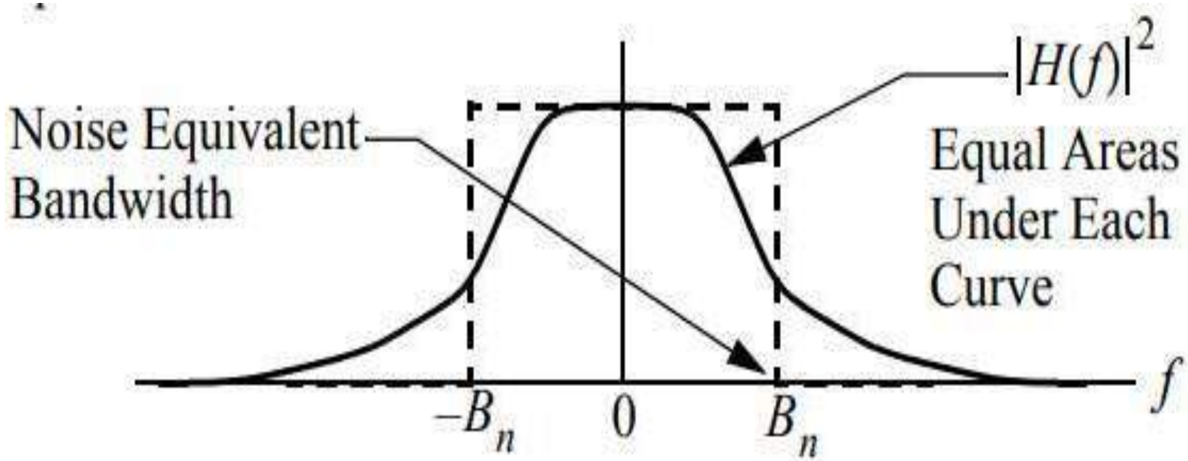


Figure of Merit

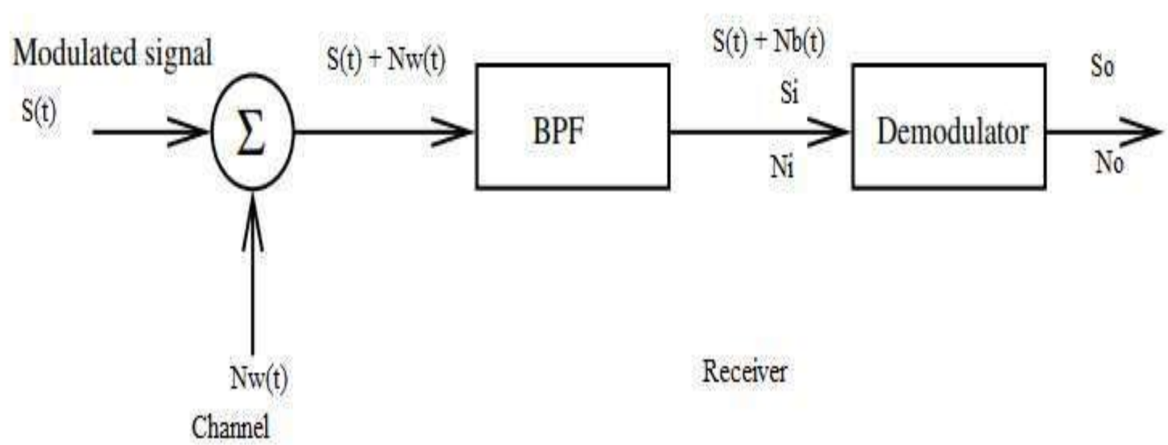
Figure of Merit (FOM): It is ratio of output SNR to input SNR of a communication system.

$$\text{FOM} = \frac{S_o/N_o}{S_i/N_i}$$

Where S0= Output Signal Power &N0= Output Noise Power

Si= Input Signal Power &Ni= Input Noise Power

Receiver model for noise calculation:



1. The message power is the same as the modulated signal power of the modulation scheme under study.
2. The baseband low-pass filter passes the message signal and rejects out-of-band noise. Accordingly, we may define the reference signal-to-noise ratio, SNR_{ref}, as

$$\text{SNR}_{\text{ref}} = \frac{\text{average power of the modulated message signal}}{\text{average power of noise mesured in the message bandwidth}} \qquad (9.11)$$

- A Figure of merit

$$\text{Figure of merit} = \frac{\text{post-detection SNR}}{\text{reference SNR}}$$

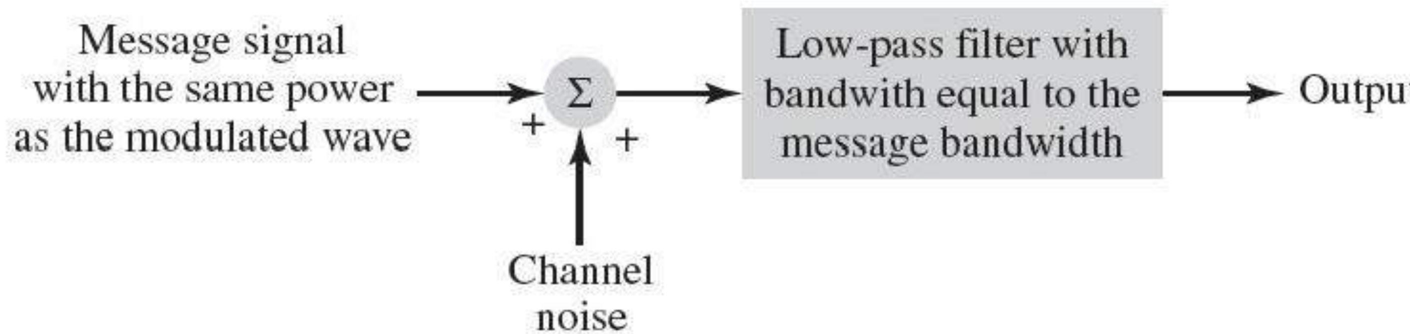


FIGURE 9.4 Reference transmission model for analog communications.

- The higher the value that the figure of merit gas, the better the noise performance of the receiver will be.

- To summarize our consideration of signal-to-noise ratios:
  - The pre-detection SNR is measured before the signal is demodulated.
  - The post-detection SNR is measured after the signal is demodulated.
  - The reference SNR is defined on the basis of a baseband transmission model.
  - The figure of merit is a dimensionless metric for comparing sifferent analog modulation-demodulation schemes and is defined as the ratio of the post- detection and reference SNRs.

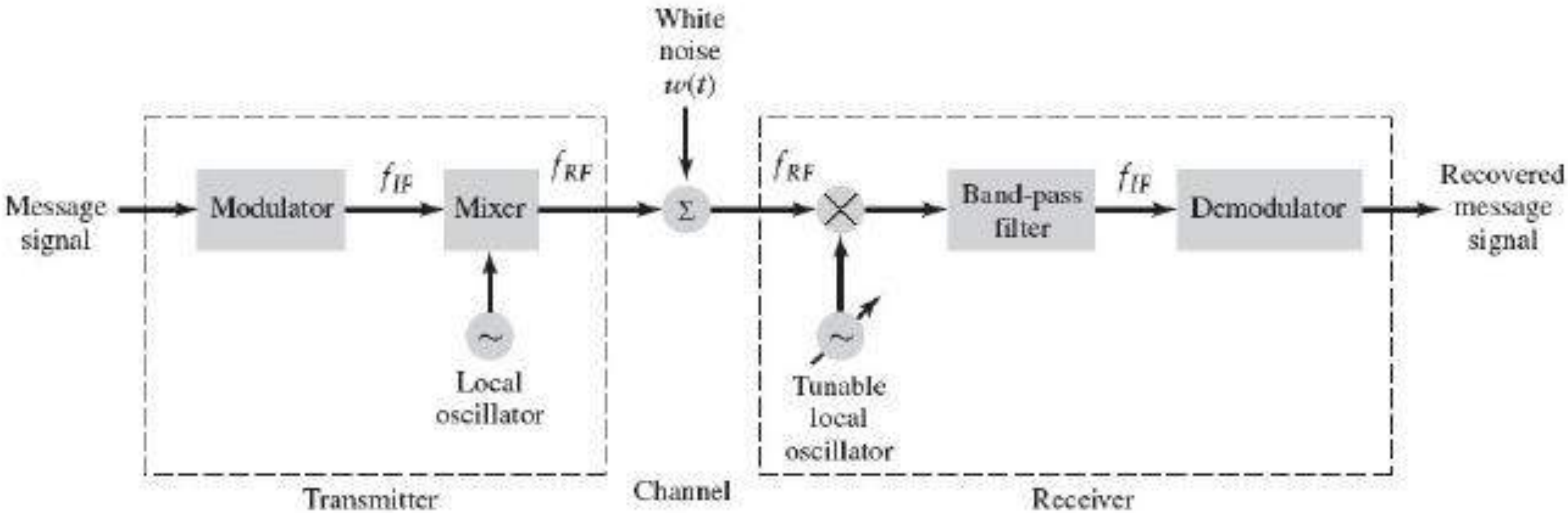


FIGURE 9.5 Block diagram of band-pass transmission showing a superheterodyne receiver.

Noise in Linear Receivers Using Coherent Detection

- Double-sideband suppressed-carrier (DSB-SC) modulation, the modulated signal is represented as

$$s(t) = A_c m(t) \cos(2\pi f_c t + \theta) \quad (9.13)$$

- f_c is the carrier frequency
- $m(t)$ is the message signal
- The carrier phase θ
- In Fig. 9.6, the received RF signal is the sum of the modulated signal and white Gaussian noise $w(t)$
- After band-pass filtering, the resulting signal is

$$x(t) = s(t) + n(t) \quad (9.14)$$

Fig. 9.6

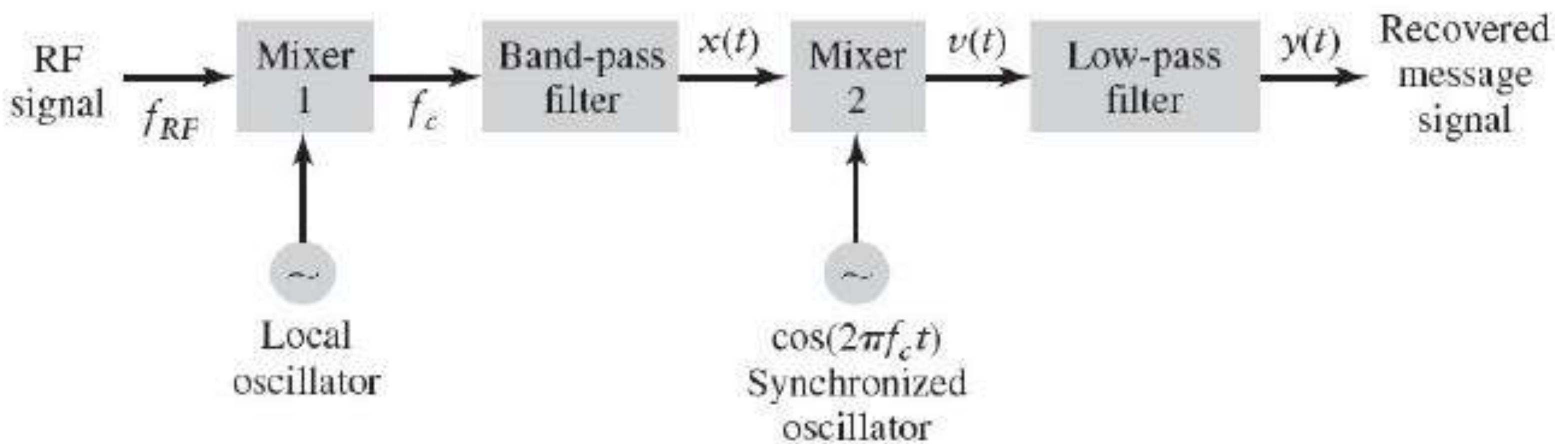


FIGURE 9.6 A linear DSB-SC receiver using coherent demodulation.

- In Fig.9.7
  - The assumed power spectral density of the band-pass noise is illustrated
- For the signal $s(t)$ of Eq. (9.13), the average power of the signal component is given by expected value of the squared magnitude.
- The carrier and modulating signal are independent

$$\mathrm{E}[s^2(t)] = \mathrm{E}[(A_c \cos(2\pi f_c t + \theta))^2]\mathrm{E}[m^2(t)] \quad (9.15)$$

$$P = \mathrm{E}[m^2(t)] \quad (9.16)$$

$$\mathrm{E}[s^2(t)] = \frac{A_c^2 P}{2} \quad (9.17)$$

- Pre-detection signal-to-noise ratio of the DSB-SC system
  - A noise bandwidth B_T
  - The signal-to-noise ratio of the signal is

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2 N_0 B_T} \quad (9.18)$$

Fig. 9.7

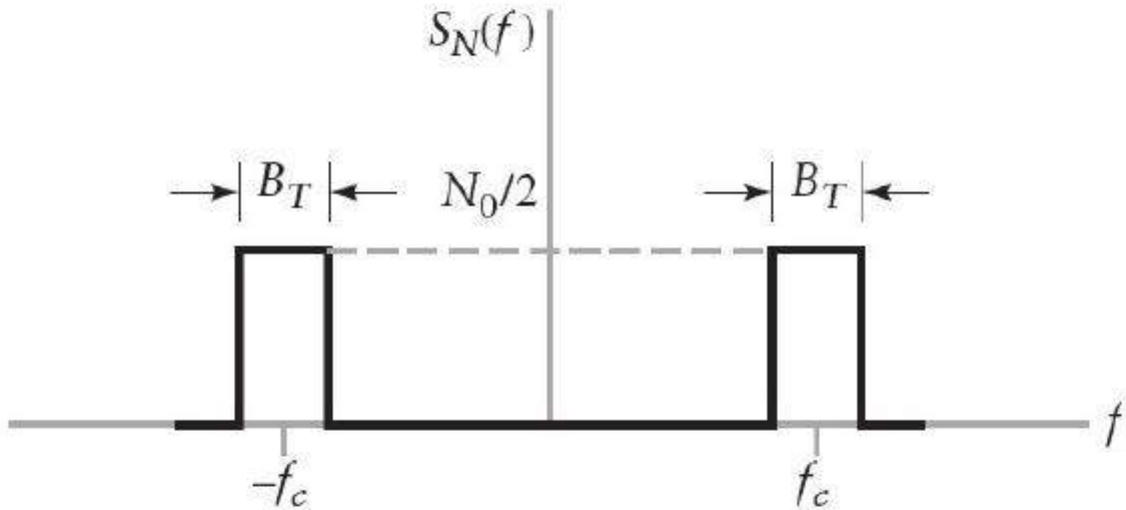


FIGURE 9.7 Power spectral density of band-pass noise.

- The signal at the input to the coherent detector of Fig. 9.6

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (9.19)$$

$$\begin{aligned} v(t) &= x(t)\cos(2\pi f_c t) \\ &= \tfrac{1}{2}(A_c m(t) + n_I(t)) \\ &\quad + \tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2}n_Q(t)\sin(4\pi f_c t) \quad (9.20) \end{aligned}$$

$$\cos A \cos A = \frac{1+\cos 2A}{2} \quad \text{and} \quad \sin A \cos A = \frac{\sin 2A}{2}$$

- These high-frequency components are removed with a low-pass filter

$$y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t)) \quad (9.21)$$

- The message signal $m(t)$ and the in-phase component of the filtered noise $n_I(t)$ appear additively in the output.
- The quadrature component of the noise is completely rejected by the demodulator. Post-detection signal to noise ratio
- The message component is $\frac{1}{2}A_c m(t)$, so analogous to the computation of the predetection signal power, the post-detection signal power is $\frac{1}{4}A_c^2 P$ where P is the average message power as defined in Eq. (9.16).
- The noise component is $\frac{1}{2}n_I(t)$ after low-pass filtering. As described in Section 8.11, the in-phase component has a noise spectral density of N_0 over the bandwidth from $-B_T/2 \text{ to } B_T/2$. If the low-pass filter has a noise bandwidth W, corresponding to the message bandwidth, which is less than or equal to $B_T/2$, then the output noise power is

$$\mathrm{E}[n_I^2(t)] = \int_{-\mathrm{W}}^{\mathrm{W}} N_0 \, df$$

$$= 2N_0\mathrm{W} \quad (9.22)$$

- Post-detection SNR of

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} = \frac{\frac{1}{4}(A_c^2)P}{\frac{1}{4}(2N_0\mathrm{W})}$$

$$= \frac{A_c^2 P}{2N_0\mathrm{W}} \quad (9.23)$$

- Post-detection SNR is twice pre-detection SNR.
- Figure of merit for this receiver is

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$

- We lose nothing in performance by using a band-pass modulation scheme compared to the baseband modulation scheme, even though the bandwidth of the former is twice as wide.

Noise in AM Receivers Using Envelope Detection

- The envelope-modulated signal

$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t) \quad (9.24)$$

- The power in the modulated part of the signal is

$$\begin{aligned} \mathrm{E}[(1 + k_a m(t))^2] &= \mathrm{E}[1 + 2k_a m(t) + k_a^2 m^2(t)] \\ &= 1 + 2k_a \mathrm{E}[m(t)] + k_a^2 \mathrm{E}[m^2(t)] \\ &= 1 + k_a^2 P \qquad (9.25) \end{aligned}$$

- The pre-detection signal-to-noise ratio is given by

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2N_0 \mathrm{B_T}} \quad (9.26)$$

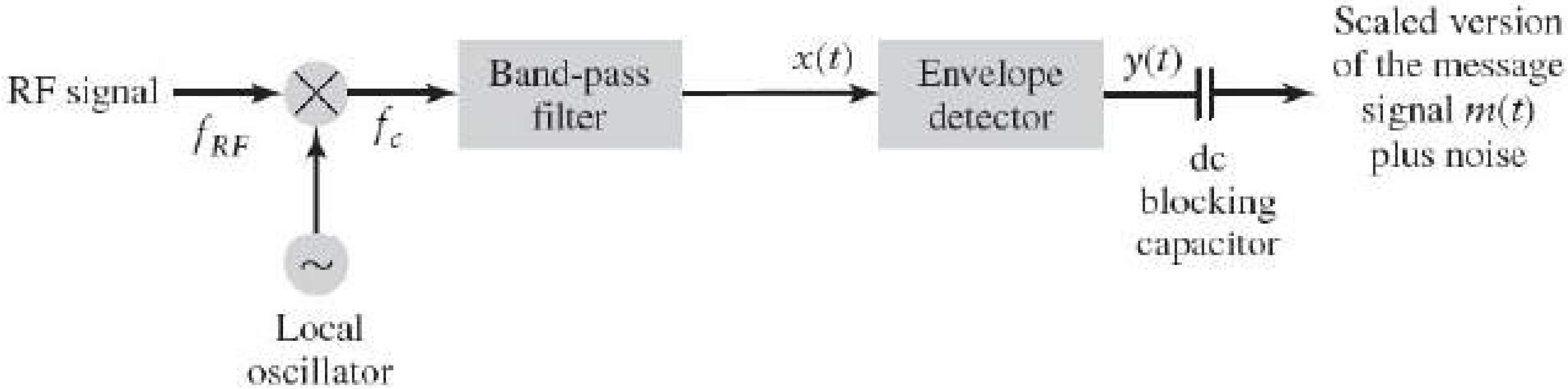


FIGURE 9.8 Model of AM receiver using envelope detection.

- Model the input to the envelope detector as

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= [A_c + A_c k_a m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (9.27) \end{aligned}$$

- The output of the envelope detector is the amplitude of the phasor representing $x(t)$ and it is given by

$$\begin{aligned} y(t) &= \text{envelope of } x(t) \\ &= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2} \quad (9.28) \end{aligned}$$

- Using the approximation $\sqrt{A^2 + B^2} \approx A \quad when \quad A >> B,$

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t) \quad (9.29)$$

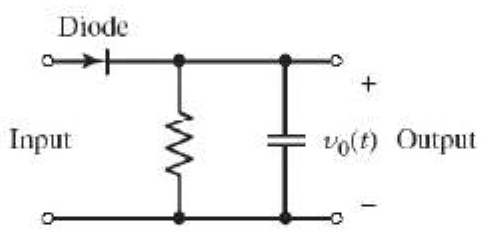


FIGURE 9.9 Circuit diagram of envelope detector.

- The post-detection SNR for the envelope detection of AM,

$$\text{SNR}_{\text{post}}^{\text{AM}} = \frac{A_c^2 k_a^2 P}{2N_0 \text{W}} \quad (9.30)$$

- This evaluation of the output SNR is only valid under two conditions:
  - The SNR is high.
  - is adjusted for 100% modulation or less, so there is no distortion of the signal envelope.
- The figure of merit for this AM modulation-demodulation scheme is

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{AM}}}{\text{SNR}_{\text{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P} \quad (9.31)$$

Band-Pass Receiver Structures

- Fig. 9.5 shows an example of a superheterodyne receiver
- AM radio transmissions
  - Common examples are AM radio transmissions, where the RF channels' frequencies lie in the range between 510 and 1600 kHz, and a common IF is 455 kHz
- FM radio
  - Another example is FM radio, where the RF channels are in the range from 88 to 108 MHz and the IF is typically 10.7 MHz.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad (9.12)$$

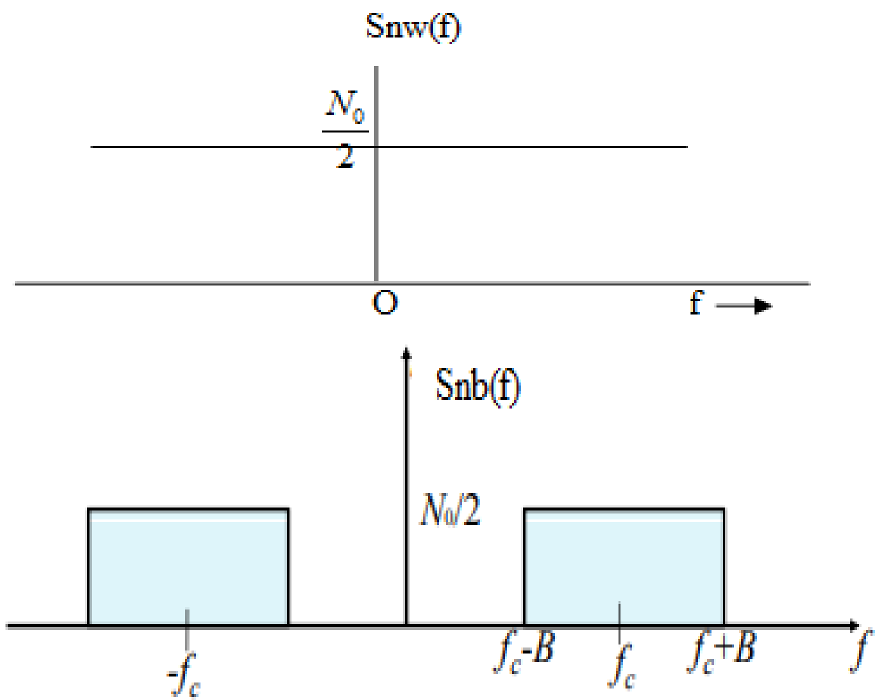
- The filter preceding the local oscillator is centered at a higher RF frequency and is usually much wider, wide enough to encompass all RF channels that the receiver is intended to handle.
- With the same FM receiver, the band-pass filter after the local oscillator would be approximately 200kHz wide; it is the effects of this narrower filter that are of most interest to us.

Receiver model for noise calculation

- The receiver is combination of Band Pass Filter (BPF) and Demodulator.
- The BPF is combination of RF Tuned Amplifier, Mixer and Local Oscillator whose band width is equal to band width of modulated signal at transmitter.
- Channel Inter connects transmitter & receiver. Channel adds noise to the modulated signal while transmitting and it is assumed to be white noise whose Power Spectral Density is uniform.
- BPF converts white noise in to color or Band pass noise or narrow band pass noise.

Receiver model for noise calculation

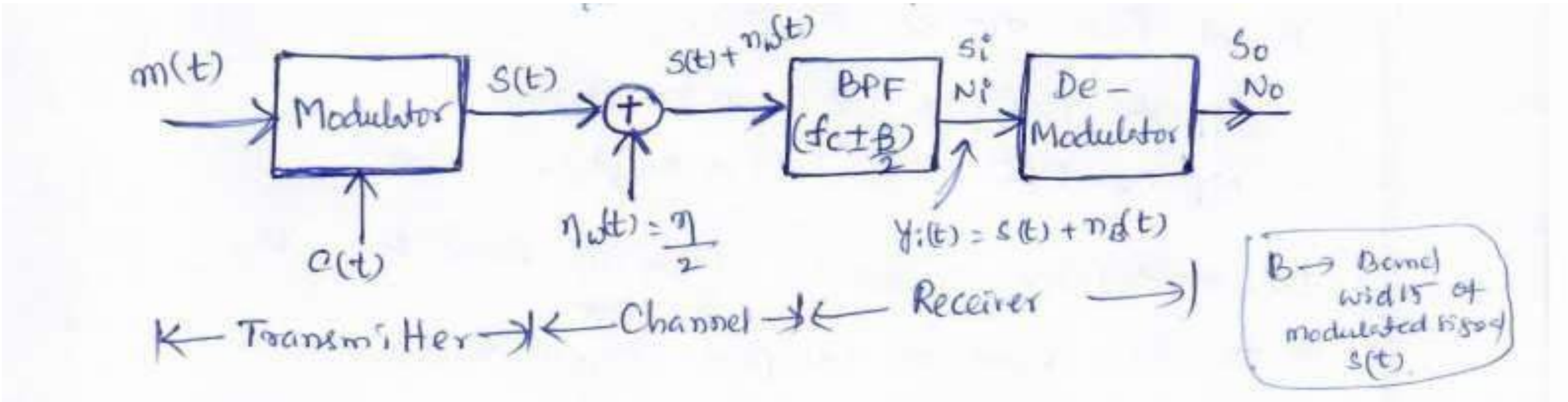
PSD of white noise and Narrow band pass noise are,



Power of band pass noise P $= \int_{f_c-B}^{f_c+B} \frac{N_0}{2}\, df = N_0 B$

Where B = Band width noise.

Communication system model for noise calculation



- The communication system model for noise calculation contains transmitter, channel and receiver.
- Transmitter is replaced by modulator which converts low frequency modulating signal x(t) into high frequency bandpass signal with the help of carrier signal.
- Channel is replaced or modelled as additive noise which adds white noise with PSD η/2 and it contains all frequencies.

Communication system model for noise calculation

- Receiver is modelled as BPF followed by demodulator.
- BPF is combination of RF tuned amplifier, mixer , local oscillator.
- Passband or badnwidth of BPF is equal to bandwidth of modulated signal.
- BPF converts white noise into color or bandpass noise $\eta_B(t)$.

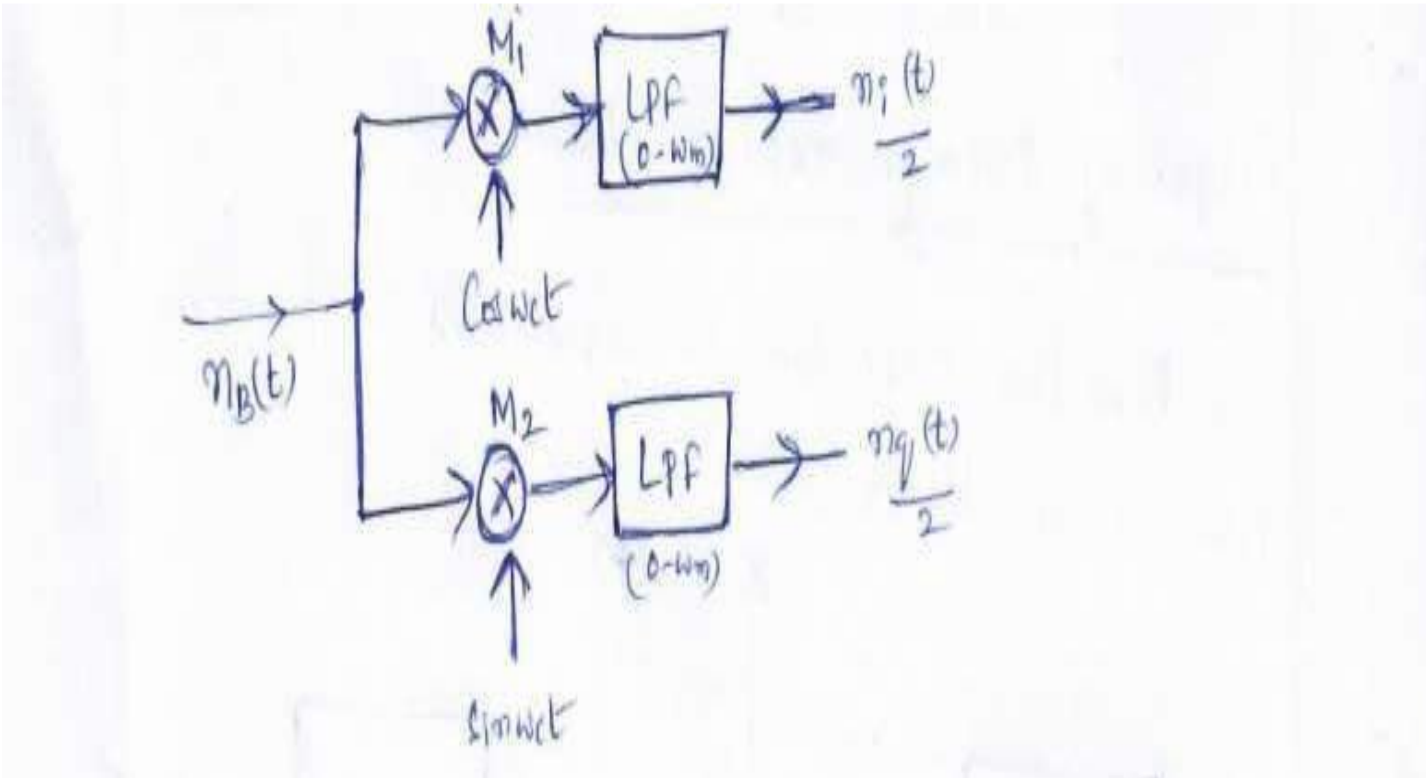
$$\text{Input to BPF is } s(t) + \eta_w(t)$$

$$\text{Output of BPF is } s(t) + \eta_B(t)$$

- Demodulator converts high frequency or bandpass signal into low frequency or baseband signal.

Quadrature representation

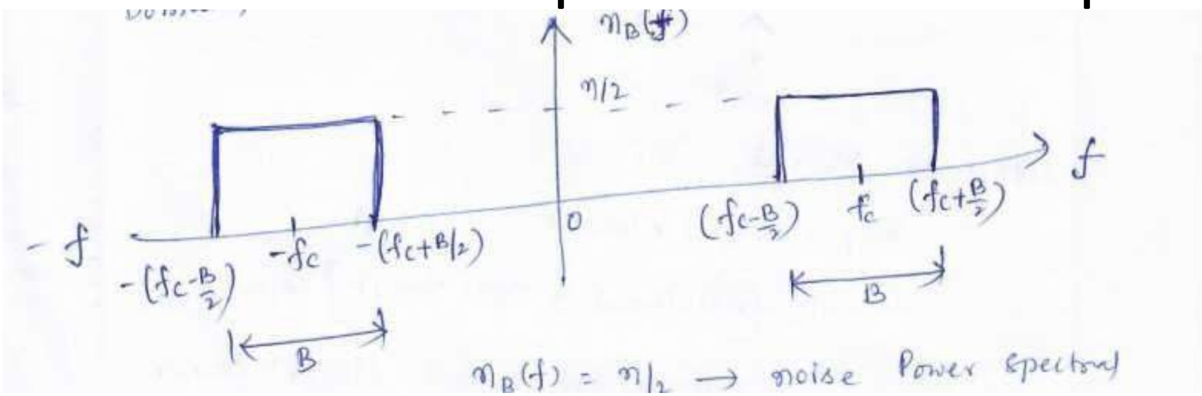
$\eta_i(t)$ and $\eta_q(t)$ can be recovered from $\eta_B(t)$,



Bandpass noise representation

Frequency domain representation:

Bandpass noise can be represented in frequency domain as,



Properties of $\eta_B(t)$:

- $\eta_B(t)$, $\eta_i(t)$, $\eta_q(t)$ will have same power.
- The PSD of $\eta_i(t)$ & $\eta_q(t)$ is,

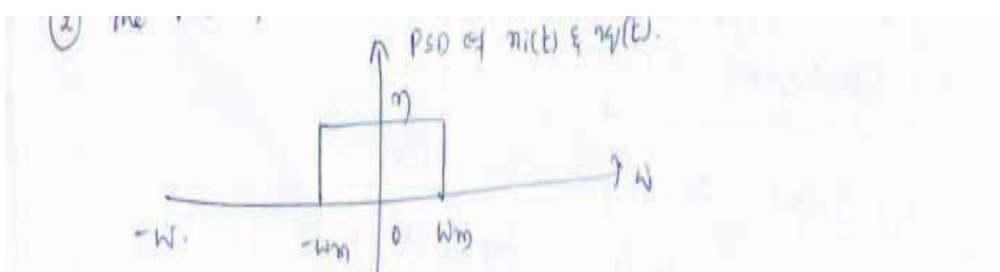
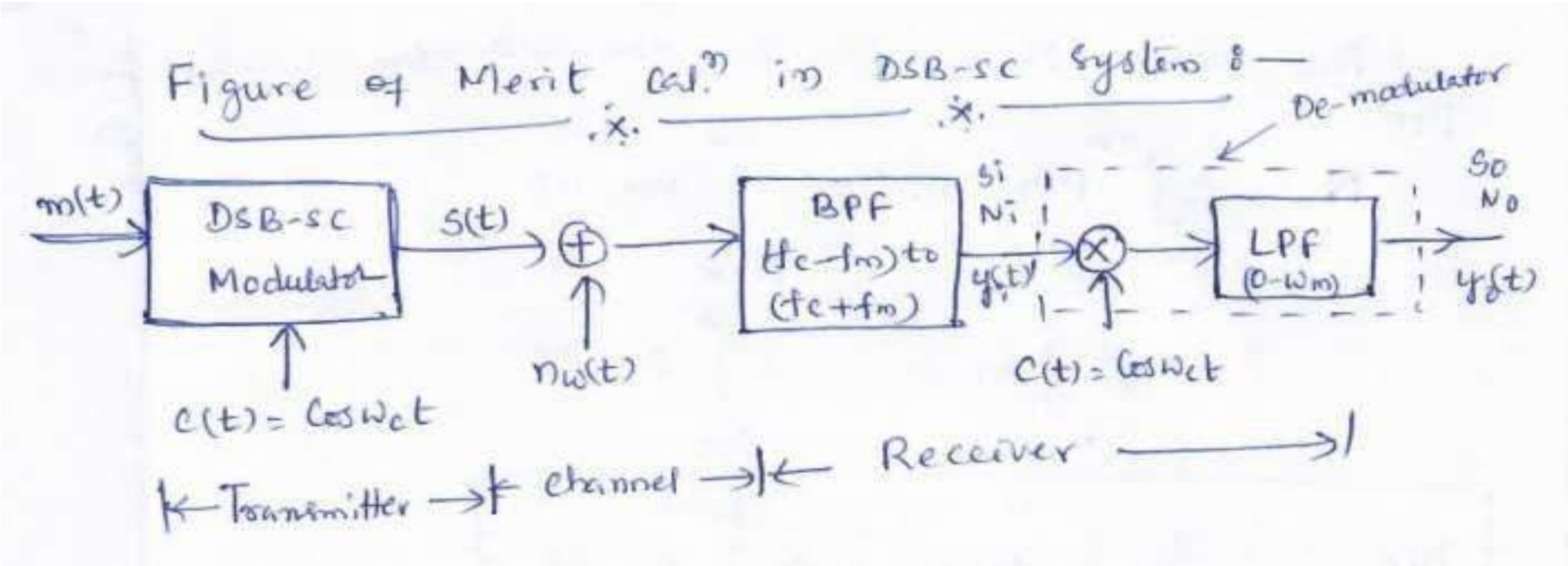


Figure of Merit calculation in DSB-SC



- Transmitter contains DSB-SC modulator, whose output s(t) = m(t) coswct.
- Noise generated by the channel is considered as white noise $\eta_w(t)$ with uniform noise power spectral density $\eta/2$.

Figure of Merit calculation in DSB-SC

- Band pass filter's bandwidth is equal to modulated signal bandwidth.
- BPF allows DSB-SC signal and converts white noise into color noise or bandpass noise $\eta_B(t)$.

  Therefore, o/p of the BPF is $yi(t) = s(t) + \eta_B(t)$.
- Synchronous detector is used to extract modulating signal m(t) which contain multiplier followed by low pass filter.

Input signal power is , $Si = m^2(t)/2$, Input noise power, $Ni = \eta.\ 2fm$, Output signal power, $S0 = [m(t)/2]^2$, Output noise power, $N0 = \eta.\ fm/2$

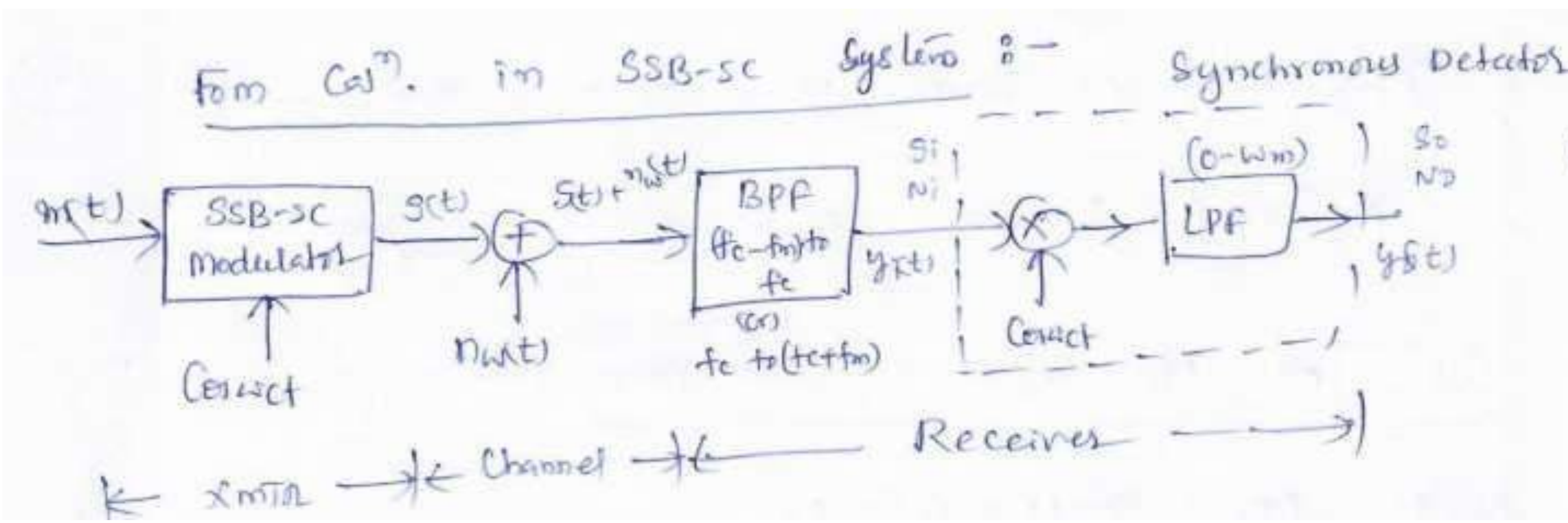
Substituting these values, FOM=(S0/N0)/(Si/Ni)= 2

Figure of Merit calculation in SSB-SC

SSB-SC signal, $s(t) = m(t)coswct \pm mh(t)sinwct$

Output of BPF, $yi(t) = s(t) + \eta_B(t)$

Bandpass noise, $\eta_B(t) = \eta_i(t)$ ocs $W_c t$ $\eta_q(t) Sin W_c t$



Input signal power is , $Si = m^2(t)$, Input noise power, $Ni = \eta . fm$

Output signal power, $S0 = m^2(t)/4$, Output noise power, $N0 = \eta . fm/4$

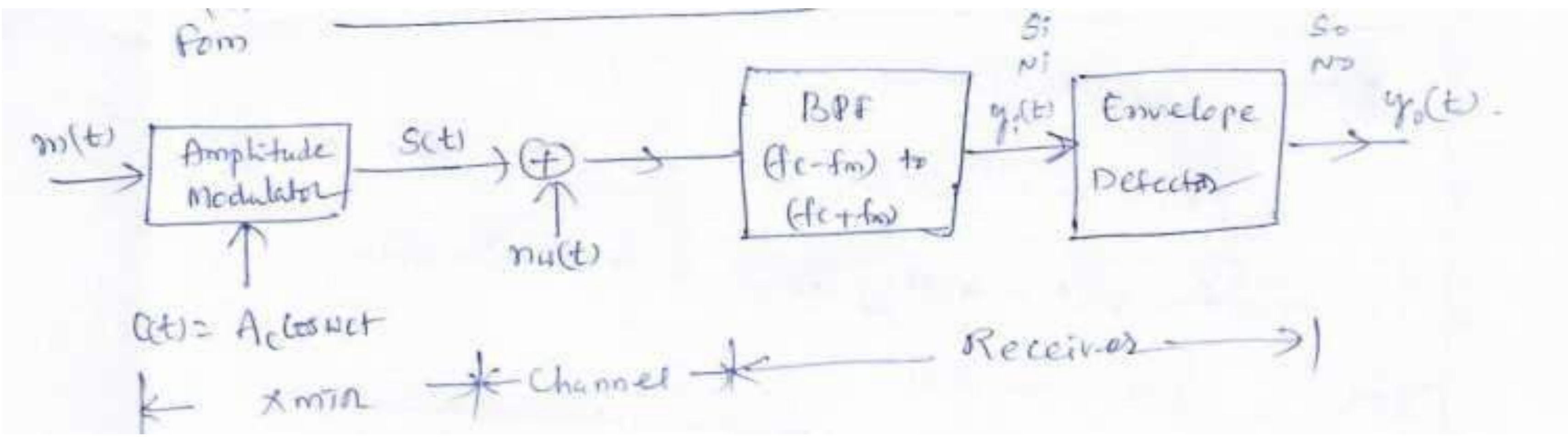
Substituting these values, $FOM = (S0/N0)/(Si/Ni) = 1$

Noise calculation in AM system

AM signal, $S(t) = [Ac+m(t)]\ Cosw_ct$

Output of BPF is, $yi(t) = s(t) + \eta_B(t)$

$= [Ac+m(t)]\ Cosw_ct + \eta_B(t)$



Input signal power is , $Si = [Ac^2/2]+[m^2(t)/2]$

Input noise power, $Ni = 2\ \eta.\ Fm$, Output signal power,$S0 = m^2(t)$

Output noise power, $N0 = 2\eta.\ Fm$

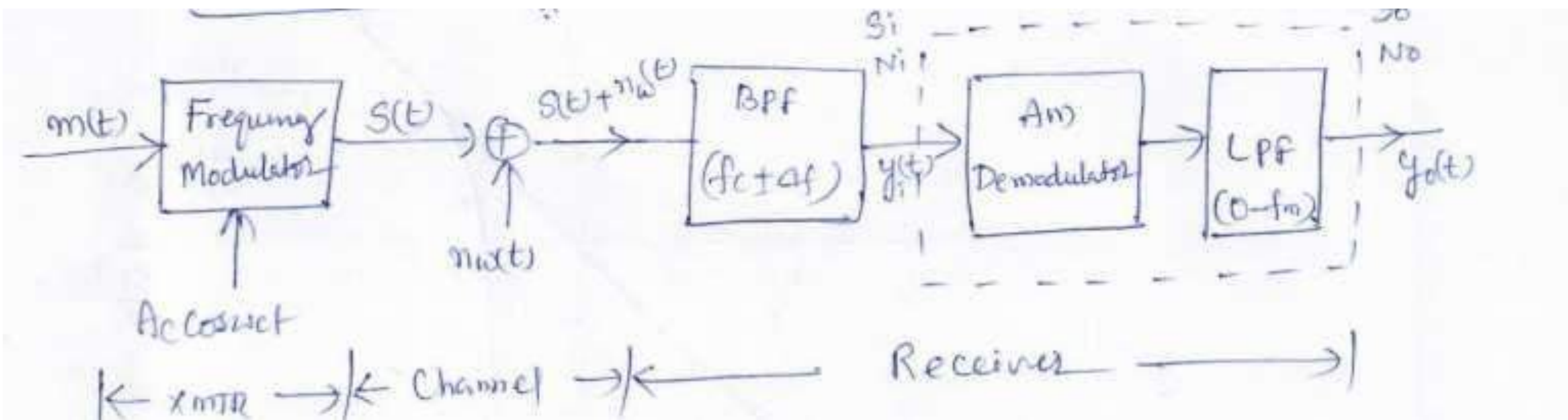
Using these values, FOM= 2.

Noise calculation in FM system

Frequency modulated signal $s(t) = A_c \text{Cos} [\, w_c t + Kf\int m(t)\, dt]$

Output of BPF is, $yi(t) = s(t) + \eta_B(t)$

$$= A_c \text{Cos} [\, w_c t + Kf\int m(t)\, dt] + \eta_B(t)$$



Input signal power is , $Si = Ac^2/2$, Input noise power, $Ni = 2\ \eta.\ \Delta f$

Output signal power, $S0 = \gamma^2 K_f^2 m^2(t)$

Substituting these values, FOM= (S0/N0)/(Si/Ni)

$$FOM = (3/4\pi^2) mf^3, \quad \text{Where } m_f = \Delta f/f_m$$

Detection of Frequency Modulation (FM)

- The frequency-modulated signal is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right] \quad (9.40)$$

- Pre-detection SNR
  - The pre-detection SNR in this case is simply the carrier power $A_C^2/2$ divided by the noise passed by the bandpass filter, $N_0 B_T$; namily,

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2}{2N_0 B_T}$$

1. A slope network or differentiator with a purely imaginary frequency response that varies linearly with frequency. It produces a hybrid-modulated wave in which both amplitude and frequency vary in accordance with the message signal.
2. An envelope detector that recovers the amplitude variation and reproduces the message signal.

- Post-detection SNR

$$x(t) = s(t) + n(t) \quad (9.41)$$

  - The noisy FM signal after band-pass filtering may be represented as

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (9.42)$$

  - We may equivalently express $n(t)$ in terms of its envelope and phase as

$$n(t) = r(t)\cos[2\pi f_c t + \phi_n(t)] \quad (9.43)$$

  - Where the envelope is

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2} \quad (9.44)$$

  - And the phase is

$$\phi_n(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \quad (9.45)$$

Fig. 9.14

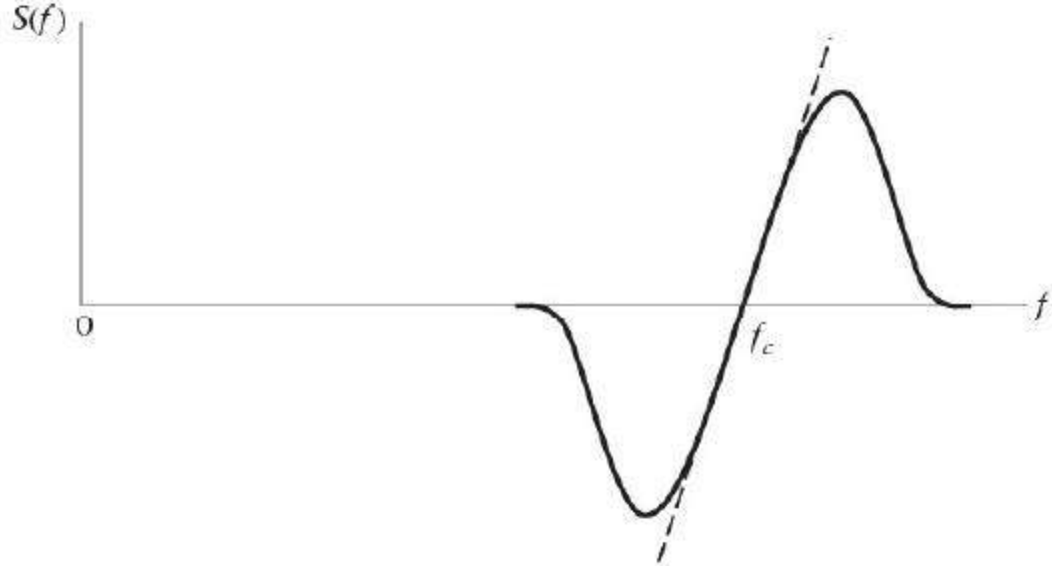


FIGURE 9.14 Amplitude response of slope network used in FM discriminator.

- We note that the phase of $s(t)$ is

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)d\tau \quad (9.46)$$

- The noisy signal at the output of the band-pass filter may be expressed as

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \phi_n(t)] \quad (9.47) \end{aligned}$$

- The phase $\theta(t)$ of the resultant is given by

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi(t))}{A_c + r(t)\cos(\psi(t))}\right\} \quad (9.48)$$

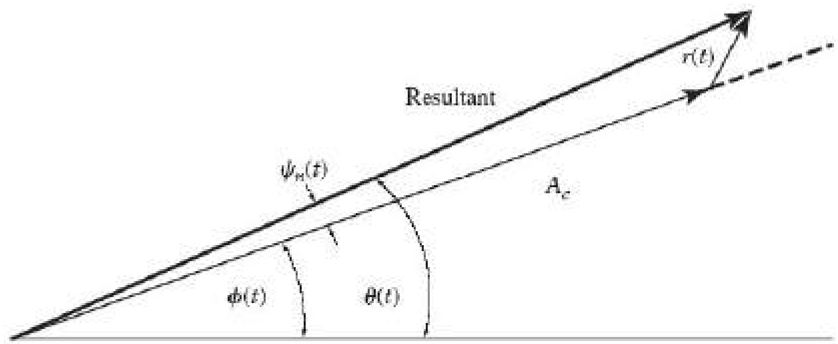


FIGURE 9.15 Phasor diagram for FM signal plus narrowband noise assuming high carrier-to-noise ratio.

- Under this condition, and noting that $\tan^{-1}\xi \approx \xi \sin ce \xi << 1$, the expression for the phase simplifies to

$$\theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t)] \quad (9.49)$$

- Then noting that the quadrature component of the noise is

$n_Q(t) = r(t)\sin[\phi_n(t)]$, we may simplify Eq.(9.49) to

$$\theta(t) = \phi(t) + \frac{n_Q(t)}{A_c} \quad (9.50)$$

$$\theta(t) \approx 2\pi k_f \int_0^t m(\tau)d\tau + \frac{n_Q(t)}{A_c} \quad (9.51)$$

- The ideal discriminator output

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$
$$= k_f m(t) + n_d(t) \quad (9.52)$$

- The noise term $n_d(t)$ is defined by

$$n_d(t) = \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt} \quad (9.53)$$

- The additive noise at the discriminator output is determined essentially by the quadrature component $n_Q(t)$ of the marrowband noise $n(t)$.

$$G(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c} \quad (9.54)$$

- The power spectral density $S_{N_Q}(f)$ of the quadrature noise compinent $n_Q(t)$ as follows;

$$S_{N_d}(f) = |G(f)|^2 S_{N_Q}(f)$$

$$= \frac{f^2}{A_c^2} S_{N_Q}(f) \quad (9.55)$$

- Power spectral density of the noise $n_d(t)$ is shown in Fig.9.16

$$S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}, & |f| < \dfrac{B_T}{2} \\ 0, & \text{otherwise} \end{cases} \quad (9.56)$$

- Therefore, the power spectral density $S_{N_o}(f)$ of the noise $n_0(t)$ appearing at the receiver output is defined by

$$S_{N_o}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}, & |f| < \mathrm{W} \\ 0, & \text{otherwise} \end{cases} \quad (9.57)$$

$$\text{Average post - detection noise power} = \frac{N_0}{A_c^2}\int_{-\mathrm{W}}^{\mathrm{W}} f^2 df$$

$$= \frac{2N_0\mathrm{W}^3}{3A_c^2} \quad (9.58)$$

Fig. 9.16

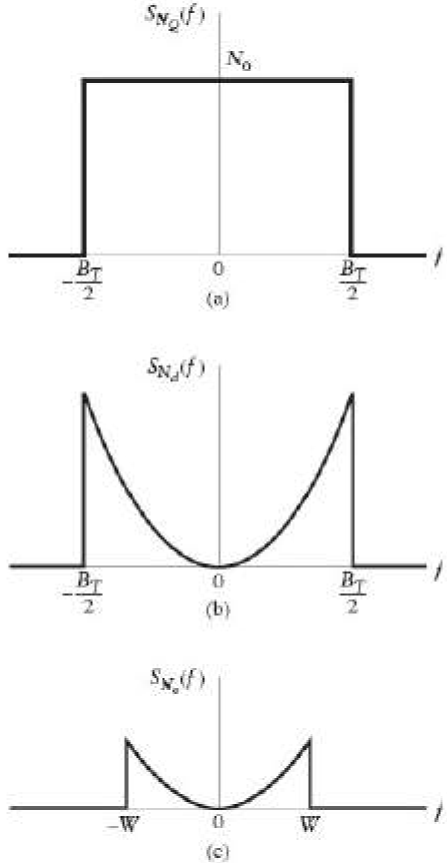


FIGURE 9.16 Noise analysis of FM receiver. (*a*) Power spectral density of quadrature component $n_Q(t)$ of narrowband noise $n(t)$. (*b*) Power spectral density $n_d(t)$ at discriminator output. (*c*) Power spectral density of noise $n_o(t)$ at receiver output.

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$$\text{SNR}_{\text{post}}^{\text{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 \text{W}^3} \quad (9.59)$$

- Figure of merit

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}} = \frac{\dfrac{3A_c^2 k_f^2 P}{2N_0 \text{W}^3}}{\dfrac{A_c^2}{2N_0 \text{W}}}$$

$$= 3\left(\frac{k_f^2 P}{\text{W}^2}\right)$$

$$= 3D^2 \qquad (9.60)$$

- The figure of merit for an FM system is approximately given by

$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{\text{W}}\right)^2 \quad (9.61)$$

- Thus, when the carrier to noise level is high, unlike an amplitude modulation system an FM system allows us to trade bandwidth for improved performance in accordance with square law.

Comparison between different Modulation Systems with respect to FOM

S.No.	Modulation System	FOM
1.	DSB-SC	2
2.	SSB-SC	1
3.	AM with Envelope Detector	$2\ \frac{\overline{m^2(t)}}{A_c^2+\overline{m^2(t)}}$
4.	AM with Square Law Detector	$2\ \frac{\overline{m^2(t)}}{A_c^2+\overline{m^2(t)}} \times \frac{1}{1+\frac{\overline{m^2(t)}}{A_c^2}}$
5.	FM	$\frac{3}{4\pi^2} m_f^3$