

The objects which make up a set are called its members or elements.

If the number of elements in a set is finite then the set is finite set.

If a set has infinitely many elements, then it is infinite set.

A set having only one element is called singleton set.

* A Set is any well defined collection of objects. The sets are denoted by capital letters

Ex - $A = \{ \text{Students of MCA I Semesters} \}$
 $B = \{ 1, 2, 3, 4 \}$

- If x is an element of the set A , it is written as $x \in A$.
- If x does not belong to A , it is written as $x \notin A$.

Representation of a set -

Let A be a set. $P(x)$ be a property that elements of the set have.

then $A = \{ x / P(x) \}$.

i.e. every element of the set A is identified by variable x .

Ex: - $A = \{x \mid x \text{ is a positive integer less than } 5\}$
or $A = \{1, 2, 3, 4\}$

$A = \{x \mid x \text{ is a vowels}\}$
or $A = \{a, e, i, o, u\}$

Let $C = \{1, 8, 27, 64, 125\}$
 $C = \{x \mid x \text{ is a positive integer and } x^3 < 216\}$

NULL SET.

A set which has no elements is called null set. It is denoted by $\phi = \{\}$

Ex. $\phi = \{x \mid x \text{ is a positive integer } < 10 \text{ and divisible by } 11\}$
 $\therefore \phi = \{\}$

Ex. $\phi = \{x \mid x \text{ is a real number and } x^2 = -1\}$
 $\therefore \phi = \{\}$

EQUALITY OF SETS

Two sets A & B are equal if they have same elements. It is written as $A = B$.

Ex. Let $A = \{\text{Dos, windows, Unix}\}$
 $B = \{\text{window, Unix, Dos}\}$

$\therefore A = B$

Ex If $A = \{1, 2, 3, 4\}$
 $\& B = \{x \mid x \text{ is a true integer \& less than } 5\}$
 then $A = B$.

SUB SETS :- Given 2 sets $A \& B$, A is a subset of B or we say that A is contained in B if every element of A is an element of B .

- If A contains an element which is not in B then A is not a subset of B .
- Subset is denoted as " $A \subseteq B$ "
- A is not a subset of B then - " $A \not\subseteq B$ ".

Ex Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$ $C = \{2, 3, 5\}$

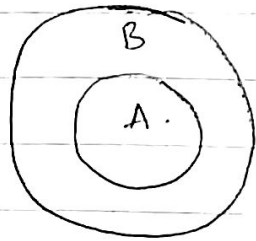
$\Rightarrow \underline{A \subseteq B}$ and $\underline{C \subseteq B}$ But
 $\underline{A \not\subseteq C}$.

PROPER Subset :- A set A is a proper subset of B if (i) $A \subseteq B$ (ii) B posses atleast one element that is not in A .

We can denote proper subset as " $\underline{A \subset B}$ "

VENN DIAGRAMS.

Relationships between sets can be depicted in diagrams called "Venn diagrams".



A is a subset of B
 $A \subset B$

Universal set :-

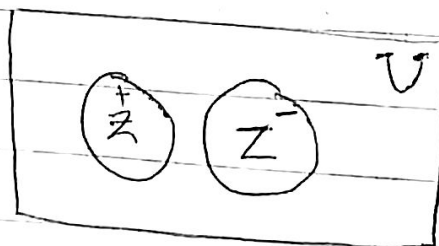
A set which contains all the objects which are in discussion is called a Universal set.

It is denoted by U .

Let $U = \{x \mid x \text{ is a number or integer}\}$
then $\mathbb{Z}^+ =$ Set of all +ve integers
 $\mathbb{Z}^- =$ Set of all -ve integers.

$\therefore \mathbb{Z}^+ \& \mathbb{Z}^-$ are subsets of U .

Venn diagram is denoted as :-



FINITE SET :- If set A has finite number of elements say n , then it is called "finite set".

Cardinality of Set :-

Cardinality of a set A gives the number of distinct elements of A . It is denoted by $|A|$.
- If the set A has n distinct elements, then $|A| = n$.

EX Let $A_1 = \{a, b, c, d\}$ $A_2 = \{a, d, e, f, g\}$

$A_3 = \{a, d, c\}$

$|A_1| = 4$, $|A_2| = 5$, $|A_3| = 3$.

$\therefore \{a, d, c\} = A_3$ is the set of smallest cardinality of A .

POWER SET

If A is a set, the set of all subsets of A is called the power set of A .
- It is denoted by $P(A)$.

EX Let $A = \{a, b, c\}$

Power set $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

EQUIPOTENT SETS

Let A & B are two sets. A and B are called equipotent sets if they have same cardinality.

It is denoted as $A \sim B$.

OPERATIONS ON SETS.

The important operations on sets are Union, Intersection, Symmetric difference and Complement.

Union of two sets.

Let A and B are two sets.

The Union of A & B is defined as the set containing all elements that belongs to A or B .

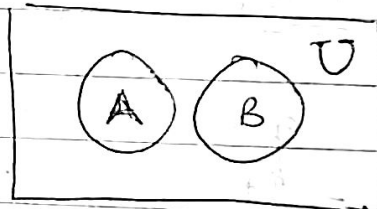
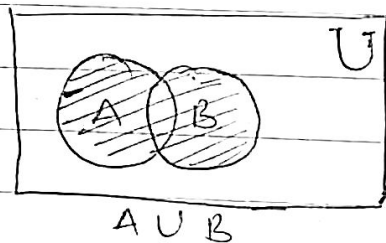
- It is denoted by $A \cup B$.

$$\text{ie } A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Ex $A = \{a, b, c, d, e, f\}$ $B = \{b, d, e, s\}$

$$A \cup B = \{a, b, c, d, e, f, s\}$$

Venn diagram:



INTERSECTION OF TWO SETS

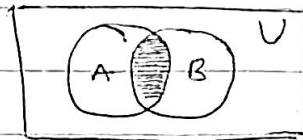
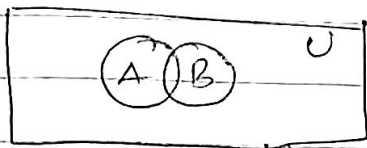
Let A and B be two sets. The intersection of A and B is defined as the set containing all elements that belongs to both A and B .

It is denoted by $A \cap B$.

Let $A = \{a, b, c, e, f\}$ $B = \{b, e, f, s\}$

$$A \cap B = \{b, e, f\}$$

Venn Diagrams



$A \cap B$

DISJOINT SETS

If two sets A and B have no common elements then they are called Disjoint sets.

Ex $A = \{1, 2, 3, 4\}$ $B = \{6, 7, 8, 9\}$

$$A \cap B = \{\phi\}$$

$\therefore A$ & B are disjoint sets.

i.e. If 2 sets are disjoint then their intersection set is an empty set.

SYMMETRIC DIFFERENCE OF TWO SETS

Let A and B be two sets. The symmetric difference of A & B is the set of all elements that belong to A or B but not both.

It is denoted by $A \oplus B$.

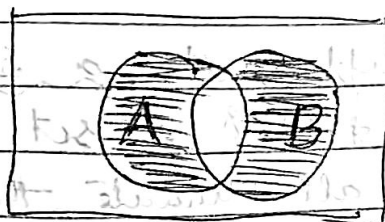
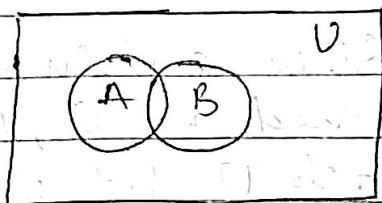
$$\text{i.e. } A \oplus B = \{x / (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$$

Ex Let $A = \{1, 2, 3, 4, 5\}$ $B = \{4, 5, 6, 7\}$

$$A \oplus B = \{1, 2, 3, 6, 7\}$$

$$B \oplus A = \{6, 7, 1, 2, 3\}$$

Venn Diagram



$A \oplus B$

(Difference of 2 sets).

Relative COMPLEMENT OF TWO SETS

Let $A \subset B$ be 2 sets. Complement of B with respect to A is the set of all elements that belong to A but not to B .

It is denoted by $A - B$.

$$\text{i.e. } A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

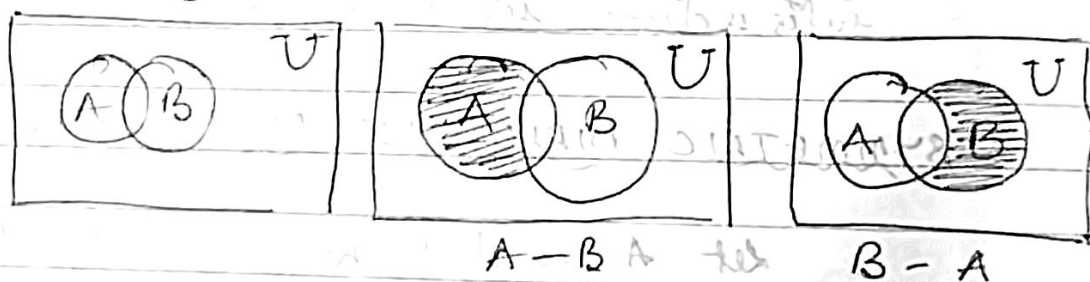
$$\& B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

Ex: Let $A = \{a, b, c\}$ & $B = \{b, c, d, e\}$

$$A - B = \{a\}$$

$$B - A = \{d, e\}$$

Venn diagram



Symmetric Difference

For 2 sets A & B , the Relative Complement of $A \cap B$ in $A \cup B$ is called symmetric difference of A & B & denoted by $A \Delta B$.

$$\therefore A \Delta B = (A \cup B) - (A \cap B)$$

$$= \{x \mid x \in A \cup B \text{ and } x \notin A \cap B\}$$



Complement of a set :- Given a Universal set U & a set A contained in U , the set of all elements that belong to U but not A is called complement of A & denoted as $\bar{A}$.

$$\text{i.e. } \bar{A} = \{x \mid x \in U \text{ and } x \notin A\}$$

DE MORGAN'S LAW

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Let A & B are 2 sets, Prove that

$$(i) \overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$(ii) \overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$(i) \overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\text{As RHS} = \bar{A} \cap \bar{B} \Rightarrow \{x \mid x \in \bar{A} \text{ and } x \in \bar{B}\}$$

$$\Rightarrow \{x \mid x \notin A \text{ and } x \notin B\}$$

$$\Rightarrow \{x \mid x \notin (A \cup B)\}$$

$$\Rightarrow \overline{A \cup B} = \text{LHS}$$

$$\therefore \overline{A \cup B} = \bar{A} \cap \bar{B}$$

((OR))

$$(ii) \text{ Let } x \in \overline{A \cap B}$$

By the definition of complement of a set.

$$x \notin A \cap B \Rightarrow x \notin A \text{ and } x \notin B$$

$$\Rightarrow x \in \bar{A} \text{ and } x \in \bar{B}$$

$$\Rightarrow x \in (\bar{A} \cap \bar{B})$$

$$\Rightarrow \overline{A \cap B} \subseteq \bar{A} \cap \bar{B} \longrightarrow (1).$$

$$(ii) \overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$\text{as RHS} = \bar{A} \cup \bar{B}$$

$$= \{x \mid x \in \bar{A} \text{ or } x \in \bar{B}\}$$

$$= \{x \mid x \notin A \text{ or } x \notin B\}$$

$$= \{x \mid x \notin (A \cap B)\}$$

$$= \overline{A \cap B} = \text{LHS}$$

$$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$$

((OR))

$$\text{Let } x \in \overline{A \cap B}$$

$$\Rightarrow x \in \bar{A} \text{ and } x \in \bar{B}$$

$$\Rightarrow x \notin A \text{ and } x \notin B$$

$$\Rightarrow x \notin (A \cup B)$$

$$\Rightarrow x \in (\overline{A \cup B})$$

$$\text{Thus } \overline{A \cap B} \subseteq \overline{A \cup B} \longrightarrow \textcircled{2}$$

from ① & ②

$$\overline{A \cup B} = \overline{A \cap B}$$

$$\overline{A \cap B} = \overline{A \cup B} \therefore$$

((or))

$$\overline{A \cup B} = \overline{A \cap B}$$

Let the definition of intersection with set

$$x \in A \cap B \Leftrightarrow x \in A \text{ and } x \in B$$

$$\Rightarrow x \in A \text{ and } x \in B \Leftrightarrow$$

$$\Rightarrow x \in (A \cap B)$$

$$\textcircled{1} \leftarrow \overline{A \cap B} \subseteq \overline{A \cup B} \Leftrightarrow$$

$$\overline{A \cup B} = \overline{A \cap B} \quad \textcircled{ii}$$

$$\overline{A \cup B} = \overline{A \cap B}$$

$$\{x \in U \mid x \in A \text{ and } x \in B\} =$$

$$\{x \in U \mid x \in A \cap B\} =$$

Ex

Among 100 students in a class,
32 study mathematics, 20 study physics
45 study biology, 7 study maths &
physics, 10 study physics & biology; 30
do not study any one of them, 15 study
maths & biology.

- i) Find no. of students studying all subjects
- ii) Find no. of students studying exactly one subject.

Sol

Let $|A| = 32$ — study maths

$|B| = 20$ — " physics

$|C| = 45$ — " biology

$|A \cap B| = 7$ — maths & physics

$|A \cap C| = 15$ — maths & biology

$|B \cap C| = 10$ — physics & biology

30 = students who do not study any one subject

$\therefore$ No. of students studying at least one subject

$$= 100 - 30 = 70$$

$$\therefore |A \cup B \cup C| = 70$$

$$i) |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$70 = 32 + 20 + 45 - 7 - 15 - 10 + |A \cap B \cap C|$$

$$\Rightarrow |A \cap B \cap C| = 5 \quad (\text{No. of students study all 3 subjects})$$

ii) No. of students studying exactly one subject = (No. of students study at least one subject) - (No. of students study at least 2 subjects)

$$= |A \cup B \cup C| - (|A \cap B| + |A \cap C| + |B \cap C| - 2|A \cap B \cap C|)$$

$$= 70 - \{7 + 15 + 10 - 2 \cdot 5\}$$

$$= 70 - \{7 + 15 + 10 - 10\}$$

$$= 48 //$$

The Laws of Set Theory

1) Commutative laws.

$$A \cup B = B \cup A \quad \& \quad A \cap B = B \cap A.$$

2) Associative laws.

$$A \cup (B \cap C) = (A \cup B) \cap C \quad \& \quad A \cap (B \cup C) = (A \cap B) \cup C$$

3) Distributive laws.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

4) Idempotent laws

$$A \cup A = A \quad \& \quad A \cap A = A.$$

5) Identity laws.

$$A \cup \phi = A \quad \& \quad A \cap U = A \quad \& \quad A \cap \phi = \phi.$$

6) Law of double complement

$$\overline{\overline{A}} = A \quad \& \quad A \cap \phi = U \quad \& \quad \overline{U} = \phi.$$

7) Inverse laws.

$$A \cup \overline{A} = U \quad \& \quad A \cap \overline{A} = \phi.$$

8) De Morgan's laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

9) Universal set Properties

$$A \cup U = U \quad \& \quad A \cap U = A$$

$$A \cap U = A \quad \& \quad A \cup \phi = A$$

For any 2 sets $A \neq B$, prove the following

i) $A - B = A \cap \bar{B}$

$$A - B = \{x \in U \mid x \in A \text{ \& \& } x \notin B\}$$

$$= \{x \in U \mid x \in A \text{ \& \& } x \in \bar{B}\}$$

$$= A \cap \bar{B} = \text{RHS.}$$

ii) $\overline{A - B} = (\bar{A} \cup B) \cup (\bar{A} \cap A) = (\bar{A} \cup B) \cap A$

from (ex ①): $\bar{A} \cup B = A \cap \bar{B}$

LHS $\Rightarrow \overline{A - B} = A \cap \bar{B}$
 $\Rightarrow \overline{A \cap \bar{B}} \Rightarrow \bar{A} \cup \bar{\bar{B}} \quad (\text{By De Morgan's law})$

$$\Rightarrow \bar{A} \cup B = \text{RHS.}$$

iii) $A - B = \bar{A} \cup (A \cap \bar{B})$

from eqn ① $A - B = A \cap \bar{B}$

Let RHS $\Rightarrow \bar{A} \cup (A \cap \bar{B})$

$$\Rightarrow \bar{A} \cup (A \cap \bar{B})$$

$$\Rightarrow (\bar{A} \cup \bar{A}) \cup (A \cap \bar{B})$$

$$\Rightarrow \bar{A} \cup (A \cap \bar{B}) \quad \bar{A} \cup \bar{A} = \bar{A}$$

$$\Rightarrow A \cap \bar{B} \Rightarrow A - B = \text{LHS.}$$

(iv) $\overline{A - B} = \bar{A} \cup (A \cap \bar{B})$

from (iii) $\overline{A - B} = \bar{A} \cup (A \cap \bar{B})$

$$\Rightarrow \bar{A} \cup (A \cap \bar{B}) \cup \bar{A}$$

$$\Rightarrow \bar{A} \cup (A \cap \bar{B}) \cup \bar{A}$$

$$= \text{RHS.}$$

Ex Prove - the following

$$(A-B)-C = A-(B \cup C) = (A-C)-(B-C)$$

Sol $\Rightarrow (A-B)-C = (A \cap \bar{B}) - C$

$$\Rightarrow A \cap (\bar{B} \cap \bar{C}) \quad (\because \text{associative})$$

$$\Rightarrow A \cap (\overline{B \cup C}) \quad (\because \text{De Morgan's law})$$

$$\Rightarrow A - (B \cup C) = \text{RHS.}$$

Also let $(A-C) - (B-C)$ $(\because A-B = A \cap \bar{B})$

$$\Rightarrow (A-C) \cap (\overline{B-C})$$

$$\Rightarrow (A \cap \bar{C}) \cap (\overline{B \cap \bar{C}})$$

$$\Rightarrow (A \cap \bar{C}) \cap (\bar{B} \cup C) \quad [\text{using distributive law}]$$

$$\Rightarrow [(A \cap \bar{C}) \cap \bar{B}] \cup [(A \cap \bar{C}) \cap C]$$

$$\Rightarrow [A \cap (\bar{C} \cap \bar{B})] \cup [A \cap (\bar{C} \cap C)]$$

$$\Rightarrow [A \cap (\overline{C \cup B})] \cup [A \cap \phi] \quad (1)$$

$$\Rightarrow [A \cap (\overline{B \cup C})] \cup \phi$$

$$\Rightarrow A \cap (\overline{B \cup C})$$

$$\Rightarrow \underline{\underline{A - (B \cup C)}}$$

$$(A \cap \bar{B}) \cap (\bar{A} \cap \bar{A}) \Leftarrow$$

$$(A \cap \bar{B}) \cap \bar{A} \Leftarrow$$

$$(A \cap \bar{B}) \cap A \Leftarrow$$

$$(A \cap \bar{B}) \cap (A \cap A) \Leftarrow$$

$$(A \cap \bar{B}) \cap A \Leftarrow$$

Using laws of set-theory, simplify following

i) $A \cap (B - A)$ ii) $(A - B) \cup (A \cap B)$

iii) $(\bar{A} \cup \bar{B}) \cup (A \cap B \cap \bar{C})$

iv) $(A \cup B) \cap C \cup \bar{B}$

psq:- i) $A \cap (B - A)$

$$\Rightarrow A \cap (B \cap \bar{A}) \quad (\because A - B = A \cap \bar{B})$$

$$\Rightarrow A \cap (\bar{A} \cap B) \quad (\because \text{commutative law})$$

$$\Rightarrow (A \cap \bar{A}) \cap B$$

$$\Rightarrow \phi \cap B$$

$$\Rightarrow \phi$$

ii) $(A - B) \cup (A \cap B)$

$$\Rightarrow (A \cap \bar{B}) \cup (A \cap B)$$

$$\Rightarrow A \cap (B \cup \bar{B})$$

$$\Rightarrow A \cap U$$

$$\Rightarrow A$$

→ [using distributive law]

iii) $(\bar{A} \cup \bar{B}) \cup (A \cap B \cap \bar{C})$

$$\Rightarrow (\bar{A} \cup \bar{B}) \cup [(A \cap B) \cap \bar{C}]$$

$$\Rightarrow [(\bar{A} \cup \bar{B}) \cup (A \cap B)] \cap [(\bar{A} \cup \bar{B}) \cup \bar{C}]$$

$$\Rightarrow U \cap (\bar{A} \cup \bar{B} \cup \bar{C})$$

$$\Rightarrow \bar{A} \cup \bar{B} \cup \bar{C}$$