

RULES OF INFERENCE

Consider a set of propositions $P_1, P_2, \dots, P_n$ and a proposition Q . Then a compound proposition of the form

$$(P_1 \wedge P_2 \wedge P_3 \wedge P_4 \dots \wedge P_n) \rightarrow Q.$$

is called argument.

$P_1, P_2, \dots, P_n$ are called premises of the argument and Q is called conclusion of the argument.

→ Arguments written in tabular form as:

P_1

P_2

P_3

$\vdots$

P_n

$$\hline \therefore Q$$

The preceding argument is valid if each of the premises $P_1, P_2, \dots, P_n$ is true; then the conclusion Q is true.

Rules of Inference: for testing valid arguments.

Rule of Conjunctive Simplification.

1)

For any 2 propositions P & Q , if $P \wedge Q$ is true then P is true i.e.

$$(P \wedge Q) \Rightarrow P \quad (P \wedge Q) \Rightarrow Q$$

(Rule of conjunction)

2) Rule of Disjunctive Amplification

For any 2 propositions p & q , if p is true then $p \vee q$ is true i.e

$$p \Rightarrow p \vee q$$

(Rule of disjunction)

3) Rule of Syllogism:-

For any 3 propositions, p, q, r , if $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true i.e

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$$

(Rule of transitivity) This can be expressed in tabular form as

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

4) Modus Ponens (Rule of Detachment)

If p is true and $p \rightarrow q$ is true then q is true i.e $[p \wedge (p \rightarrow q)] \Rightarrow q$

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

5) Modus Tollens

If $p \rightarrow q$ is true and q is false, then p is false i.e

$$[(p \rightarrow q) \wedge \sim q] \Rightarrow (\sim p)$$

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

→
tabular
form →

(contradiction of p)

6) Rule of Disjunctive Syllogism.

If $p \vee q$ is True and p is false,
then q is true. i.e.

$$\therefore [(p \vee q) \wedge \sim p] \Rightarrow q$$

Can express in Tabular form as:

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

Ex 1 Test whether following is valid.

a) If Sachin hits a Century then he gets a free car.

Sachin hits a Century

$\therefore$ Sachin gets a free car.

Sol

Let p : Sachin hits a Century

q : Sachin gets a free car

the argument can be read as

$$p \rightarrow q$$

$$p$$

$$\therefore q$$

From Rule of Modus Ponens, the given statement is valid.

b) If Sachin hits a century, he gets a free car.
Sachin gets a free car

$\therefore$ Sachin hits a century.

Sol Let p : Sachin hits century

q : Sachin gets a free car.

argument can be read as

$$\begin{array}{c} p \rightarrow q \\ q \end{array}$$

$$\hline \therefore p$$

If $p \rightarrow q$ and q are true, then it is not that p must be true.

ie

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$
0	1	1	1

$[(p \rightarrow q) \wedge q]$ is not a tautology.

$\therefore$ given argument is not a valid argument.

Ex I will become famous or I will not become musician.

I will become a musician.

$\therefore$ I will become famous.

Sol

Let

p : I will become famous

q : I will become musician.

Given argument can be read as

$$p \vee \neg q$$

$$q$$

$$\hline \therefore p$$

as $p \vee \neg q \iff \neg q \vee p \iff q \rightarrow p$.

$\therefore$ we can write $q \rightarrow p$

$$q$$

$$\hline \therefore p$$

From Modus ponens rule.

argument is valid.

Ex If I study, then I do not fail in Exam
 If I do not fail in Exam, my father gives me a gift.
 $\therefore$ If I study, my father gives me a gift

$$\begin{array}{l} \text{ie } p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array} \quad \text{Rule of Syllogism.}$$

Ex I will get FCD in MCA or I will ^{not} join s/w comp
 If I do not join s/w comp, I will do business
 I got FCD
 $\therefore$ I will not do business P:

check for validity

Sol Let p : I will get FCD in MCA
 q : I will not join s/w company
 r : I will do business

Now, argument can be read as

$$\begin{array}{l} p \vee q \\ q \rightarrow r \\ p \end{array}$$

$$\therefore r$$

$p \vee q$ is equivalent to $q \vee p \Rightarrow \neg q \rightarrow p$

$$\neg q \rightarrow p$$

$$\therefore \neg r \rightarrow \neg q \quad (\text{using contrapositive})$$

Construct truth table

p	q	r	$s = (p \vee q)$	$t = (q \rightarrow r)$	st	$st \wedge p$	$\neg st$
0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	1
0	1	1	1	1	1	0	0
1	0	0	1	1	1	1	0
1	0	1	1	1	1	1	0
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	0

It is clear that from truth table when

$(p \vee q)$ and $(q \rightarrow r)$ and p are 1, it may not be that $\neg st$ is always true.

Hence the given statement is not valid.

$$\begin{array}{l} \text{Ex} \\ \underline{p \rightarrow r} \\ \underline{q \rightarrow r} \end{array}$$

$$\therefore (p \vee q) \rightarrow r$$

$$\begin{aligned} \text{Let } (p \rightarrow r) \wedge (q \rightarrow r) &\Leftrightarrow (\neg p \vee r) \wedge (\neg q \vee r) \\ &\Leftrightarrow (r \vee \neg p) \wedge (r \vee \neg q) \quad [\text{comm. law}] \\ &\Leftrightarrow r \vee (\neg p \wedge \neg q) \quad [\text{distr. law}] \\ &\Leftrightarrow \neg(p \vee q) \vee r \quad [\because \text{de m. law}] \\ &\Leftrightarrow (p \vee q) \rightarrow r \quad [\because p \rightarrow q \Leftrightarrow \neg p \vee q] \end{aligned}$$

Hence the given argument is valid.

Prove the validity of given arguments

$$\begin{array}{l} \text{Ex} \\ p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

$$\begin{aligned} \text{Sol} \quad & \text{Let } (p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) \\ & \Leftrightarrow (p \rightarrow q) \wedge (\neg p \rightarrow s) \wedge (s \rightarrow s) \\ & \Leftrightarrow (p \rightarrow q) \wedge (\neg p \rightarrow s) \quad [\because \text{Rule of syllogism}] \\ & \Leftrightarrow (\neg q \rightarrow \neg p) \wedge (\neg p \rightarrow s) \quad [\because \text{contrapositive}] \\ & \Leftrightarrow (\neg q \rightarrow s) \quad [\because \text{Rule of syllogism}] \\ & \checkmark \Leftrightarrow (q \vee s) \end{aligned}$$

$$\begin{array}{l} \text{Ex} \\ p \rightarrow q \\ r \rightarrow s \\ \hline \neg q \vee \neg s \\ \hline \therefore \neg(p \wedge r) \end{array}$$

$$\begin{array}{l} \text{Ex} \\ p \rightarrow r \\ \neg p \rightarrow q \\ \hline q \rightarrow s \\ \hline \therefore \neg r \rightarrow s \end{array}$$

$$\begin{array}{l} \text{Ex} \\ (\neg p \vee \neg q) \rightarrow (r \wedge s) \\ r \rightarrow t \\ \hline \neg t \\ \hline \therefore p \end{array}$$

$$\begin{aligned} \text{Sol} \quad & \text{Let } [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge (r \rightarrow t) \wedge (\neg t) \\ & \Leftrightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge \neg r \vee (t \wedge \neg t) \quad [\text{by using Tollen Rule}] \\ & \Leftrightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge \neg r \\ & \Leftrightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge (\neg r \vee \neg s) \\ & \Leftrightarrow [(\neg p \vee \neg q) \rightarrow (r \wedge s)] \wedge \neg(r \wedge s) \quad [\because \text{Rule of disjunctive addition}] \\ & \Leftrightarrow \neg(\neg p \vee \neg q) \quad [\text{by rule of modus Tollens}] \\ & \Leftrightarrow \neg(\neg(p \wedge q)) \Rightarrow p \wedge q \Rightarrow p \quad \checkmark \\ & \quad (\text{Rule of conjunctive simplification}) \end{aligned}$$

$$\begin{array}{l} \text{Ex} \\ p \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ \hline p \\ \therefore r \end{array}$$

$$\begin{aligned} \text{Sol} \quad & [p \rightarrow (q \rightarrow r)] \wedge (\neg q \rightarrow \neg p) \wedge p \\ & \Leftrightarrow [[p \rightarrow (q \rightarrow r)] \wedge p] \wedge [\neg q \rightarrow \neg p] \\ & \Leftrightarrow (q \rightarrow r) \wedge (p \rightarrow q) \quad [\text{By modus ponens}] \\ & \Leftrightarrow q \rightarrow (p \rightarrow q) \wedge (q \rightarrow r) \quad [\text{contrapositive}] \\ & \Leftrightarrow q \rightarrow (p \rightarrow q) \wedge (q \rightarrow r) \quad [\text{commutative}] \\ & \Leftrightarrow p \rightarrow r \quad [\text{By Rule of syllogism}] \\ & \Leftrightarrow r \quad [\text{is a conclusion, since } p \text{ is true}] \end{aligned}$$

$$\begin{array}{l} \text{Ex} \\ (\neg p \leftrightarrow q) \\ q \rightarrow r \\ \hline \neg r \\ \therefore p \end{array}$$

$$\begin{aligned} \text{Sol} \quad & (\neg p \leftrightarrow q) \wedge (q \rightarrow r) \wedge (\neg r) \\ & \Leftrightarrow (\neg p \leftrightarrow q) \wedge \neg q \quad [\text{Modus Tollens}] \\ & \Leftrightarrow [(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)] \wedge \neg q \\ & \checkmark \Leftrightarrow [(\neg p \rightarrow q) \wedge \neg q] \wedge (q \rightarrow \neg p) \quad [\text{asso. in}] \\ & \Leftrightarrow [(\neg p \rightarrow q) \wedge \neg q] \quad [\text{Rule of conj. simplif.}] \\ & \Leftrightarrow \neg(\neg p) \quad [\text{modus Tollens Rule}] \\ & \Leftrightarrow p \end{aligned}$$

Here the given argument is valid.

Ex

$$(\neg P \vee q) \rightarrow r$$

$$r \rightarrow (s \vee t)$$

$$\neg s \wedge \neg u$$

$$\neg u \rightarrow \neg t$$

$$\therefore p$$

Ex

$$p \rightarrow r$$

$$r \rightarrow s$$

$$t \vee \neg s$$

$$\neg t \vee u$$

$$\neg u$$

$$\therefore \neg p$$

Ex

$$p \rightarrow q$$

$$q \rightarrow (r \wedge s)$$

$$\neg r \vee (\neg t \vee u)$$

$$p \wedge t$$

$$\therefore u$$

Ex

$$p \rightarrow r$$

$$r \rightarrow t$$

$$\neg t \rightarrow (\neg p \vee \neg q)$$

Ex

If Ravi goes out with friends, he will not study.
If Ravi does not study, his father becomes angry.
His father is not angry.

$\therefore$ Ravi has not gone out with friends.

Ex

If there is strike by students, the examination will be postponed.

The exam was not postponed.

$\therefore$ There was no strike by students.

Ex

If Ravi studies, then he will pass in DM.

If Ravi does not play cricket, then he will study.
Ravi failed in DM.

$\therefore$ Ravi played cricket.