Signals and Systems UNIT 2

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- Used to predict long-term behavior in a system
- The behavior of an LTI system is completely defined by its impulse response

Impulse Function

The discrete version of impulse function is defined by

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

The continuous time version of impulse function,

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Impulse Response

- The impulse response" of a system, $h[n]$, is the output that it produces in response to an impulse input. Definition: if and only if $x[n] = \delta[n]$ then $y[n] = h[n]$

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- Given the system equation, the impulse response can be found out just by feeding $x[n] = \delta[n]$ into the system.

Impulse Response Example

- Consider the system

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

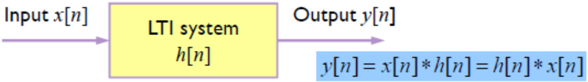
- Suppose we insert an impulse:

$$x[n] = \delta[n]$$

- Then whatever we get at the output, by Definition, is the impulse response. In this case it is

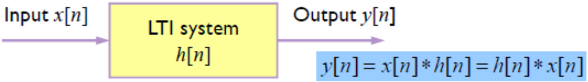
$$h[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) = \begin{cases} 0.5, & n = 0, 1 \\ 0, & otherwise \end{cases}$$

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
  x[n]=Input Signal

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
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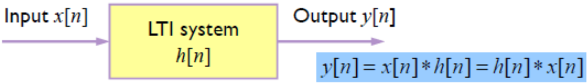
- $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
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- $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

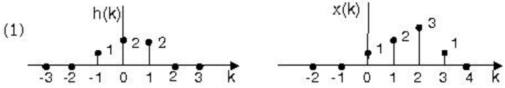
- input-excitation output-response

Convolution Sum Example

Find the response y[n] of following LTI system.

$$x(n)=[0,\underset{\uparrow}{1},2,3,1,0] \text{ and } h(n)=[0,1,\underset{\uparrow}{2},2,0]$$

Convolution Sum (Graphical method)



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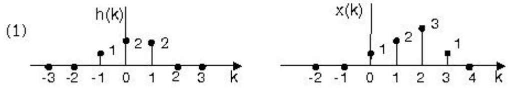
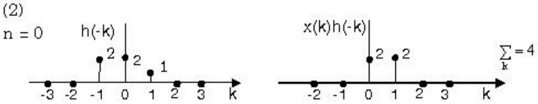
- 

- 

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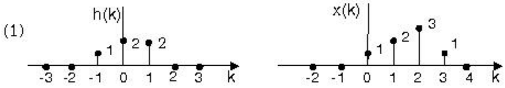
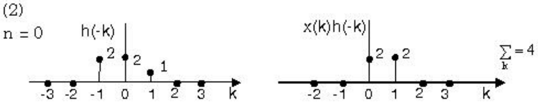
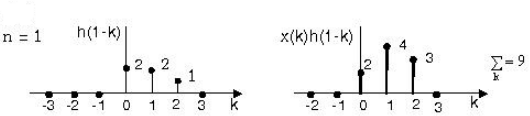
- 

- 

- 

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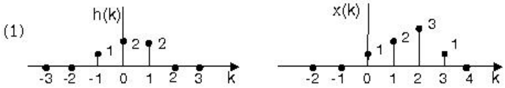
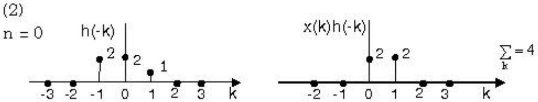
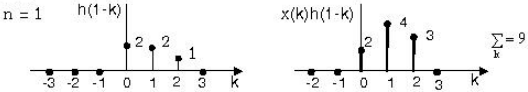
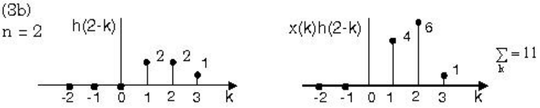
- 

- 

- 

- 

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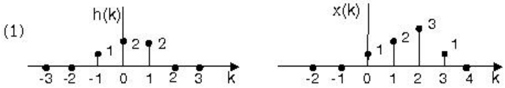
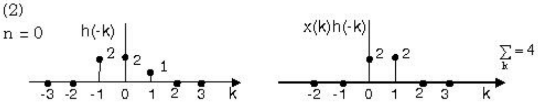
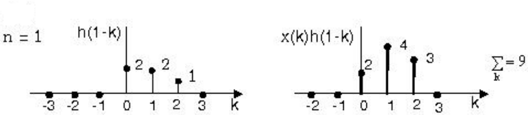
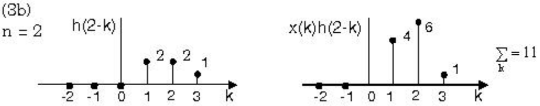
- 

- 

- 

- 

- $y(n) = [...0, 1, \underset{\uparrow}{4}, 9, 11, 8, 2, 0, ...]$

Convolution Sum (Analytical method)

- Size of x(n)=A=4, Size of h(n)=B=3
  Length of y(n)=A+B-1=4+3-1=6

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  $n_1 + n_2 = -1$, range of n=-1 to 4

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- For n=-1

$$y[-1] = \sum_{k=0}^{3} x[k]h[-1-k] =$$

$$x[0]h[-1] + x[1]h[-2] = (1x1) + (2x0) = 1$$

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- Likewise for all the values of n
  $y(n) = [...0, 1, \underset{\uparrow}{4}, 9, 11, 8, 2, 0, ...]$

Convolution Sum (Analytical method)

$$x_1(n) = [\underset{\uparrow}{1}, 2, 3]$$

$$x_2(n) = [\underset{\uparrow}{2}, 1, 4]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

- Size of $x_1(n)$=A=3, Size of $x_2(n)$=B=3
  Length of y(n)=A+B-1=3+3-1=5

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- Size of $x_1(n)$=A=3, Size of $x_2(n)$=B=3
  Length of y(n)=A+B-1=3+3-1=5
- $x_1(n)$ is starting from 0 index n_1=0
  $x_2(n)$ is starting from 0 index n_2=0
  $n_1 + n_2 = 0$, range of n=0 to 4

Convolution Sum (Analytical method)

- For n=0

$$y[0] = \sum_{k=0}^{2} x_1[k]x_2[-k] =$$

$$x[0]x_2[0] + x_1[1]x_2[-1] + x_1[2]x_2[-2]$$

$$= (1x2) + (2x0) + (3x0) = 2$$

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- Likewise for all the values of n $y(n) = [\underset{\uparrow}{2}, 5, 12, 11, 12]$

Convolution Sum

Convolute the given sequences
$x_1[n] = \alpha^n u[n]$ and $x_2[n] = \beta^n u[n]$

- $y[n] = x_1[n] * x_2[n]$

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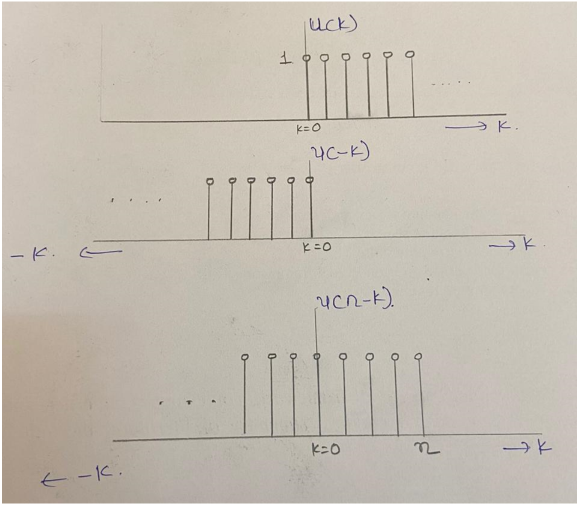
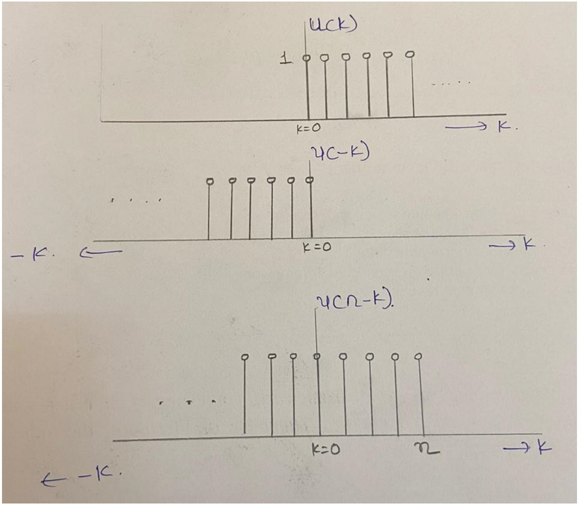
- $= \beta^n \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{-k} u[n-k]$
- $= \beta^n \sum_{k=-\infty}^{\infty} (\frac{\alpha}{\beta})^k u[k] u[n-k]$

Convolution Sum

- 

Convolution Sum

- $u[k]u[n-k] = 1, 0 \leq k \leq n, n \geq 0$



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- $$\frac{1}{\beta - \alpha}\left[\beta^{n+1} - \alpha^{n+1}\right]$$

Convolution Sum

The system is characterized by an impulse response

$$h[n] = (\frac{3}{4})^n u[n]$$

Find the step response of the system. Also evaluate the output of the system at $n = \pm 5$

- $y[n] = x[n] * h[n] = h[n] * x[n]$

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- $= \sum_{k=-\infty}^{\infty} (\frac{3}{4})^k u[k]u[n-k]$
- $= \sum_{k=0}^{n} \left(\frac{3}{4}\right)^k$
- $= \frac{(\frac{3}{4})^{n+1} - 1}{\frac{3}{4} - 1}$

Convolution Integral

- Convolution Integral between two continuous signals $x(t)$ and $h(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

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- $y(t)) = x(t) * h(t) = h(t) * x(t)$

Convolution Integral (Graphical Methods)

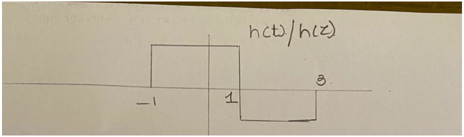
Suppose the input $x(t)$ and impulse response $h(t)$ of a LTI system are given by

$$x(t) = 2u(t-1) - 2u(t-3)$$

$$h(t) = u(t+1) - 2u(t-1) + u(t-3)$$

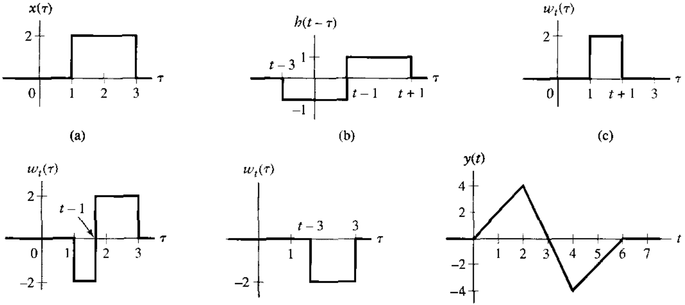
Convolution Integral

h(t)



Convolution Integral

Convolution :



Convolution Integral (Analytical Methods)

$$x(t) = e^{-2t}u(t)$$

$$h(t) = u(t+2)$$

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- $\int_{-\infty}^{\infty} e^{-2\tau}u(\tau)u(t-\tau+2)d\tau$
- Case 1: $t+2 < 0$, do not overlap hence zero.

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- $\int_{-\infty}^{\infty} e^{-2\tau}u(\tau)u(t-\tau+2)d\tau$
- Case 1: $t+2<0$, do not overlap hence zero.
- Case 2 $= \int_{0}^{t+2} e^{-2\tau} x 1 d\tau$
- $= [\frac{e^{-2\tau}}{-2}]_0^{t+2} = \frac{1}{2} - \frac{1}{2}e^{-2(t+2)}$

Convolution Integral (Analytical Methods)

$$x(t) = 2u(t-1) - 2u(t-3)$$

$$h(t) = u(t+1) - 2u(t-1) + u(t-3)$$

- $x(t) = 1, t = 1, 2, 3$

Convolution Integral (Analytical Methods)

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- $x(t) = 1, t = 1, 2, 3$
- $h(t) = \begin{cases} 1 & \text{for} \quad 1 \leq t \leq 3 \\ -1 & \text{for} \quad -1 \leq t < 1 \\ 0 & \quad\quad \textit{otherwise} \end{cases}$

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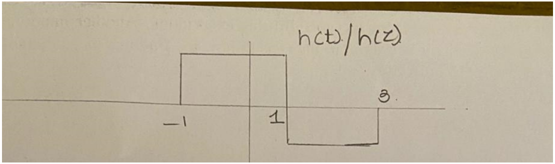
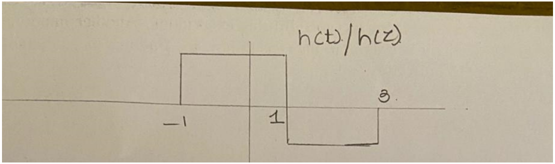
$$h(t) = u(t+1) - 2u(t-1) + u(t-3)$$

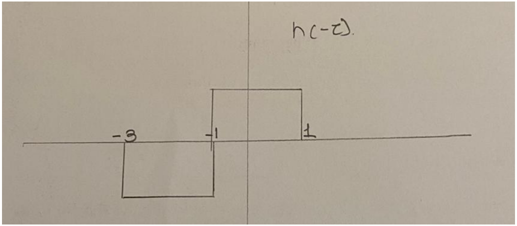
- $x(t) = 1, t = 1, 2, 3$
- $h(t) = \begin{cases} 1 & \text{for} \quad 1 \leq t \leq 3 \\ -1 & \text{for} \quad -1 \leq t < 1 \\ 0 & \quad\quad otherwise \end{cases}$
- $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Convolution Integral

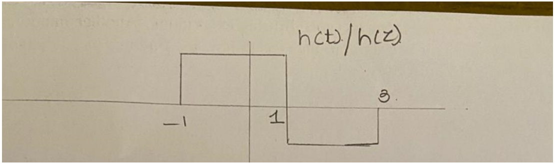
- 

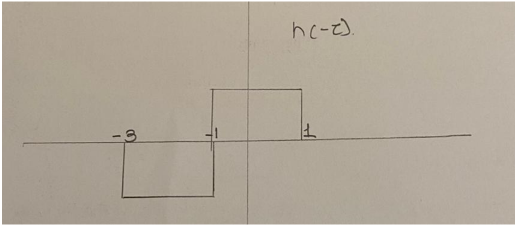
Convolution Integral

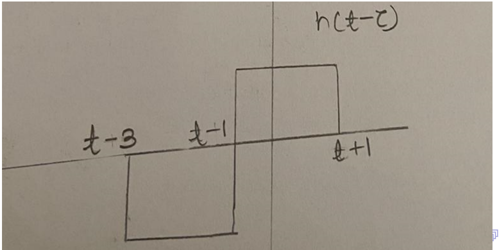
- 

- 

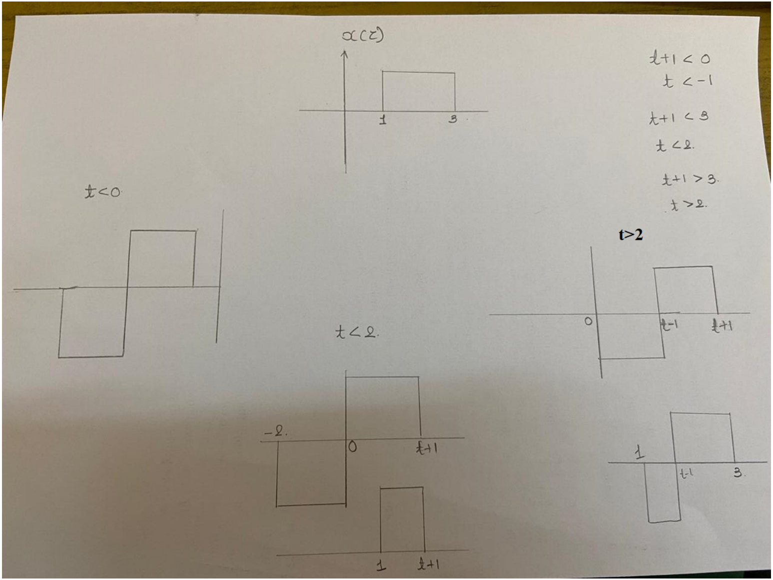
Convolution Integral

- 



- 

Convolution Integral



Step Response

Characterizes the response to sudden changes in input.

$$s[n] = h[n] * u[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]u[n-k]$$

u[n-k] exist from $-\infty$ to n

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

h[k] is the running sum of impulse response.

Step Response for Continuous time system

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

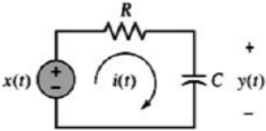
Relation b/w step response and impulse response.

$$h(t) = \frac{d}{dt}s(t)$$

$$h[n] = s[n] - s[n-1]$$

Eg. Step response of an RC ckt

$$h(t) = \frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$



Step Response Example

Find the step response for LTI system represented by impulse response $h[n] = (\frac{1}{2})^n u[n]$.

Ans:

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

$$s[n] = \sum_{k=0}^{n} (\frac{1}{2})^k$$

$$= \frac{1-(\frac{1}{2})^{n+1}}{1-\frac{1}{2}}, n \geq 0$$

Step Response Example

$$h(t) = tu(t)$$

Ans:

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

$$s(t) = \int_{0}^{t} \tau d\tau$$

$$= \frac{t^2}{2}, t \geq 0$$

Differential and Difference equation representation of LTI systems

Linear constant coefficient differential equation:

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

Linear constant coefficient difference equation:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Order of the equation is (N,M), representing the number of energy storage devices in the system.

Often N>M and the order is described using only 'N'.

Differential and Difference equation representation of LTI systems

Second order difference equation:

$$y[n] + y[n-1] + \frac{1}{4} y[n-2] = x[n] + 2x[n-1]$$

Difference equation are easily arranged to obtain recursive formulas for computing the current o/p of the system.

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \frac{1}{a_0} \sum_{k=0}^{M} b_k x[n-k] - \frac{1}{a_0} \sum_{k=1}^{N} a_k y[n-k] \qquad (1)$$

(1) Shows how to obtain y[n] from present and past values of the input.

Recursive evaluation of difference equation

Find the first two o/p values y[0] and y[1] for the system described by $y[n] = x[n] + 2x[n-1] - y[n-1] - \frac{1}{4}y[n-2]$.

Assuming that the input is $x[n] = (\frac{1}{2})^n u[n]$ and the initial conditions are y[-1]=1 and y[-2]=-2.

$$y[0] = x[0] + 2x[-1] - y[-1] - \frac{1}{4}y[-2]$$

$$y[0] = 1 + 2\times 0 - 1 - \frac{1}{4}\times(-2) = \frac{1}{2}$$

$$y[1] = x[1] + 2x[0] - y[0] - \frac{1}{4}y[-1]$$

$$y[1] = \frac{1}{2} + 2\times 1 - \frac{1}{2} - \frac{1}{4}\times(1) = 1\frac{3}{4}$$

Solving Differential and Difference equations

Output of LTI system described by differential or difference equation has two components

y^h - homogeneous solution

y^p - particular solution

Complete solution

$$y = y^h + y^p$$

Eg.

A system is described y the difference equation

$$y[n]-1.143y[n-1]+0.4128y[n-2]=0.0675x[n]+0.1349x[n-1]+0.675x[n-2]$$

$y[-1] = 1$ and $y[-2] = 2$,

Homogeneous Solution for CT system

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$$

Set all terms involving input to zero

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = 0$$

Solution

$$y^h(t) = \sum_{i=1}^{N} c_i e^{r_i t}$$

r_i are the N roots of the *characteristic equation*

Homogeneous Solution for DT system

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Set all terms involving input to zero

$$\sum_{k=0}^{N} a_k y[n-k] = 0$$

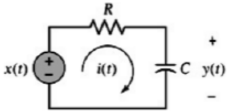
Solution

$$y^h[n] = \sum_{i=1}^{N} c_i {r_i}^n$$

r_i are the N roots of the characteristic equation.

Homogeneous Solution Example

RC ckt depicted in figure is described by the differential equation $y(t) + RC\frac{d}{dt}y(t) = x(t)$. Determine the homogenous solution.



The homogenous equation is

$$y(t) + RC\frac{d}{dt}y(t) = 0 \qquad (1)$$

Solution

$$y^h(t) = \sum_{i=1}^{N} c_i e^{r_i t}$$

$$y^h(t) = c_1 e^{r_1 t}$$

Characteristic equation

$$(1 + RCr_1) = 0$$

$$r_1 = -\frac{1}{RC}$$

Homogenous solution of the system is

$$y^h(t) = c_1 e^{-\frac{1}{RC}t}$$

Homogeneous Solution Example

Determine the homogenous solution.

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = 2x(t) + \frac{d}{dt}x(t)$$

Ans:

Homogenous equation

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = 0 \quad (1)$$

Homogenous solution

$$y^h(t) = \sum_{i=1}^{N} c_i e^{r_i t}$$

$$y^h(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

To find r_1 and r_2 solve CE.

Put $\frac{d^n}{dt^n}y(t) = r^n$

(1) becomes

$$r^2 + 5r + 6 = 0$$

$$r = -3, -2$$

$$y^h(t) = c_1 e^{-3t} + c_2 e^{-2t}$$

Homogeneous Solution Example

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n-1]$$

Ans:

Set input to zero

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=0 \quad (1)$$

N=2

$$y^h[n]=c_1{r_1}^n+c_2{r_2}^n$$

(1) Becomes

$$1-\frac{1}{4}r^{-1}-\frac{1}{8}r^{-2}=0$$

$$r^2-\frac{1}{4}r-\frac{1}{8}=0$$

$$r=\frac{1}{2},-\frac{1}{4}$$

$$y^h[n]=c_1(\frac{1}{2})^n+c_2(-\frac{1}{4})^n$$

The Particular Solution

Solution of difference or differential equation for a given input.

Assumption : output is of same general form as the input.

CT System	
Input	Particular solution
1	c
t	$c_1 t + c_2$
e^{-at}	ce^{-at}
$\cos(\omega t + \Phi)$	$c_1 \cos(\omega t) + c_2 \sin(\omega t)$

DT System	
Input	Particular solution
1	c
n	$c_1 n + c_2$
α^n	$c\alpha^n$
$\cos(\Omega n + \Phi)$	$c_1 \cos(\Omega n) + c_2 \sin(\Omega n)$

The End