Signals and Systems UNIT 2

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What is System?

A continuous-time (discrete-time) system H is an operator that transfer the input x(t) (x[n]) into the output y(t) (y[n]). We denote the process by

$$x(t) \Longrightarrow \left[\ H\ \right] \Longrightarrow y(t)$$

$$x(n) \Longrightarrow \left[\ H\ \right] \Longrightarrow y(n)$$

$$y(t) = H[x(t)]$$

Example: $y(t) = x(t) + 1$

$$y(n) = H[x(n)]$$

Example: $y(n) = x(n)^2$

Memory vs. Memoryless

- A system H is memoryless if the value $y(t_0)$ (i.e.,$y(t = t_0)$) only depends on the value $x(t_0)$ for any t_0.

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  Current input: x(t)
  Past input: x(t-1), x(t-2),...x(t-k)
  Future input: x(t+1), x(t+2),...x(t+k)

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- Example: $y(t) = x^2(t)$ is memoryless since $y(t_0) = x^2(t_0)$ for t_0.

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- Example: $y(t) = x^2(t)$ is memoryless since $y(t_0) = x^2(t_0)$ for t_0.
- Example: $y(t) = x(t - 1)$ is a system with memory since $y(t_0) = x(t_01)$, e.g., $y(0) = x(1)$. $y(t_0)$ depends on $x(t)$ at $t = t_01$, not at t_0.

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- Example: $y(t) = x^2(t)$ is memoryless since $y(t_0) = x^2(t_0)$ for t_0.
- Example: $y(t) = x(t - 1)$ is a system with memory since $y(t_0) = x(t_0 1)$, e.g., $y(0) = x(1)$. $y(t_0)$ depends on $x(t)$ at $t = t_0 1$, not at t_0.
- In other words, output y(t) at current time $t = t_0$ is only affected by input $x(t)$ at current time $t = t_0$

Memory vs. Memoryless EXAMPLES

- Problem: $y(n) = nx(n)$

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- Answer: Memory
- Problem: $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$

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- Problem: $y(t) = \frac{d}{dt}x(t)$

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Causal vs. Non-causal

- A system H is causal if the value $y(t_0)$ only depends on $x(t) : t \leq t_0$.
- I.e., current output $y(t)$ is produce by current input $x(t)$ and past input $x(t-1), x(t-2), ...x(t-k)$, not future input $x(t+1), x(t+2), ...x(t+k)$.
- The system $y[n] = x[n-1]$ is causal $(y[0] = x[-1])$
- The system $y[n] = x[n+1]$ is non-causal $(y[0] = x[1])$
- The system $y(t) = x(t+a)$ is causal if $a \leq 0$ and is non-causal if $a > 0$

Causal vs. Non-causal EXAMPLES

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Causal vs. Non-causal EXAMPLES

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- Problem: $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$
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- Problem: $y(t) = \frac{d}{dt}x(t)$

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- Problem: $y(t) = \frac{d}{dt}x(t)$
- Answer: Causal

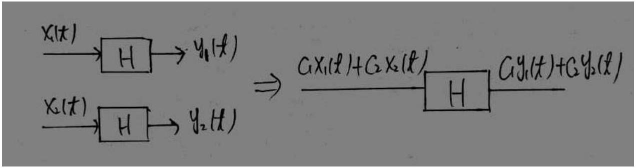
Linear vs. Nonlinear

- H is called linear if H has the superposition property:

$$H[a_1x_1(t) + a_2x_2(t)] = a_1H[x_1(t)] + a_2H[x_2(t)]$$

  where, $y_1(t) = H[x_1(t)]$ and $y_2(t) = H[x_2(t)]$

- The response of a weighted sum of input signals is equal to the same as weighted sum of output signals.



Linear vs. Nonlinear Example

$$y(n) = x(n-3)$$

- Two output signals

$$y_1(n) = H[x_1(n)] = x_1(n-3)$$

$$y_2(n) = H[x_2(n)] = x_2(n-3)$$

-

$$H[a_1x_1(n) + a_2x_2(n)] = a_1x_1(n-3) + a_2x_2(n-3)$$

$$= a_1H[x_1(n)] + a_2H[x_2(n)]$$

$$a_1y_1(n) + a_2y_2(n)$$

Hence, system is LINEAR.

Linear vs. Nonlinear Example

$$y(t) = x^2(t)$$

- Two output signals

$$y_1(n) = H[x_1(t)] = x_1^2(t)$$

$$y_2(n) = H[x_2(t)] = x_2^2(t)$$

- LHS:

$$H[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$$

- RHS:

$$a_1H[x_1(t)] + a_2H[x_2(t)] = a_1x_1^2(t) + a_2x_2^2(t)$$

LHS $\neq$ RHS. Hence, system is NON-LINEAR.

Linear vs. Nonlinear Example

$$y(n) = nx(n)$$

- Two output signals

$$y_1(n) = H[x_1(n)] = nx_1(n)$$

$$y_2(n) = H[x_2(n)] = nx_2(n)$$

-

$$H[a_1x_1(n) + a_2x_2(n)] = a_1nx_1(n) + a_2nx_2(n)$$

$$= a_1H[x_1(n)] + a_2H[x_2(n)]$$

$$a_1y_1(n) + a_2y_2(n)$$

Hence, system is LINEAR.

Linear vs. Nonlinear Example

$$y(n) = x(n)x(n-1)$$

- Two output signals

$$y_1(n) = H[x_1(n)] = x_1(n)x_1(n-1)$$

$$y_2(n) = H[x_2(n)] = x_2(n)x_2(n-1)$$

-

$$H[a_1x_1(n) + a_2x_2(n)] = a_1x_1(n)x_1(n-1) + a_2x_2(n)x_2(n-1)$$

$$= a_1H[x_1(n)] + a_2H[x_2(n)]$$

$$a_1y_1(n) + a_2y_2(n)$$

Hence, system is LINEAR.

Linear vs. Nonlinear Example

$$y(t) = \frac{d}{dt}x(t)$$

- Two output signals

$$y_1(n) = H[x_1(t)] = \frac{d}{dt}x_1(t)$$

$$y_2(n) = H[x_2(t)] = \frac{d}{dt}x_2(t)$$

- LHS:

$$H[a_1x_1(t) + a_2x_2(t)] = \frac{d}{dt}[a_1x_1(t) + a_2x_2(t)]$$

$$= a_1\frac{d}{dt}x_1(t) + a_2\frac{d}{dt}x_2(t)$$

- RHS:

$$a_1H[x_1(t)] + a_2H[x_2(t)] = a_1\frac{d}{dt}x_1(t) + a_2\frac{d}{dt}x_2(t)$$

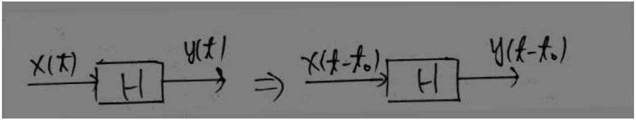
LHS = RHS. Hence, system is LINEAR.

Time-invariant vs. Time-variant

- H is called time-invariant if the following is true:

$$H[x(t)] = y(t) \Longrightarrow H[x(t - t_0)] = y(t - t_0)$$

- I.e., a time-shift to in the input x(t) results in an identical time-shift to in the output.



Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = e^{x(t)}$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = e^{x(t)}$$
- $$H(x(t-t_0) = e^{x(t-t_0)})$$
  $$y(t-t_0) = e^{x(t-t_0)}$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = e^{x(t)}$$
- $$H(x(t-t_0) = e^{x(t-t_0)})$$
  $$y(t-t_0) = e^{x(t-t_0)}$$
- Time-invariant

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = g(t)x(t)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = g(t)x(t)$$
- $$H(x(t-t_0) = g(t)x(t-t_0))$$
  $$y(t-t_0) = g(t-t_0)x(t-t_0)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = g(t)x(t)$$
- $$H(x(t-t_0) = g(t)x(t-t_0))$$
  $$y(t-t_0) = g(t-t_0)x(t-t_0)$$
- Time-variant

Time-invariant vs. Time-variant EXAMPLE

- $$y(n) = x(n-k)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(n) = x(n-k)$$
- $$H(x(n-k-n_0)) = x(n-k-n_0)$$
  $$y(n-k-n_0) = x(n-k-n_0)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(n) = x(n-k)$$
- $$H(x(n-k-n_0)) = x(n-k-n_0)$$
$$y(n-k-n_0) = x(n-k-n_0)$$
- Time-Invariant

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x(2t)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x(2t)$$
- $$H(x(t - t_0) = x(2t - t_0))$$
  $$y(t - t_0) = x(2(t - t_0))$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x(2t)$$
- $$H(x(t-t_0) = x(2t-t_0))$$
  $$y(t-t_0) = x(2(t-t_0))$$
- Time-variant

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \sin[x(t)]$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \sin[x(t)]$$

- $$H(x(t - t_0) = \sin[x(t - t_0)]$$
  $$y(t - t_0) = \sin[x(t - t_0)]$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \sin[x(t)]$$
- $$H(x(t - t_0) = \sin[x(t - t_0)]$$
  $$y(t - t_0) = \sin[x(t - t_0)]$$
- Time-invariant

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x[\cos t]$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x[\cos t]$$
- $$H(x(t - t_0) = x[\cos t - t_0]$$
$$y(t - t_0) = x[\cos(t - t_0)]$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = x[\cos t]$$
- $$H(x(t-t_0) = x[\cos t - t_0]$$
  $$y(t-t_0) = x[\cos(t-t_0)]$$
- Time-variant

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \cos t . x(t)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \cos t.x(t)$$
- $$H(x(t - t_0) = \cos t.x(t - t_0))$$
  $$y(t - t_0) = \cos(t - t_0).x(t - t_0)$$

Time-invariant vs. Time-variant EXAMPLE

- $$y(t) = \cos t . x(t)$$
- $$H(x(t-t_0) = \cos t . x(t-t_0))$$
  $$y(t-t_0) = \cos(t-t_0) . x(t-t_0)$$
- Time-variant

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

$$\begin{aligned} y_d(t) &= \int_{-\infty}^{t/2} x(\tau - d) d\tau \\ &= \int_{-\infty}^{t/2-d} x(s) ds \\ &= \int_{-\infty}^{(t-2d)/2} x(s) ds \\ &= y(t-2d). \end{aligned}$$

-

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \int_{-\infty}^{t/2} x(\tau) d\tau$

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-

- Time-variant: Therefore, it does not obey the time-invariance condition.

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

The integrator system is also a time-invariant system. To prove this, we replace $x(\tau)$ in (9.30) by $x(\tau - t_0)$ obtaining the output $w(t)$

$$w(t) = \int_{-\infty}^{t} x(\tau - t_0) d\tau \tag{9.35}$$

Now, to prove time-invariance, we must manipulate the integral in (9.35) into a form that is recognizable in terms of the original output $y(t)$. This is done by changing the "dummy variable" of integration to $\sigma = \tau - t_0$. In this substitution, $d\tau$ is replaced by $d\sigma$, the lower limit $\tau = -\infty$ becomes $\sigma = -\infty$, and the upper limit $\tau = t$ becomes $\sigma = t - t_0$. Therefore (9.35) becomes

$$w(t) = \int_{-\infty}^{t-t_0} x(\sigma) d\sigma$$

- and it is now clear that $w(t) = y(t - t_0)$, so the integrator system is seen to be time-invariant. ■

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \frac{d}{dt}x(t)$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \frac{d}{dt}x(t)$
- The derivative of a time-shifted signal is

$$y_d(t) = \frac{d}{dt}[x(t-d)] = \frac{dx}{dt}(t-d)\frac{d}{dt}(t-d) = \frac{dx}{dt}(t-d) = y(t-d).$$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = \frac{d}{dt}x(t)$
- The derivative of a time-shifted signal is

$$y_d(t) = \frac{d}{dt}[x(t-d)] = \frac{dx}{dt}(t-d)\frac{d}{dt}(t-d) = \frac{dx}{dt}(t-d) = y(t-d).$$

- Time-invariant

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = 2x(n)u(n)$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = 2x(n)u(n)$
- Not Time Invariant. The time-invariance condition does not hold, because the signal that is being multiplied by $x(n)$ varies with time.

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = x(2 - t)$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = x(2 - t)$
- $$H(x(t - t_0) = x(2 - t - t_0))$$
  $$y(t - t_0) = x(2 - (t - t_0)) = x(2 - t + t_0)$$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(t) = x(2 - t)$
- $$H(x(t - t_0) = x(2 - t - t_0))$$ $$y(t - t_0) = x(2 - (t - t_0)) = x(2 - t + t_0)$$
- Time-variant

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = nx(n)$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = nx(n)$
- Answer: Time-variant

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = nx(n)$
- Answer: Time-variant
- Problem: $y(n) = x(n)x(n-1)$

Time-invariant vs. Time-variant EXAMPLE

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Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = nx(n)$
- Answer: Time-variant
- Problem: $y(n) = x(n)x(n-1)$
- Answer: Time-invariant
- Problem: $y(t) = x(2-t)$

Time-invariant vs. Time-variant EXAMPLE

- Problem: $y(n) = nx(n)$
- Answer: Time-variant
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- Problem: $y(t) = x(2-t)$
- Answer: Time-variant

Stable Vs Non-stable

- A system is said to be Bounded Input Bounded Output (BIBO) stable if and only if every bounded input results in bounded output.
- System H is BIBO stable if $|x(t)| \leq M_x < \infty$ then $|y(t)| \leq M_y < \infty$ for all t

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(t - t_0)$

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(t - t_0)$
- Bounded Input

$$|x(t)| \leq M_x < \infty$$

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(t - t_0)$
- Bounded Input

$$|x(t)| \leq M_x < \infty$$

- By shifting input $x(t)$ by t_0

$$|x(t - t_0)| \leq M_x < \infty$$

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(t - t_0)$
- Bounded Input

$$|x(t)| \leq M_x < \infty$$

- By shifting input $x(t)$ by t_0

$$|x(t - t_0)| \leq M_x < \infty$$

- Taking magnitude on both sides of input-output relationship,

$$|y(t)| = |x(t - t_0)| \leq M_x < \infty$$

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(t - t_0)$
- Bounded Input

$$|x(t)| \leq M_x < \infty$$

- By shifting input $x(t)$ by t_0

$$|x(t - t_0)| \leq M_x < \infty$$

- Taking magnitude on both sides of input-output relationship,

$$|y(t)| = |x(t - t_0)| \leq M_x < \infty$$

- BIBO condition satisfied. Hence, stable.

Stable Vs Non-stable EXAMPLE

- Problem: $y(n) = x(-n)$

Stable Vs Non-stable EXAMPLE

- Problem: $y(n) = x(-n)$
- Bounded Input

$$|x(n)| \leq M_x < \infty$$

Stable Vs Non-stable EXAMPLE

- Problem: $y(n) = x(-n)$
- Bounded Input

$$|x(n)| \leq M_x < \infty$$

- By time reversing input $x(n)$ by

$$|x(-n)| \leq M_x < \infty$$

Stable Vs Non-stable EXAMPLE

- Problem: $y(n) = x(-n)$
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$$|x(-n)| \leq M_x < \infty$$

- Taking magnitude on both sides of input-output relationship,

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$$|y(n)| = |x(-n)| \leq M_x < \infty$$

- BIBO condition satisfied. Hence, stable.

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = [\sin 6t]x(t)$

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$$|y(t)| = |\sin 6t||x(t)|$$

Now,

$$|\sin 6t| \leq 1$$

$$|x(t)| \leq M_x < \infty$$

$y(t)$ is also bounded.

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$$|y(n)| = |x(n) + n| = |x(n)| + |n|$$

As $n \to \infty$, $y(n) \to \infty$ means output is not bounded.

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- BIBO condition not-satisfied. Hence, Unstable.

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(\frac{t}{2})$

Stable Vs Non-stable EXAMPLE

- Problem: $y(t) = x(\frac{t}{2})$
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- Problem: $y(t) = x(\frac{t}{2})$
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$$|x(t)| \leq M_x < \infty$$

- By shifting input $x(t)$ by t_0

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$$|y(t)| = |\int_{-\infty}^{t} x(\tau)d\tau|$$

let's take $x(t) = u(t)$

$$|u(t)| = 1$$

$$|y(t)| = |\int_{-\infty}^{t} x(\tau)d\tau| = |\int_{-\infty}^{t} 1d\tau| = [\tau]_{-\infty}^{t} = t + \infty$$

if $t \rightarrow \infty$ then $|y(t)| \rightarrow \infty$

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Invertible Vs Not-Invertible

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- A system is said to be invertible if the input to the system may be uniquely determined from the output.
- Distinct inputs produce distinct outputs
- A system is said to be invertible, if the inverse of that system exists.

$$y(t) = T[x(t)] \Rightarrow x(t) = T_i[y(t)]$$

$$y(t) = T[x(t)] = 10x(t) \Rightarrow x(t) = T_i[y(t)] = 0.1y(t)$$

Test for Invertibility

To test for invertibiity, we use two different techniques:

- We may show that a system is invertible by designing an inverse system that uniquely recovers the input from output.

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- We may show that a system is invertible by designing an inverse system that uniquely recovers the input from output.
- We may show that system is not invertible by finding two different inputs that produce the same output.

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- $$y[n] = 2x[n]$$

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- Hence, system is invertible.

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- $$y(n) = Re[x(n)]$$
- Two different inputs can produce same output.

$$y(n) = Re[x(n)] = Re[2 - j2] = 2$$

$$y(n) = Re[x(n)] = Re[2 + j2] = 2$$

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Common Example

A continuous-time system is described by the following input-output relationship:

$$y(t) = [\sin 6t]x(t)$$

Determine whether this system is Memoryless, Time invariant, Linear, Causal and Stable?

- Memoryless?

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LTI systems

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- Used to predict long-term behavior in a system
- The behavior of an LTI system is completely defined by its impulse response

Impulse Function

The discrete version of impulse function is defined by

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

The continuous time version of impulse function,

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Impulse Response

- The impulse response" of a system, $h[n]$, is the output that it produces in response to an impulse input. Definition: if and only if $x[n] = \delta[n]$ then $y[n] = h[n]$

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- The impulse response" of a system, $h[n]$, is the output that it produces in response to an impulse input. Definition: if and only if $x[n] = \delta[n]$ then $y[n] = h[n]$
- Given the system equation, the impulse response can be found out just by feeding $x[n] = \delta[n]$ into the system.

Impulse Response Example

- Consider the system

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

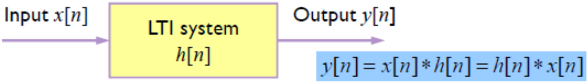
- Suppose we insert an impulse:

$$x[n] = \delta[n]$$

- Then whatever we get at the output, by Definition, is the impulse response. In this case it is

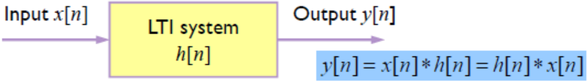
$$h[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) = \begin{cases} 0.5, & n = 0, 1 \\ 0, & otherwise \end{cases}$$

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
  x[n]=Input Signal

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
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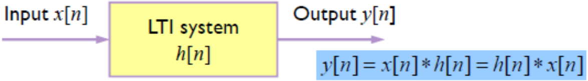
- $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Convolution Sum

- 

  where, h[n]=impulse response of LTI system
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- $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

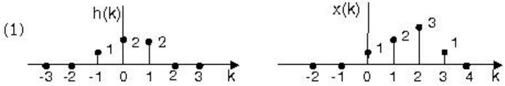
- input-excitation output-response

Convolution Sum Example

Find the response y[n] of following LTI system.

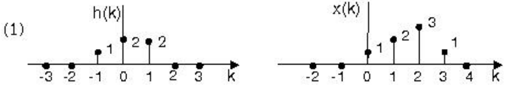
$$x(n)=[0,\underset{\Uparrow}{1},2,3,1,0] \text{ and } h(n)=[0,1,\underset{\Uparrow}{2},2,0]$$

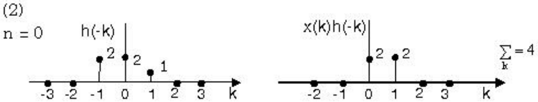
Convolution Sum (Graphical method)



-

Convolution Sum (Graphical method)

- 

- 

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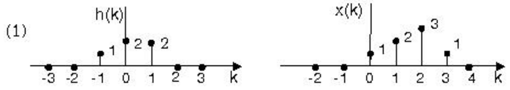
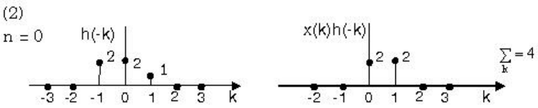
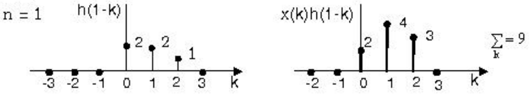
- 

- 

- 

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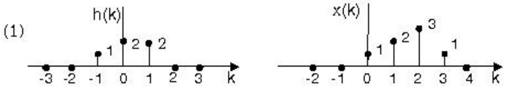
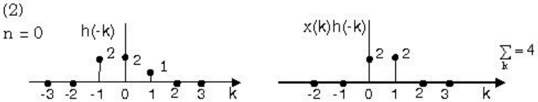
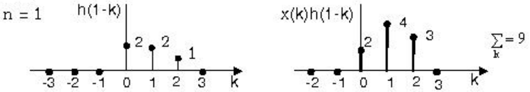
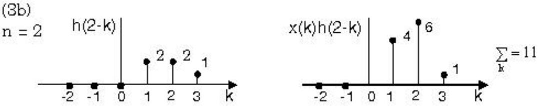
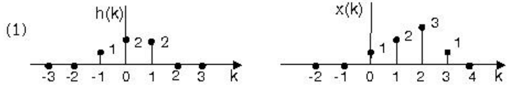
- 

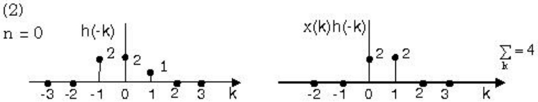
- 

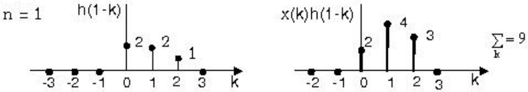
- 

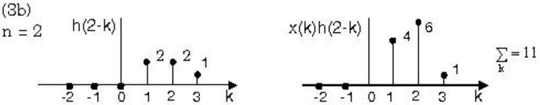
- 

Convolution Sum (Graphical method)

- 

- 

- 

- 

- $y(n) = [...0, 1, \underset{\uparrow}{4}, 9, 11, 8, 2, 0, ...]$

Convolution Sum (Analytical method)

- Size of x(n)=A=4, Size of h(n)=B=3
  Length of y(n)=A+B-1=4+3-1=6

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- Likewise for all the values of n
  $y(n) = [...0, 1, \underset{\uparrow}{4}, 9, 11, 8, 2, 0, ...]$

Convolution Sum (Analytical method)

$$x_1(n) = [\underset{\uparrow}{1}, 2, 3]$$

$$x_2(n) = [\underset{\uparrow}{2}, 1, 4]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

- Size of $x_1(n)$=A=3, Size of $x_2(n)$=B=3
  Length of y(n)=A+B-1=3+3-1=5

Convolution Sum (Analytical method)

$$x_1(n) = [\underset{\uparrow}{1}, 2, 3]$$

$$x_2(n) = [\underset{\uparrow}{2}, 1, 4]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

- Size of $x_1(n)$=A=3, Size of $x_2(n)$=B=3
  Length of y(n)=A+B-1=3+3-1=5
- $x_1(n)$ is starting from 0 index n_1=0
  $x_2(n)$ is starting from 0 index n_2=0
  $n_1 + n_2 = 0$, range of n=0 to 4

Convolution Sum (Analytical method)

- For n=0

$$y[0] = \sum_{k=0}^{2} x_1[k]x_2[-k] =$$

$$x[0]x_2[0] + x_1[1]x_2[-1] + x_1[2]x_2[-2]$$

$$= (1x2) + (2x0) + (3x0) = 2$$

Convolution Sum (Analytical method)

- For n=0

$$y[0] = \sum_{k=0}^{2} x_1[k]x_2[-k] =$$

$$x[0]x_2[0] + x_1[1]x_2[-1] + x_1[2]x_2[-2]$$

$$= (1x2) + (2x0) + (3x0) = 2$$

- Likewise for all the values of n $y(n) = [\underset{\uparrow}{2}, 5, 12, 11, 12]$

The End