Signals and Systems

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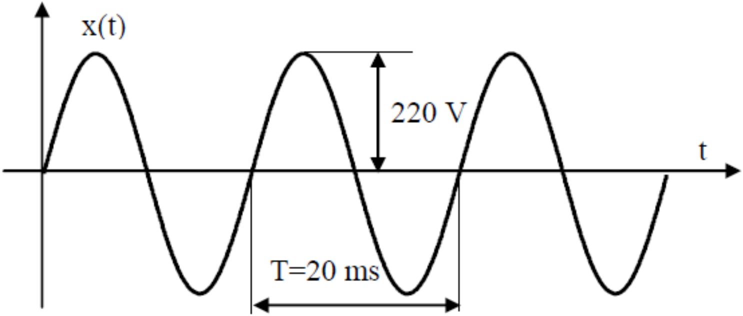
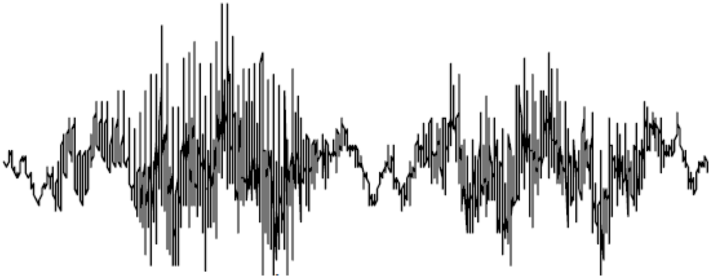
What is Signal?



What is Signal? (Continue ...)

Speech Signal



What is Signal? (Continue ...)

Image/ Meme: Student attending Post-Lunch Lectures

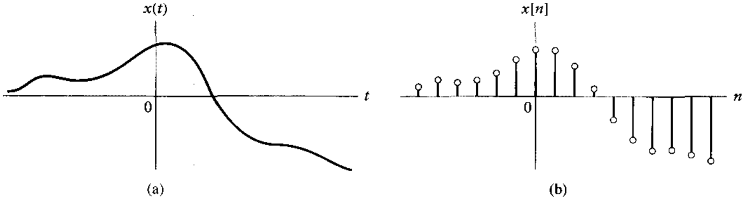
What is Signal? (Continue ...)

- A function of a set of independent variables (one variable or Multi-variables)
- Examples: temperature of Bangalore, number of COVID cases, speed of a car, Radio and TV signals, EEG, ECG, Stock market -share price

Classifications of Signals

- Continuous-time and Discrete-time signals
- Analog and Digital signals
- Deterministic and Random signals
- Even and Odd Signals
- Periodic and Non-periodic signals
- Power and Energy signals

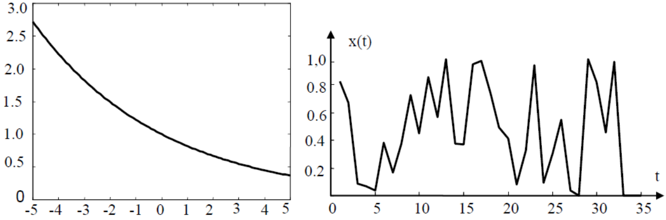
1. Continuous-time and Discrete-time signals



1. Continuous-time and Discrete-time signals (Continue ...)

- Continuous-time Signal: $x(t)$
  Independent Variable - time $t=$ Continuous
- Discrete-time Signal: $x(n)$
  Independent Variable -time $t=$ Discrete

2. Deterministic and Random signals



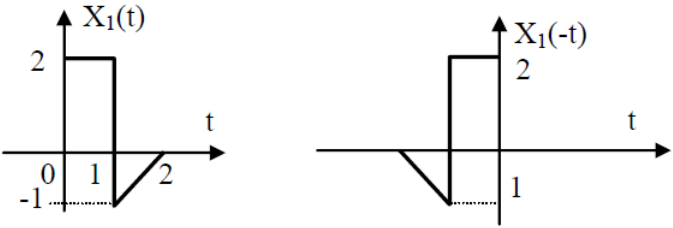
2. Deterministic and Random signals (Continue ...)

- Deterministic Signal: No uncertainty
  Behavior is known and can be represented
  $x(t) =$ number
- Random Signal:uncertainty
  Behavior is not known and can be represented as expected values like mean, variance etc.
  $x(t) =$ is a random variable

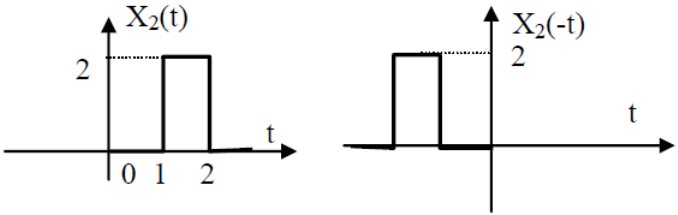
3. Analog and Digital signals

- Analog Signal: If a continuous-time signal can take on any values in a continuous time interval
  (Time and Amplitude - Continuous)
- Digital Signal: If a discrete-time signal can take on only a finite number of distinct values
  (Time and Amplitude - Discrete)

Time Reversal



Time Reversal (Continue ...)



4. Even and Odd signals

- Even Signal:

$$x(t) = x(-t)$$

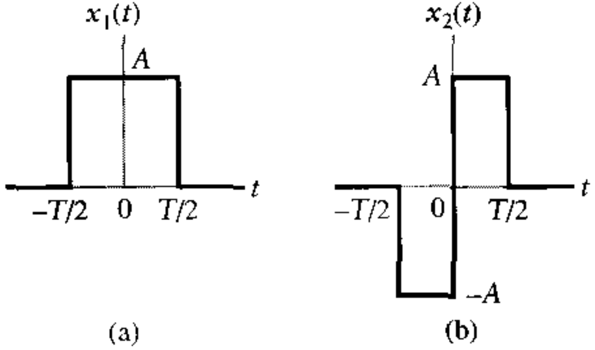
$$x(n) = x(-n)$$

- Odd Signal:

$$x(t) = -x(-t)$$

$$x(n) = -x(-n)$$

4. Even and Odd signals



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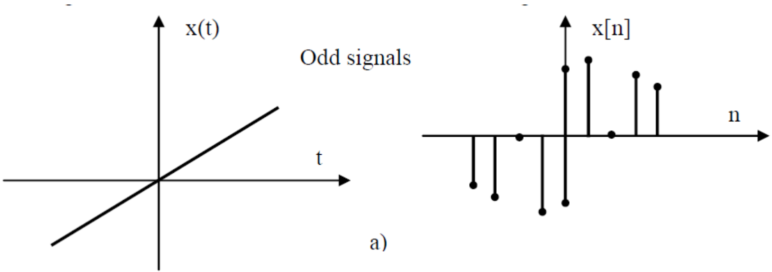

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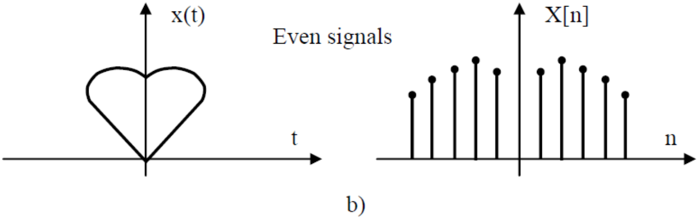

4. Even and Odd signals (Continue ...)

- Any signal can be written in terms of its even and odd components

$$x(t) = x_e(t) + x_o(t) \tag{1}$$

Where,
$x_o(t)$=Odd component of Signal
$x_e(t)$ = Even component of Signal

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- Since $x_o(t)$ is odd $x_o(-t) = -x_o(t)$ and $x_e(t)$ is even $x_e(-t) = -x_e(t)$ putting this in equation 2

$$x(-t) = x_e(t) - x_o(t) \tag{3}$$

4. Even and Odd signals (Continue ...)

- From following two equations

$$x(t) = x_e(t) + x_o(t) \tag{4}$$

$$x(-t) = x_e(t) - x_o(t) \tag{5}$$

4. Even and Odd signals (Continue ...)

- From following two equations

$$x(t) = x_e(t) + x_o(t) \tag{4}$$

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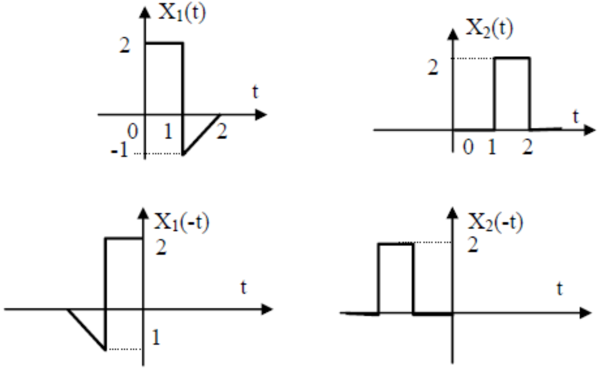
- The $x_e(t)$ and odd $x_o(t)$ components of signal x(t) are defined as

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2} \tag{6}$$

4. Even and Odd signals: EXAMPLE

Find the even and odd components of $X_1(t)$ and $X_2(t)$.

4. Even and Odd signals: EXAMPLE (Continue ...)



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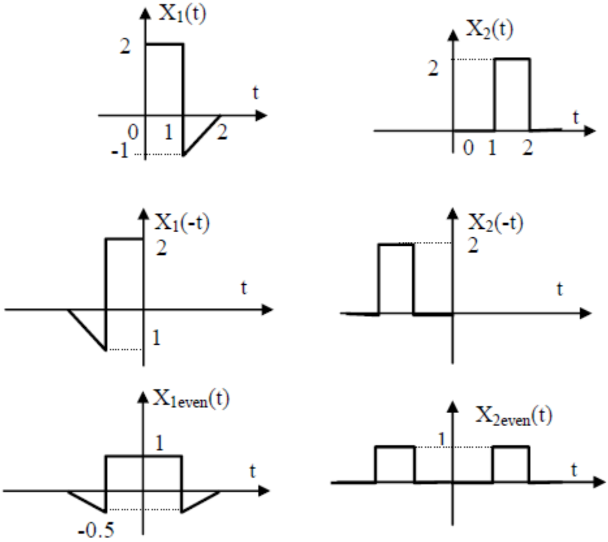

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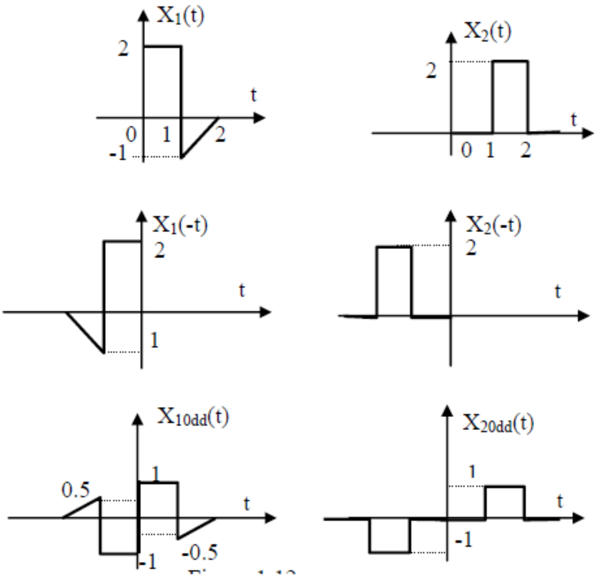

EXAMPLE: Find the even and odd components of $x(t) = e^{jt}$

$$x(-t) = e^{-jt}$$

Even Component $x_e(t)$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \frac{1}{2}[e^{jt} + e^{-jt}] = \cos t$$

Odd Component $x_o(t)$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$x_o(t) = \frac{1}{2}[e^{jt} - e^{-jt}] = j \sin t$$

5. Periodic and Nonperiodic Signals

- A periodic signal is a signal x(t) that satisfies the property

$$x(t+T) = x(t)$$

for all t where, T is positive constant.

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- The fundamental frequency $F = \frac{1}{T}$Hz and angular frequency $\omega = 2\pi F = \frac{2\pi}{T}$

5. Periodic and Nonperiodic Signals (Continuous ...)

- A periodic Discrete time signal is a signal x(n) that satisfies the property

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for all n where, N is integer.

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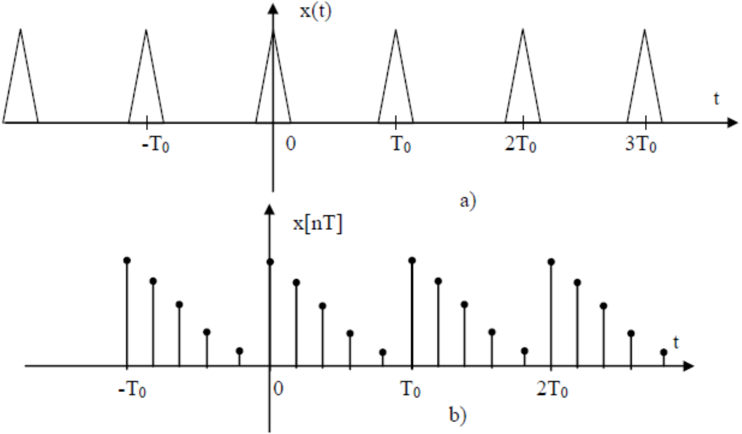

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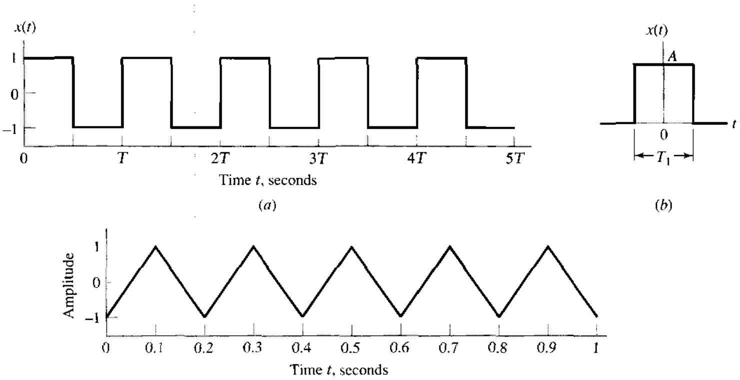

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Determine weather the following signals are periodic. If they are periodic, find the fundamental period.

- $x(t) = (\cos(2\pi t))^2$

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Determine weather the following signals are periodic. If they are periodic, find the fundamental period.

- $x(t) = (\cos(2\pi t))^2$
- $x(n) = (-1)^n$

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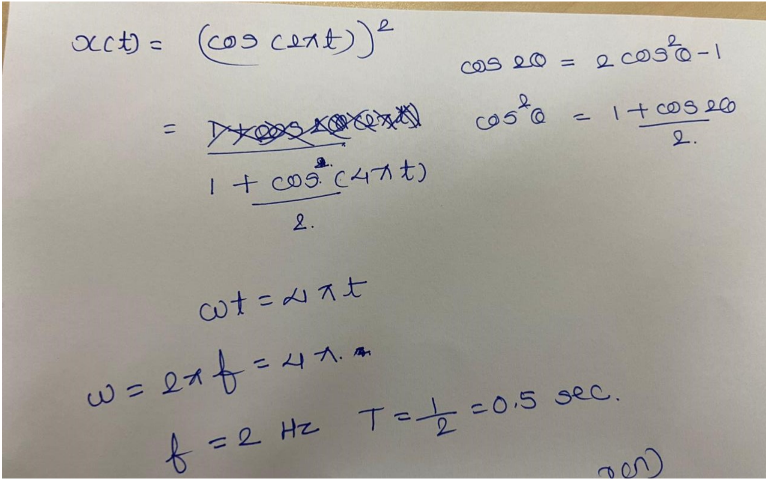
- $x(t) = (\cos(2\pi t))^2$
- $x(n) = (-1)^n$
- $x(n) = (-1)^{n^2}$
- $x(n) = \cos(2n)$

5. Periodic and Nonperiodic Signals

Solution of $x(t) = (\cos(2\pi t))^2$:



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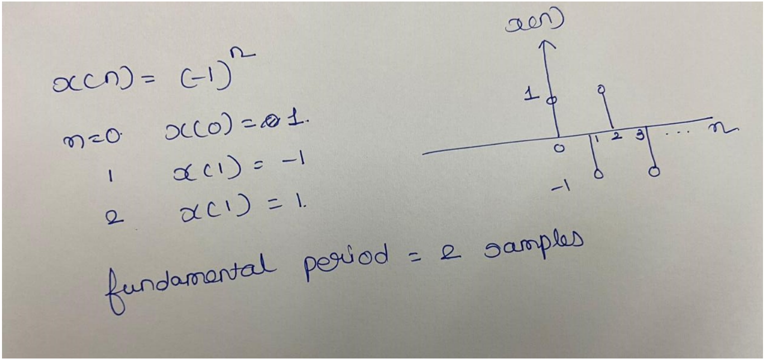

5. Periodic and Nonperiodic Signals (Continuous ...)

- If $x_1(t)$ and $x_2(t)$ are periodic signal with fundamental period T_1 and T_2, respectively. The signal $x(t) = x_1(t) + x_2(t)$ is periodic? if it is, find the fundamental period?

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- $\frac{T_1}{T_2}$ =rational number

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Determine weather the following signals are periodic. If they are periodic, find the fundamental period.

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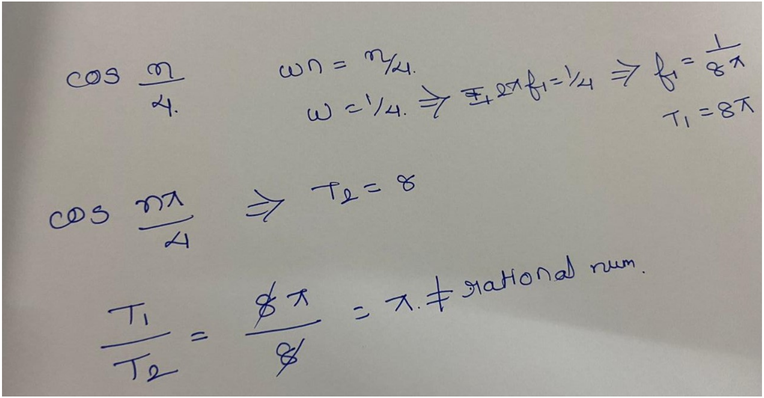


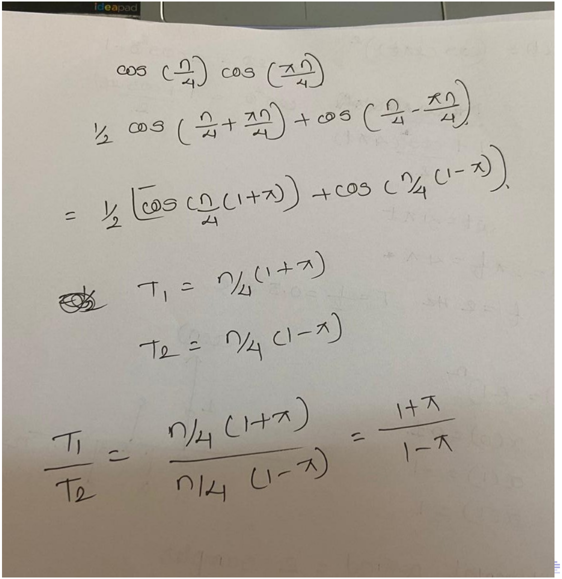
5. Periodic and Nonperiodic Signals EXAMPLE

Determine weather the following signals are periodic. If they are periodic, find the fundamental period.

- $x(n) = \cos(\frac{n}{4})\cos(\frac{\pi n}{4})$
- HINT: $\cos X \cos Y = \frac{cos(X+Y)+cos(X-Y)}{2}$

5. Periodic and Nonperiodic Signals

Solution of $x(n) = \cos(\frac{n}{4}) + \cos(\frac{\pi n}{4})$:



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Determine weather the following signals are periodic. If they are periodic, find the fundamental period.

- $x(n) = \cos(\frac{\pi n}{6})\cos(\frac{\pi n}{3})$

6. Energy and Power Signals

For continuous time signal $x(t)$:

- Energy

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

6. Energy and Power Signals

For continuous time signal $x(t)$:

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- Average Power

$$P = \lim_{t \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$$

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- If Signal is Periodic, Average Power

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For descrete time signal $x(n)$:

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- Average Power

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} |x(n)|^2$$

- If Signal is Periodic, Average Power

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)$$

6. Energy and Power Signals

Signal is either Energy or Power signal?

- A signal is referred as energy signal, if and only if the total energy of the signal satisfies the condition

$$0 < E < \infty$$

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- The energy and power classifications of signals are mutually exclusive.
- An energy signal has zero average power.
- A power signal has infinite energy.
- Periodic and Random signals are usually viewed as power signals.
- Signals that are both deterministic and non-periodic are energy signals.

6. Energy and Power Signals EXAMPLE

Calculate the energy of the signal

$$x(t) = \begin{cases} 1 & \frac{-\epsilon}{2} \leq t \leq \frac{\epsilon}{2} \\ 0 & otherwise \end{cases}$$

- Energy:

$$E = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\epsilon/2}^{\epsilon/2} x^2(t)dt = \int_{-\epsilon/2}^{\epsilon/2} 1^2 dt = \epsilon$$

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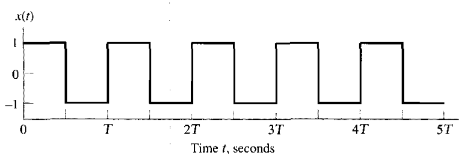
- Avg. Power

$$P = \lim_{t\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt = \lim_{t\to\infty} \frac{1}{T} \int_{-\epsilon/2}^{\epsilon/2} 1^2 dt$$

$$= \lim_{t\to\infty} \frac{1}{T}\epsilon = 0$$

6. Energy and Power Signals EXAMPLE

What is the average power of square wave shown below:



If Signal is Periodic, Average Power

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt = \frac{1}{T}\int_0^T 1^2 dt = \frac{1}{T} x T = 1$$

6. Energy and Power Signals EXAMPLE

What is the average power of the signal,

$$x(t) = A\cos(w_0 t + \theta)$$

- A sinusoidal signal $x(t)$ is periodic with $T_0 = \frac{2\pi}{w_0}$.

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What is the average power of the signal,

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- A sinusoidal signal $x(t)$ is periodic with $T_0 = \frac{2\pi}{w_0}$.
- Avg. Power

$$P = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt = \frac{w_0}{2\pi}\int_0^{\frac{2\pi}{w_0}} A^2 cos^2(w_0 t + \theta)dt$$

$$= \frac{A^2 w_0}{2\pi}\int_0^{\frac{2\pi}{w_0}} \frac{1}{2}[1 + cos(2w_0 t + 2\theta)dt = \frac{A^2}{2}$$

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- $P = \frac{A^2}{2} < \infty$, x(t) is power signal.

6. Energy and Power Signals EXAMPLE

The following signal is energy or power signal?

$$x(n) = \cos(\frac{n\pi}{2})$$

- $x(n)$ is periodic with a fundamental period N=4.

- $x(n)$ is a power signal.

6. Energy and Power Signals EXAMPLE

The following signal is energy or power signal?

$$x(n) = \cos(\frac{n\pi}{2})$$

- $x(n)$ is periodic with a fundamental period N=4.
- Hence,

$$P = \frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2 = \frac{1}{4}\sum_{n=0}^{3}1^2 = \frac{1}{4}x4 = 1$$

- $x(n)$ is a power signal.

Amplitude Scaling

Let $x(t)$ be a continuous-time signal. The resulting signal $y(t)$ from amplitude scaling is given by

$$y(t) = Cx(t)$$

where, C is the scaling factor.

- when $C < 1$, signal $y(t)$ is attenuated.

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$$y(t) = Cx(t)$$

where, C is the scaling factor.

- when $C < 1$, signal $y(t)$ is attenuated.
- when $C > 1$, signal $y(t)$ is amplified.

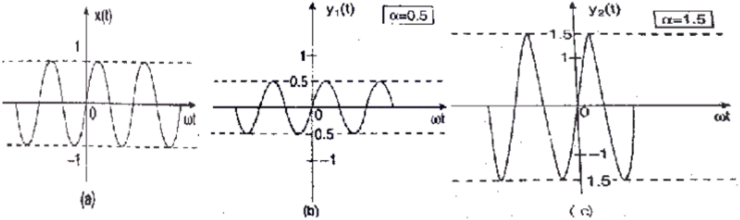
Amplitude Scaling EXAMPLE

Signals:

(a) $x(t) = \sin(wt)$

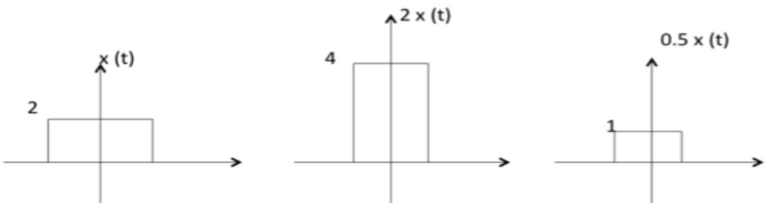
(b) $y(t) = 0.5x(t) = \sin(wt)$ C=0.5

(c) $y(t) = 2x(t) = \sin(wt)$ C=2



Amplitude Scaling EXAMPLE

Signal (a) $x(t) = \sin(wt)$
(b) $y(t) = 2x(t) = \sin(wt)$ C=2
(c) $y(t) = 0.5x(t) = \sin(wt)$ C=0.5

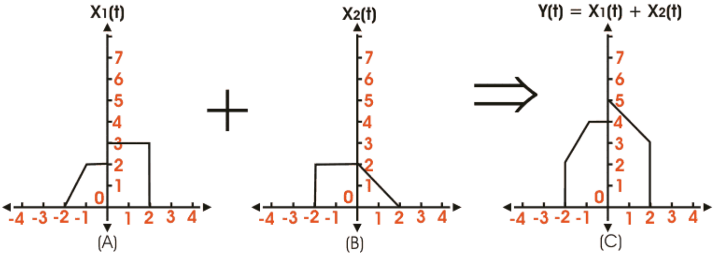


Addition of Signals

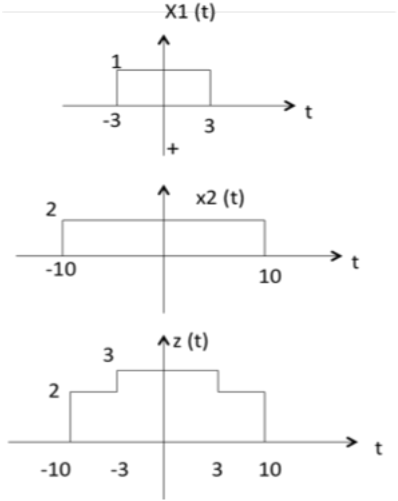
Addition of signals is illustrated in the diagram below, where $x_1(t)$ and $x_2(t)$ are two continuous-time signals, performing the additional operation on them we get,

$$y(t) = x_1(t) + x_2(t)$$

Addition of Signals EXAMPLE



Addition of Signals EXAMPLE

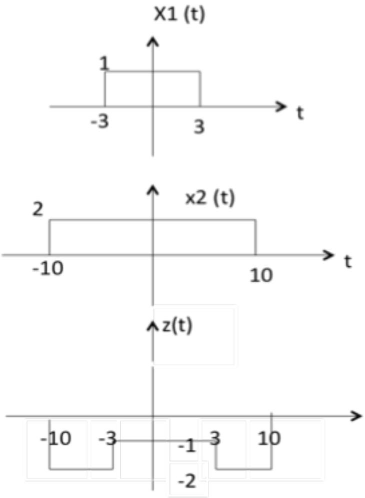


Subtraction of Signals

Subtraction of signals is illustrated in the diagram below, where $x_1(t)$ and $x_2(t)$ are two continuous-time signals, performing the Subtraction operation on them we get,

$$y(t) = x_1(t) - x_2(t)$$

Subtraction of Signals EXAMPLE

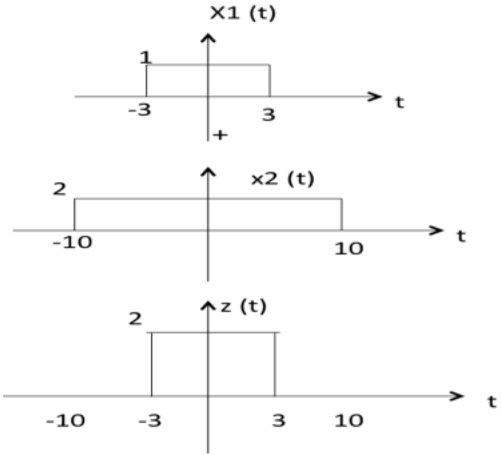


Multiplication of Signals

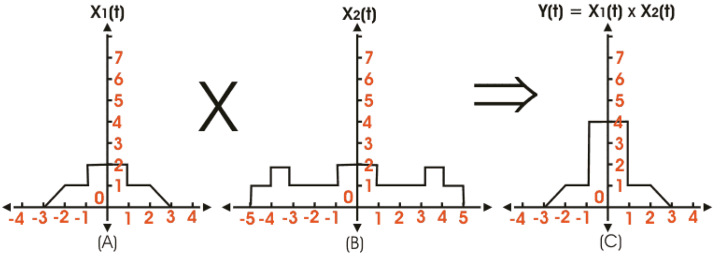
Multiplication of signals is illustrated in the diagram below, where $x_1(t)$ and $x_2(t)$ are two continuous-time signals, performing the multiplication operation on them we get,

$$y(t) = x_1(t) * x_2(t)$$

Multiplication of Signals EXAMPLE



Multiplication of Signals EXAMPLE

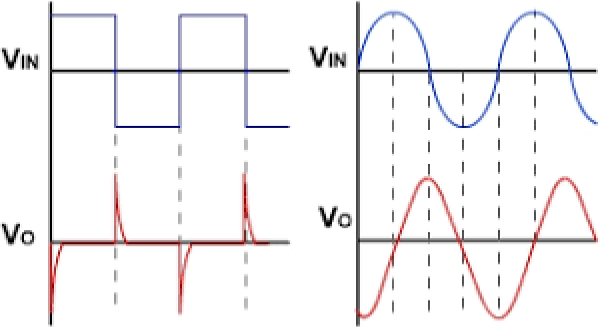


Differentiation of Signals

For differentiation of signals, it must be noted that this operation is only applicable for only continuous signals, as a discrete function cannot be differentiated.

$$y(t) = \frac{dx(t)}{dt}$$

Differentiation of Signals EXAMPLE

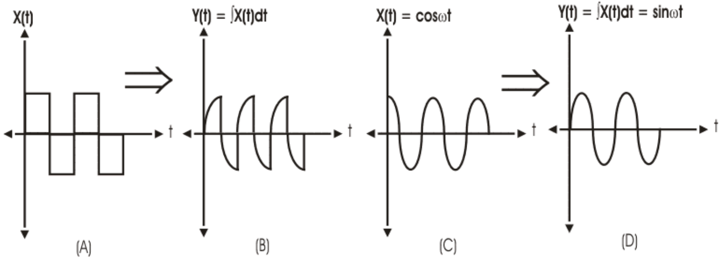


Integration of Signals

For Integration of signals, it must be noted that this operation is only applicable for only continuous signals, as a discrete function cannot be Integrated.

$$y(t) = \int_{-\infty}^{t} x(t)dt$$

Integration of Signals EXAMPLE



Time scaling of signals

Time scaling of signals of signals involves the modification of a periodicity of the signal, keeping its amplitude constant. Its mathematically expressed as,

$$y(t) = x(at)$$

where, a=scaling factor

- when $a < 1$, the signal $y(t)$ is expanded.

Time scaling of signals

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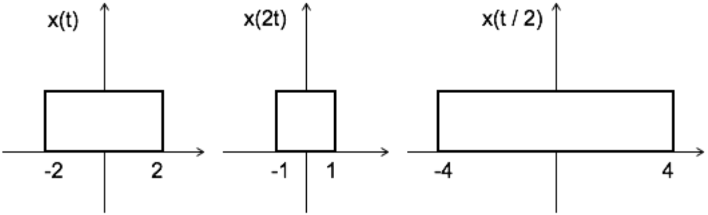
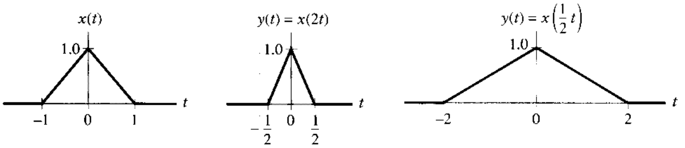
- when $a < 1$, the signal $y(t)$ is expanded.
- when $a > 1$, the signal $y(t)$ is compressed

Time scaling EXAMPLE

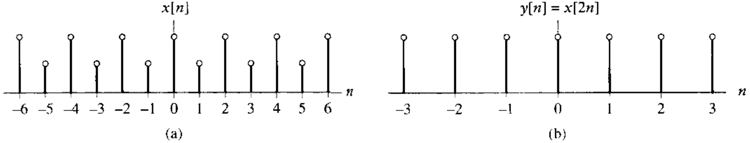


Time scaling EXAMPLE



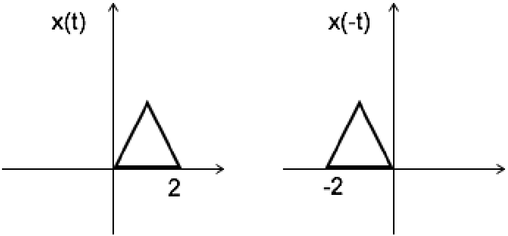
Time scaling EXAMPLE

Discrete time Signal

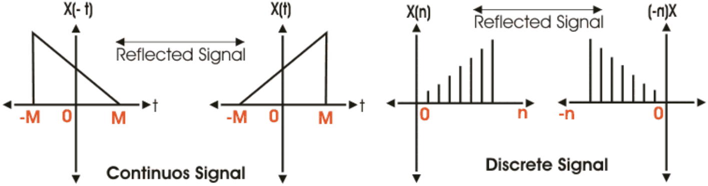


Time Reversal of signals/ Reflection of Signal

$x(-t)$ is the time reversal of the signal $x(t)$.



Time Reversal of signals/ Reflection of Signal



Time shifting of signals

Time shifting is mathematically expressed as,

$$y(t) = x(t - t_0)$$

where, t_0=represents the shift in time.

- $x(t - t_0)$-right side shift-positive shift

Time shifting of signals

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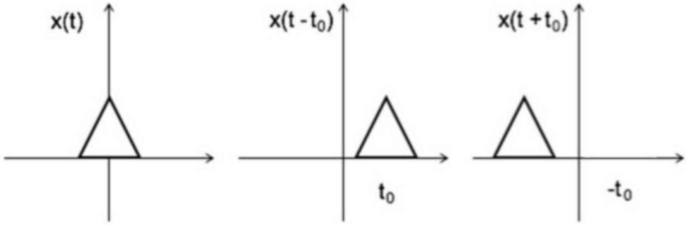
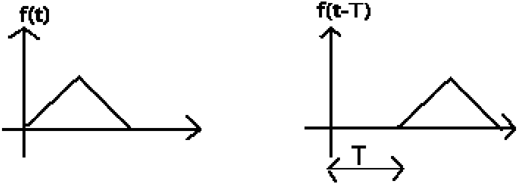
where, t_0=represents the shift in time.

- $x(t - t_0)$-right side shift-positive shift
- $x(t + t_0)$-left side shift-negative shift

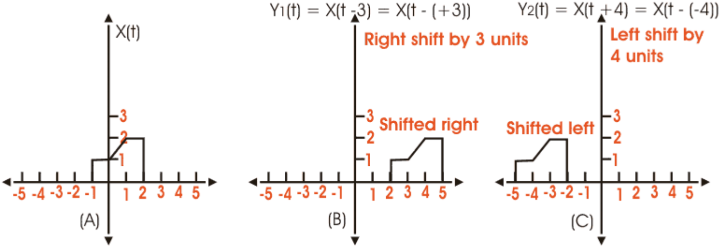
Time shifting of signals



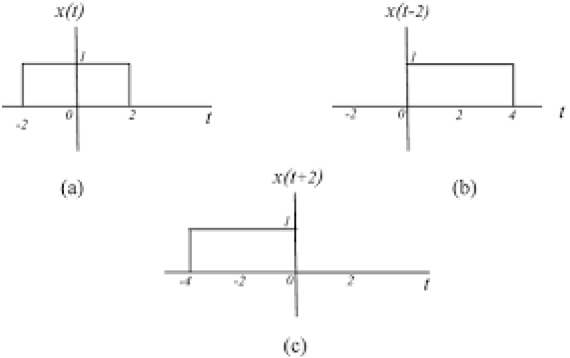
Time shifting of signals



Time shifting of signals



Time shifting of signals



Practice Example

P1.6

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t+2)$

(c) $x(2t+2)$

(d) $x(1-3t)$

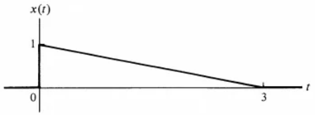


Figure P1.6

Practice Example

(a) This is just a time reversal.

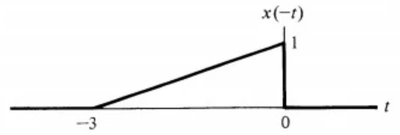


Figure S1.6-1

Practice Example

(b) This is a shift in time. At $t = -2$, the vertical portion occurs.

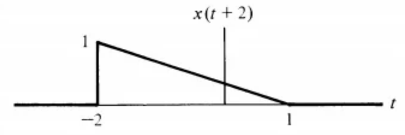


Figure S1.6-2

Combinations of Operations

Let, $y(t)$ defines the continuous time domain signal

$$y(t) = x(at - b)$$

The relation between $y(t)$ and $x(t)$ define following condition:

$$y(0) = x(-b)$$

and

$$y(b/a) = x(0)$$

- Shifting and Scaling should be performed in correct manner

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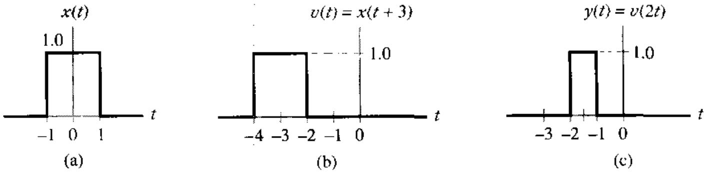
and

$$y(b/a) = x(0)$$

- Shifting and Scaling should be performed in correct manner
- First operation- shifting
  second operation-scaling

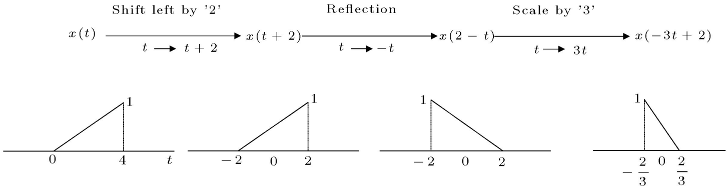
Combinations of Operations Example

Find $y(t) = x(2t + 3)$



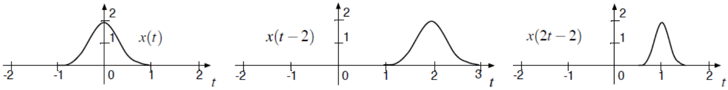
Combinations of Operations Example

Find $y(t) = x(-3t + 2)$



Combinations of Operations Example

Find $y(t) = x(2(t-1)) = x(2t-2)$



Combinations of Operations Practice Example

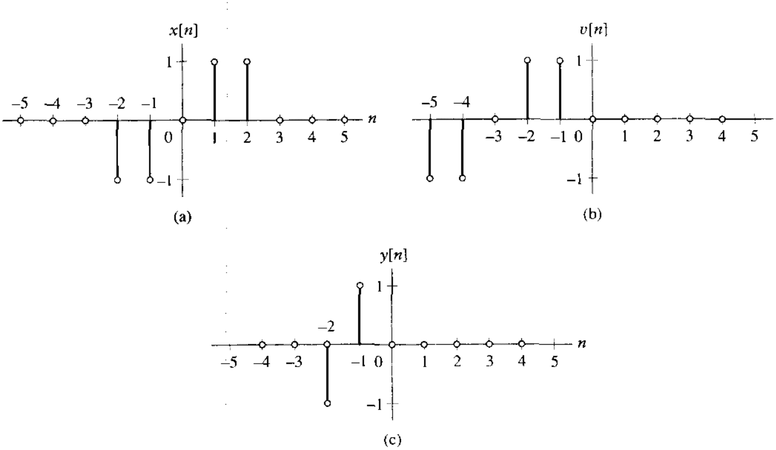
► **Drill Problem 1.12** Consider a discrete-time signal $x[n]$ defined by

$$x[n] = \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & |n| > 2 \end{cases}$$

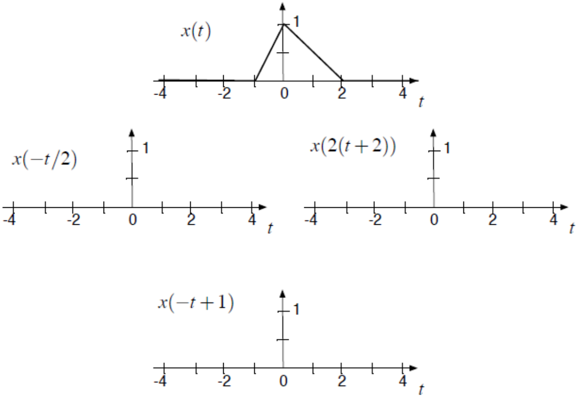
Find $y[n] = x[3n - 2]$.

Answer: $y[n] = \begin{cases} 1, & n = 0, 1 \\ 0, & \text{otherwise} \end{cases}$

Solution: Combinations of Operations Practice Example



Combinations of Operations Practice Example



Exponential signal

The continuous time complex exponential signal is of the form:

$$x(t) = Be^{at}$$

where, B and a are real parameters.

- Decaying exponential when $a < 0$

Exponential signal

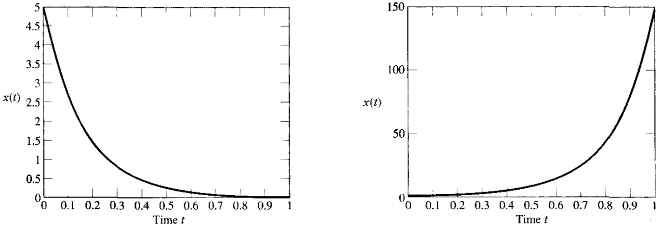
The continuous time complex exponential signal is of the form:

$$x(t) = Be^{at}$$

where, B and a are real parameters.

- Decaying exponential when $a < 0$
- Growing exponential when $a > 0$

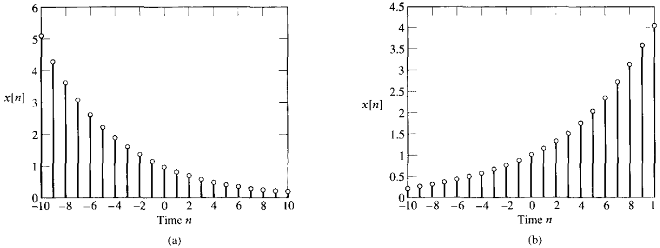
Exponential signal ...



(a) (b)

Discrete Exponential signal ...

$$x(n) = Be^{\alpha n}$$



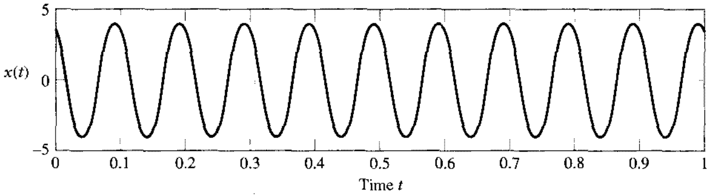
Sinusoidal signal

The sinusoidal signal is defined, in general, by

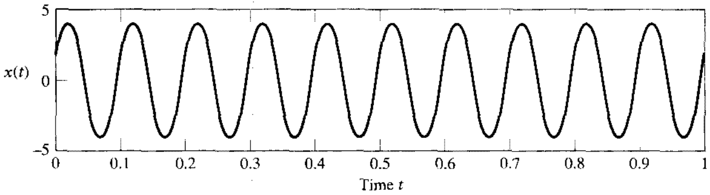
$$x(t) = A\cos(wt + \phi)$$

where, A=amplitude
w=frequency in radian
ϕ=phase angle in radian

Sinusoidal signal ...

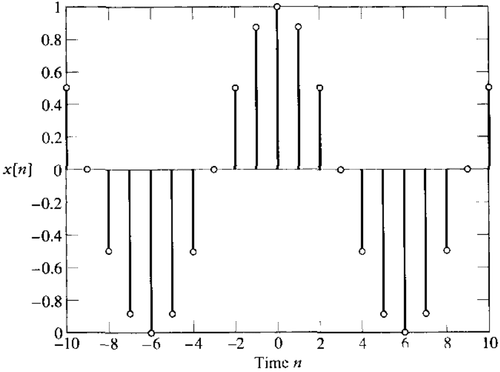


(a)



(b)

Discrete sinusoidal signal ...



Exponentially Damped Sinusoidal signal

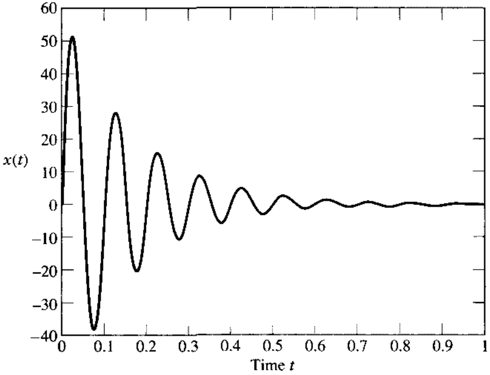
The eponentially damped sinusoidal signal is defined, in general, by

$$x(t) = Ae^{-\alpha t}\sin(wt + \phi)$$

where, A=amplitude
w=frequency in radian
ϕ=phase angle in radian

Discrete sinusoidal signal ...

A=60, α=6, ϕ=0



Step Function/Signal

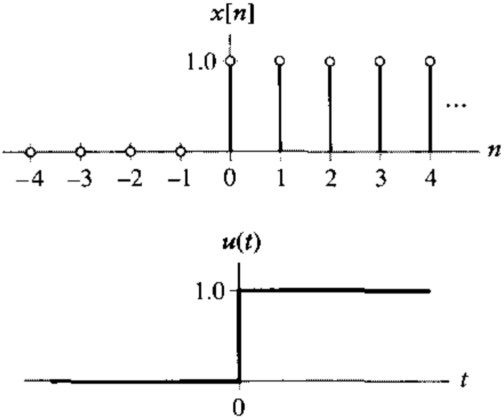
The discrete version of step function is defined by

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

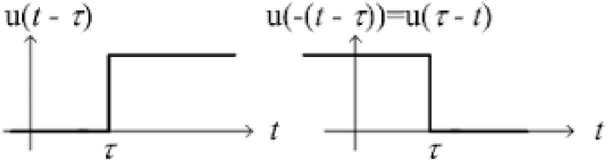
The continous time verson of step function,

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Step Function/Signal ...



Shifted Step Function/Signal ...



a. Shifted Unit Step Function

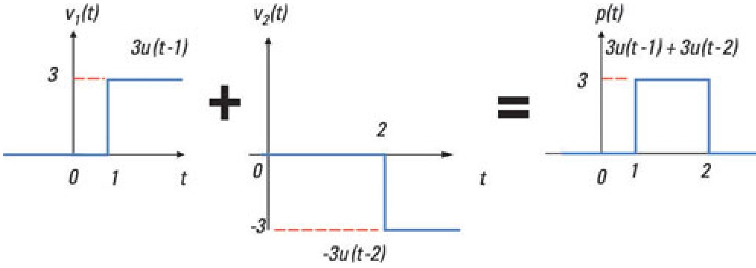
b. Shifted and Reflected Unit Step Function

Rectangle Signal using Step Function/Signal ...

- $$x(t) = \begin{cases} 3, & 1 \leq t \leq 2 \\ 0, & otherwise \end{cases}$$

Rectangle Signal using Step Function/Signal ...

- $$x(t) = \begin{cases} 3, & 1 \leq t \leq 2 \\ 0, & otherwise \end{cases}$$
- Solution:



Rectangle Signal using Step Function/Signal ...

- $$x(n) = \begin{cases} 1, & 0 \leq t \leq 9 \\ 0, & otherwise \end{cases}$$

Rectangle Signal using Step Function/Signal ...

- $$x(n) = \begin{cases} 1, & 0 \leq t \leq 9 \\ 0, & otherwise \end{cases}$$
- Solution: $x(n) = u(n) - u(n-10)$

Impulse Function/Signal

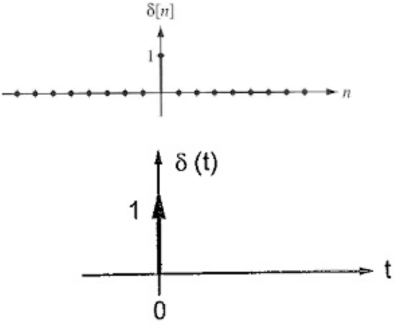
The discrete version of impulse function is defined by

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

The continous time verson of impulse function,

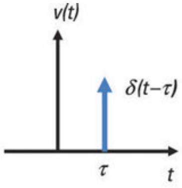
$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Impulse Function/Signal ...



Shifted Impulse Function/Signal ...

$$\delta(t-\tau) = \begin{cases} 1, & t = \tau \\ 0, & t \neq \tau \end{cases}$$



Ramp Function/Signal

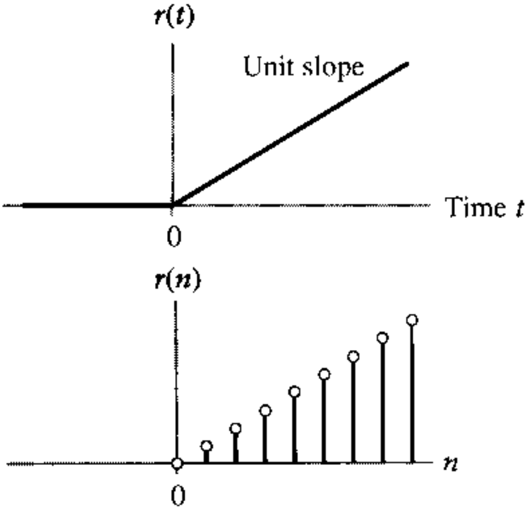
The discrete version of ramp function is defined by

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The continous time verson of ramp function,

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Ramp Function/Signal ...



Ramp Function/Signal in terms of unit step signal

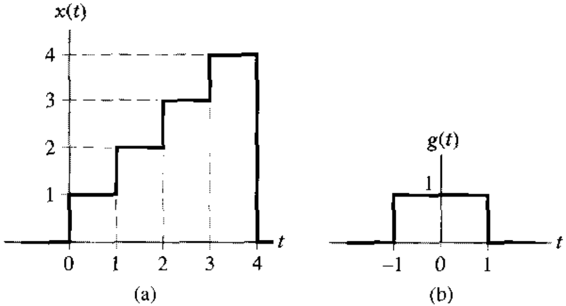
The discrete version of ramp function is defined by

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n \leq 0 \end{cases}$$

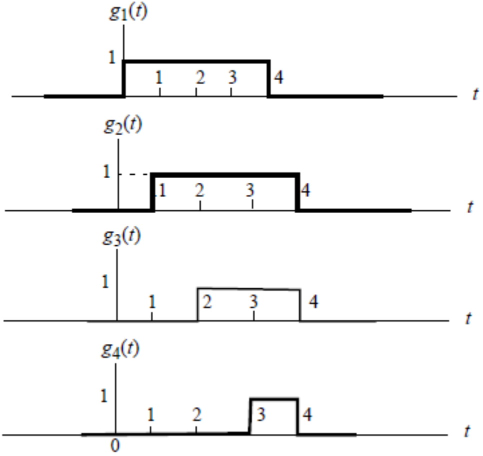
$$r(n) = nu(n)$$

Practice Problem

1.15 Figure P1.15(a) shows a staircase-like signal $x(t)$ that may be viewed as the superposition of four rectangular pulses. Starting with the rectangular pulse $g(t)$ shown in Fig. P1.15(b), construct this waveform, and express $x(t)$ in terms of $g(t)$.



Solution of Practice Problem



Solution of Practice Problem

- To generate g1(t) from the prescribed g(t), we let

$$g_1(t) = g(at - b)$$

where a and b are to be determined. The width of pulse g(t) is 2, whereas the width of pulse g1(t) is 4. We therefore need to expand g(t) by a factor of 2, which, in turn, requires that we choose $a = \frac{1}{2}$

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- The mid-point of g(t) is at t = 0, whereas the mid-point of g1(t) is at t = 2. Hence, we must choose b to satisfy the condition

$$a(2) - b = 0$$

$$b = 2a = 2(\frac{1}{2}) = 1$$

hence, $g_1(t) = g(\frac{1}{2}t - 1)$

Solution of Practice Problem

- Proceeding in a similar manner, we find that

$$g_2(t) = g(\frac{2}{3}t - \frac{5}{3})$$

$$g_3(t) = g(t - 3)$$

$$g_4(t) = g(2t - 7)$$

- Accordingly, we may express the staircase signal x(t) in terms of the rectangular pulse g(t) as follows:

$$x(t) = g_1(t) = g(\frac{1}{2}t - 1) + g(\frac{2}{3}t - \frac{5}{3}) + g(t - 3) + g(2t - 7)$$

The End