Control Systems UNIT 2

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Signal Flow Graphs

- Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

Basic Elements of Signal Flow Graph

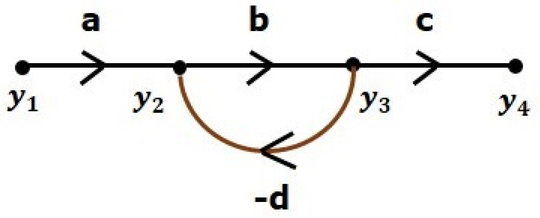
Nodes and branches are the basic elements of signal flow graph.

Node

Node is a point which represents either a variable or a signal. There are three types of nodes input node, output node and mixed node.

- Input Node It is a node, which has only outgoing branches.
- Output Node It is a node, which has only incoming branches.
- Mixed Node It is a node, which has both incoming and outgoing branches.

Example



The nodes present in this signal flow graph are y1, y2, y3 and y4.
y1 and y4 are the input node and output node respectively.
y2 and y3 are mixed nodes.

Branch

Branch is a line segment which joins two nodes. It has both gain and direction. For example, there are four branches in the above signal flow graph. These branches have gains of a, b, c and -d.

Construction of Signal Flow Graph

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

$$y_4 = a_{34}y_3$$

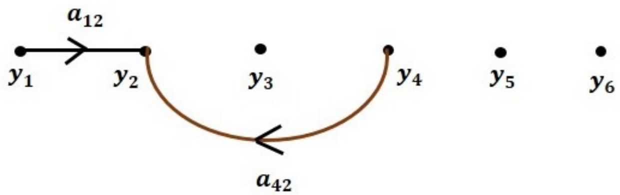
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

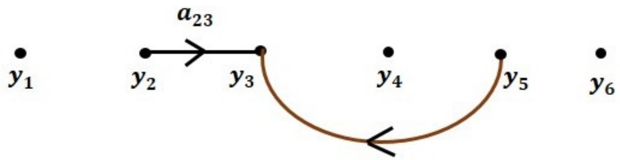
There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

Construction of Signal Flow Graph

Step 1 – Signal flow graph for $y_2 = a_{13}y_1 + a_{42}y_4$ is shown in the following figure.

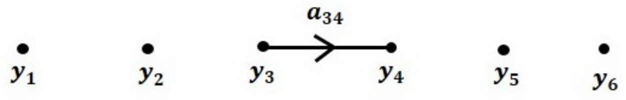


Step 2 – Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.

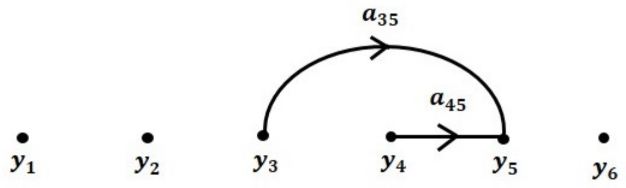


Construction of Signal Flow Graph

Step 3 – Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.



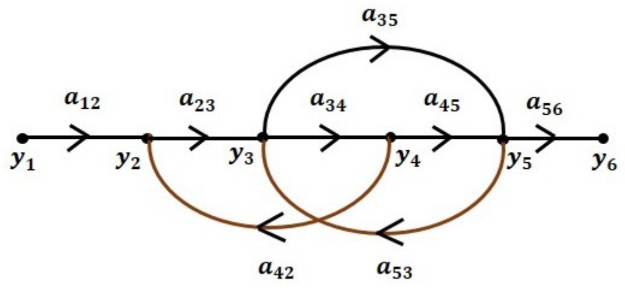
Step 4 – Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Construction of Signal Flow Graph

Step 5 – Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure.

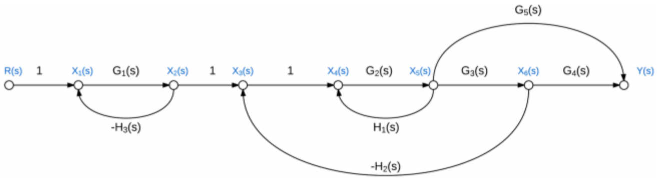
Step 6 – Signal flow graph of overall system is shown in the following figure.



Mason's Gain Formula

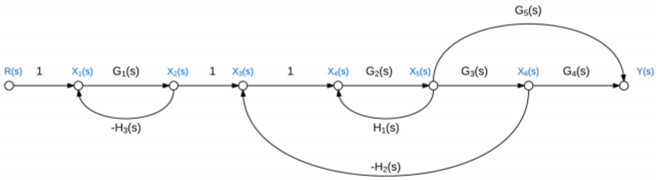
- Suppose there are N forward paths in a signal flow graph. The gain between the input and the output nodes of a signal flow graph is nothing but the transfer function of the system. It can be calculated by using Masons gain formula.
- Weve seen how to reduce a complicated block diagram to a single inputtooutput transfer function.
- Masons rule provides a formula to calculate the same overall transfer function.
- Before presenting the Masons rule formula, we need to define some terminology.

Loop Gain



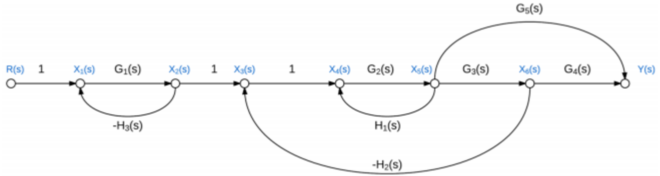
- ***Loop gain*** – total gain (product of individual gains) around any path in the signal flow graph
  - Beginning and ending at the same node
  - Not passing through any node more than once
- Here, there are three loops with the following gains:
  1. $-G_1H_3$
  2. G_2H_1
  3. $-G_2G_3H_2$

Forward Path Gain



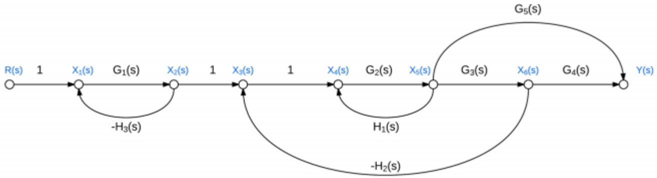
- ***Forward path gain*** – gain along any path from the input to the output
  - Not passing through any node more than once
- Here, there are two forward paths with the following gains:
  1. $G_1 G_2 G_3 G_4$
  2. $G_1 G_2 G_5$

NonTouching Loops



- ***Non-touching loops*** – loops that do not have any nodes in common
- Here,
  1. $-G_1H_3$ does not touch G_2H_1
  2. $-G_1H_3$ does not touch $-G_2G_3H_2$

NonTouching Loop Gains



- ***Non-touching loop gains*** – the *product* of loop gains from non-touching loops, taken two, three, four, or more at a time
- Here, there are only two *pairs* of non-touching loops
  1. $[-G_1H_3]\cdot[G_2H_1]$
  2. $[-G_1H_3]\cdot[-G_2G_3H_2]$

Mason's Gain Formula

Masons gain formula is

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between $R(s)$ and $C(s)$
- **P_i** is the i^{th} forward path gain

$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

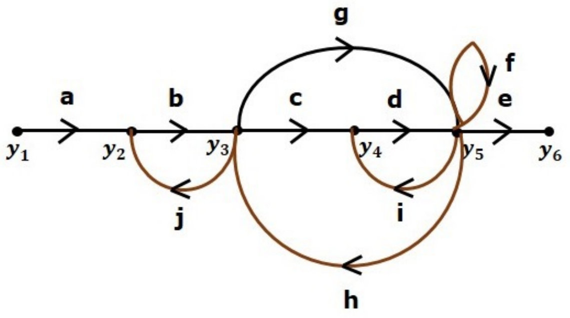
$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops) + \ldots$$

Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

Example: Mason's Gain Formula

Consider the following signal flow graph in order to understand the basic terminology involved here.



Example: Mason's Gain Formula

Path

It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.

Examples – $y_2 \to y_3 \to y_4 \to y_5$ and $y_5 \to y_3 \to y_2$

Forward Path

The path that exists from the input node to the output node is known as **forward path**.

Examples – $y_1 \to y_2 \to y_3 \to y_4 \to y_5 \to y_6$ and $y_1 \to y_2 \to y_3 \to y_5 \to y_6$.

Forward Path Gain

It is obtained by calculating the product of all branch gains of the forward path.

Examples – $abcde$ is the forward path gain of $y_1 \to y_2 \to y_3 \to y_4 \to y_5 \to y_6$

and abge is the forward path gain of $y_1 \to y_2 \to y_3 \to y_5 \to y_6$.

Example: Mason's Gain Formula

Loop

The path that starts from one node and ends at the same node is known as **loop**. Hence, it is a closed path.

Examples – $y_2 \rightarrow y_3 \rightarrow y_2$ and $y_3 \rightarrow y_5 \rightarrow y_3$.

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

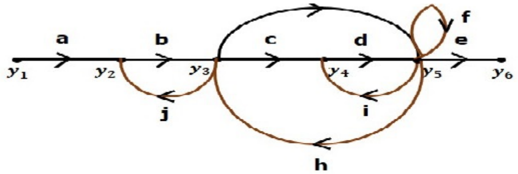
Examples – b_j is the loop gain of $y_2 \rightarrow y_3 \rightarrow y_2$ and g_h is the loop gain of $y_3 \rightarrow y_5 \rightarrow y_3$.

Non-touching Loops

These are the loops, which should not have any common node.

Examples – The loops, $y_2 \rightarrow y_3 \rightarrow y_2$ and $y_4 \rightarrow y_5 \rightarrow y_4$ are non-touching.

Calculation of Transfer Function using Masons Gain Formula



- Number of forward paths, N = 2.
- First forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$.
- First forward path gain, $p_1 = abcde$.
- Second forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.
- Second forward path gain, $p_2 = abge$.
- Number of individual loops, L = 5.

Calculation of Transfer Function using Masons Gain Formula

- Loops are - $y_2 \to y_3 \to y_2$, $y_3 \to y_5 \to y_3$, $y_3 \to y_4 \to y_5 \to y_3$, $y_4 \to y_5 \to y_4$ and $y_5 \to y_5$.
- Loop gains are - $l_1 = bj$, $l_2 = gh$, $l_3 = cdh$, $l_4 = di$ and $l_5 = f$.
- Number of two non-touching loops = 2.
- First non-touching loops pair is - $y_2 \to y_3 \to y_2$, $y_4 \to y_5 \to y_4$.
- Gain product of first non-touching loops pair, $l_1 l_4 = bjdi$
- Second non-touching loops pair is - $y_2 \to y_3 \to y_2$, $y_5 \to y_5$.
- Gain product of second non-touching loops pair is - $l_1 l_5 = bjf$

Calculation of Transfer Function using Masons Gain Formula

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops) + \dots$$

Substitute the values in the above equation,

$$\Delta = 1 - (bj + gh + cdh + di + f) + (bjdi + bjf) - (0)$$

$$\Rightarrow \Delta = 1 - (bj + gh + cdh + di + f) + bjdi + bjf$$

There is no loop which is non-touching to the first forward path.

So, $\Delta_1 = 1$.

Similarly, $\Delta_2 = 1$. Since, no loop which is non-touching to the second forward path.

Calculation of Transfer Function using Masons Gain Formula

Substitute, N = 2 in Mason's gain formula

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

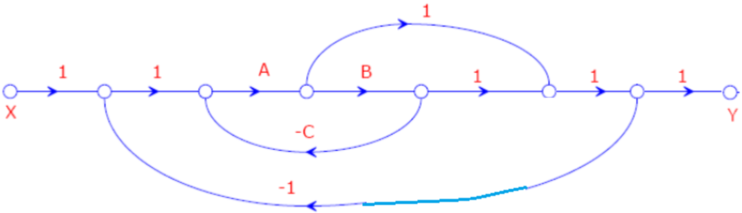
$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Example 1

Using Mason's Formula, Find the T.F. Y(s)/X(s)



Example 1 Solution

$$P_1 = AB \;;\; P_2 = A$$

$$\Delta = 1 - (-ABC - AB - A)$$

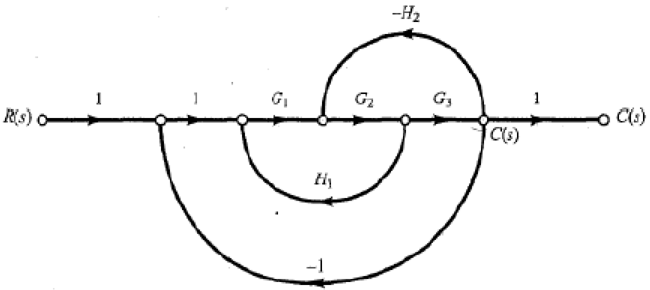
$$\Delta = 1 + ABC + AB + A$$

$$\Delta_1 = 1 \;;\; \Delta_2 = 1$$

$$\frac{Y}{X} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{A(1+B)}{1+ABC+AB+A}$$

Example 2

Using Mason's Formula, Find the T.F. C(s)/R(s)



Solution Example 2

In this system there is only one forward path between the input $R(s)$ and the output $C(s)$. The forward path gain is

$$P_1 = G_1 G_2 G_3$$

we see that there are three individual loops. The gains of these loops are

$$L_1 = G_1 G_2 H_1$$
$$L_2 = -G_2 G_3 H_2$$
$$L_3 = -G_1 G_2 G_3$$

Solution Example 2

Note that since all three loops have a common branch, there are no non-touching loops. Hence, the determinant Δ is given by

$$\Delta = 1 - (L_1 + L_2 + L_3)$$
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

The cofactor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained from Δ by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

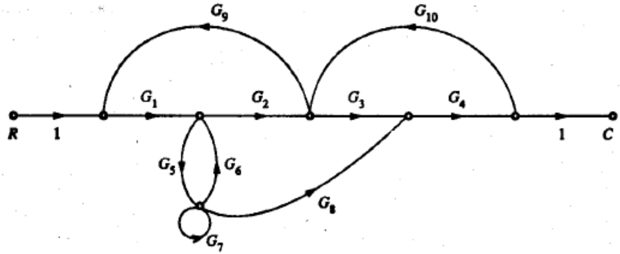
$$\Delta_1 = 1$$

Therefore, the overall gain between the input $R(s)$ and the output $C(s)$, or the closed-loop transfer function, is given by

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

Example 3

Using Mason's Formula, Find the T.F. $C(s)/R(s)$



Solution Example 3

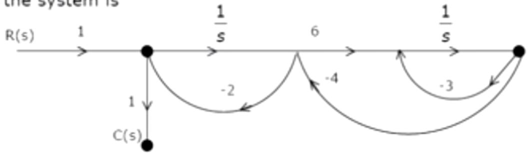
Two forward Path
Four Loops
Two non-touching loops: Three

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 (1 - G_7) + G_1 G_5 G_8 G_4}{1 - [G_1 G_2 G_9 + G_3 G_4 G_{10} + G_1 G_5 G_8 G_4 G_{10} G_9 + G_5 G_6 + G_7] + [G_1 G_2 G_9 G_7 + G_3 G_4 G_{10} G_5 G_6 + G_3 G_4 G_{10} G_7]}$$

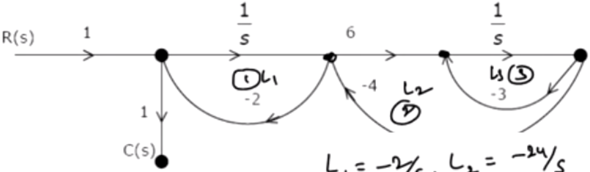
Example 4

The signal flow graph of a system is shown in figure. The transfer function $\frac{C(s)}{R(s)}$ of the system is



(a) $\frac{6}{s^2+29s+6}$ (b) $\frac{6s}{s^2+29s+6}$ (c) $\frac{s(s+2)}{s^2+29s+6}$ (d) $\frac{s(s+27)}{s^2+29s+6}$

Solution Example 4



$L_1 = -2/s$, $L_2 = -24/s$

$L_3 = -3/s$

L_1 & L_3 - Non touching loops

$L_1 L_3 = 6/s^2$

$$\Delta = 1 - \left(-2/s - 24/s - 3/s\right) + \left(6/s^2\right)$$

$$\Delta = 1 + 29/s + 6/s^2 \quad -(1)$$

Solution Example 4

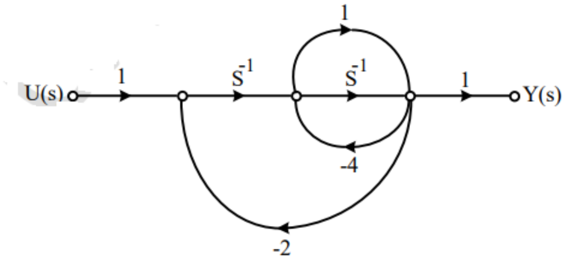
P_i = i^{th} forward path gain = $1 \cdot 1 = 1$

$$\Delta_1 = 1 - (L_2 + L_3) = 1 + 24/s + 3/s = 1 + 27/s$$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta} = \frac{(1 + 27/s)}{(1 + 29/s + 6/s^2)} = \frac{(s+27)(1/s)}{(s^2 + 29s + 6)(1/s^2)} =$$

Example 5

The signal flow graph for a system is given below. The transfer function Y(s)/U(s)



(a) $\frac{s+1}{5s^2+6s+2}$

(b) $\frac{s+1}{s^2+6s+2}$

(c) $\frac{s+1}{s^2+4s+2}$

(d) $\frac{1}{5s^2+6s+2}$

Solution Example 5

Soln. The forward path transmittance $P_1 = S^{-1} \times S^{1} = \frac{1}{S^2}$

The forward path transmittance $P_2 = S^{-1} = \frac{1}{S}$

$$\Delta_1 = 1, \Delta_2 = 1$$

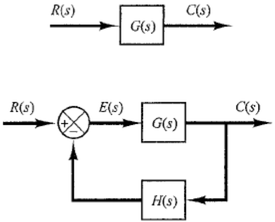
$$\Delta = 1 - (-2S^{-2} - 2S^{-1} - 4S^{-1} - 4)$$

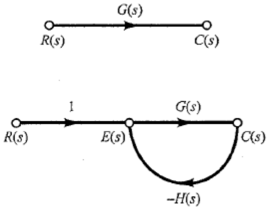
$$= 1 + \frac{2}{S^2} + \frac{2}{S} + \frac{4}{S} + 4$$

$$= 1 + \frac{2}{S^2} + \frac{2}{S} + \frac{4}{S} + 4$$

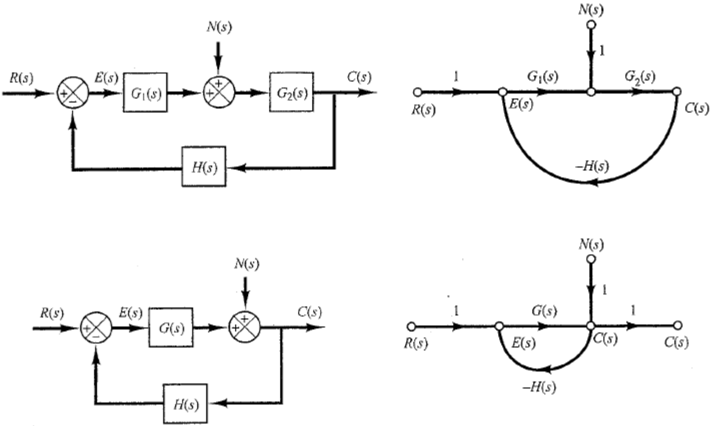
$$= (5S^2 + 6S + 2)/S^2$$

Signal flow graph from Block Diagram

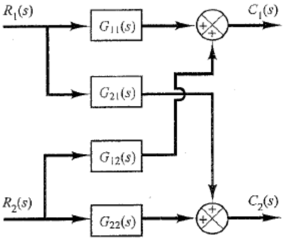


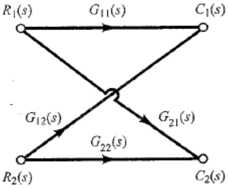


Signal flow graph from Block Diagram



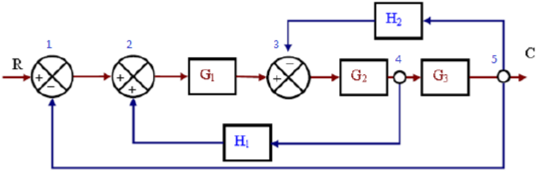
Signal flow graph from Block Diagram





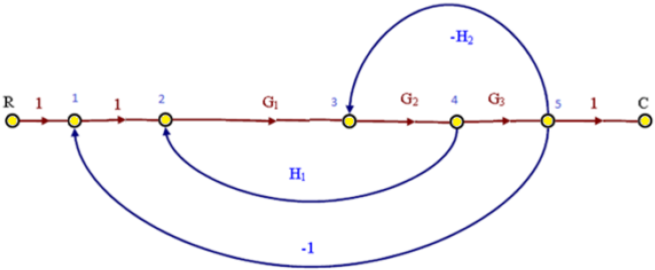
Example: Signal flow graph from Block Diagram

Draw the DFG from the block diagram given below.



Solution of Example:Signal flow graph from Block Diagram

Choose the nodes to represent the variables say 1, 2, .. 5 as shown in the block diagram above. Connect the nodes with appropriate gain along the branch. The signal flow graph is shown below.



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