Control Systems

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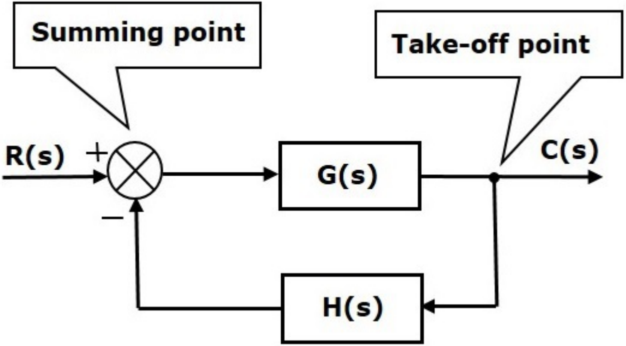
Block Diagram

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

Block, The summing point and The take-off point



Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

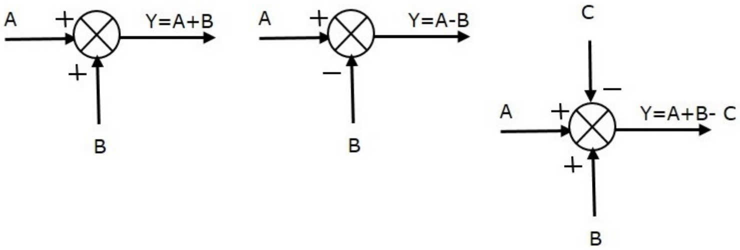
Transfer Function, $$G(s) = \frac{Y(s)}{X(s)}$$

$$\Rightarrow Y(s) = G(s)X(s)$$

Output of the block is obtained by multiplying transfer function of the block with input.

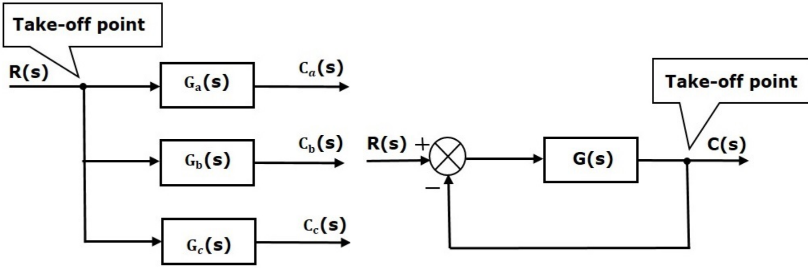
Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs.



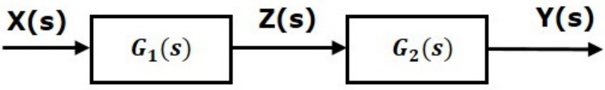
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.



Series Connection

Series connection is also called **cascade connection**. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in series.



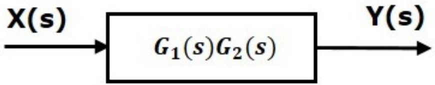
For this combination, we will get the output $Y(s)$ as

$$Y(s) = G_2(s)Z(s)$$

Where, $Z(s) = G_1(s)X(s)$

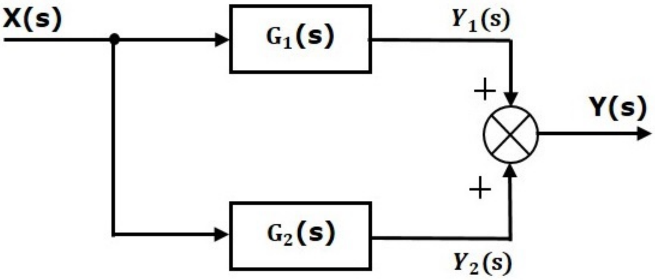
$$\Rightarrow Y(s) = G_2(s)[G_1(s)X(s)] = G_1(s)G_2(s)X(s)$$

Series Connection



Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output $Y(s)$ as

$$Y(s) = Y_1(s) + Y_2(s)$$

Parallel Connection

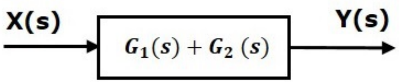
Where, $Y_1(s) = G_1(s)X(s)$ and $Y_2(s) = G_2(s)X(s)$

$$\Rightarrow Y(s) = G_1(s)X(s) + G_2(s)X(s) = \{G_1(s) + G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s) = G(s)X(s)$.

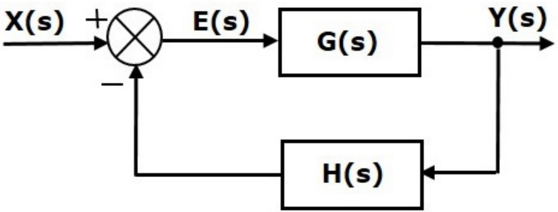
Where, $G(s) = G_1(s) + G_2(s)$.

That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure shows negative feedback control system. Here, two blocks having transfer functions $G(s)$ and $H(s)$ form a closed loop.



The output of the summing point is -

$$E(s) = X(s) - H(s)Y(s)$$

Negative Feedback

The output $Y(s)$ is -

$$Y(s) = E(s)G(s)$$

Substitute $E(s)$ value in the above equation.

$$Y(s) = \{X(s) - H(s)Y(s)\}G(s)\}$$

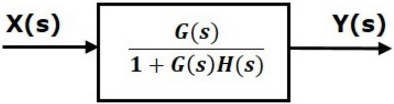
$$Y(s)\{1 + G(s)H(s)\} = X(s)G(s)\}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Feedback Connection

Positive Feedback

This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown below.



Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $\frac{G(s)}{1-G(s)H(s)}$

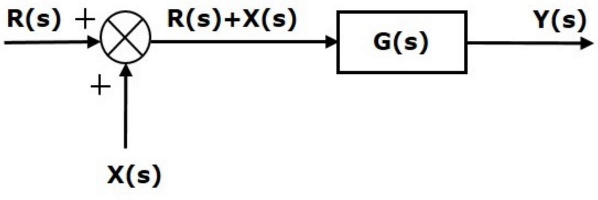
Block Diagram Algebra for Summing Points

There are two possibilities of shifting summing points with respect to blocks:

- Shifting summing point after the block
- Shifting summing point before the block

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure. Here, the summing point is present before the block.



Summing point has two inputs $R(s)$ and $X(s)$. The output of it is $\{R(s) + X(s)\}$.

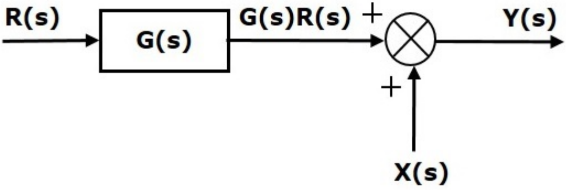
So, the input to the block $G(s)$ is $\{R(s) + X(s)\}$ and the output of it is –

$$Y(s) = G(s)\{R(s) + X(s)\}$$

$$\Rightarrow Y(s) = G(s)R(s) + G(s)X(s) \qquad \textbf{(Equation 1)}$$

Shifting Summing Point After the Block

Now, shift the summing point after the block. This block diagram is shown in the following figure.



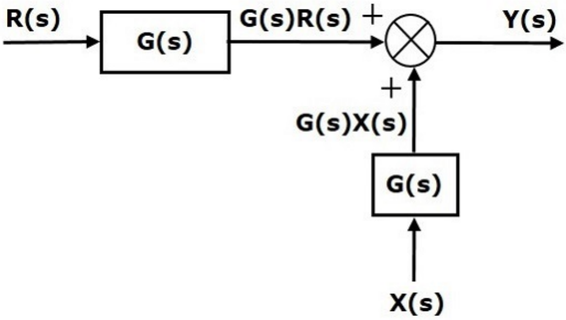
Output of the block $G(s)$ is $G(s)R(s)$.

The output of the summing point is

$$Y(s) = G(s)R(s) + X(s)$$ **(Equation 2)**

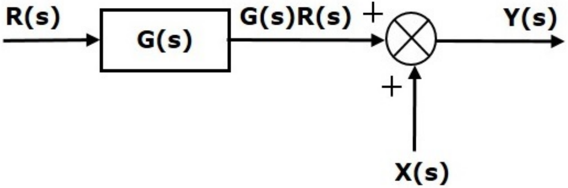
Shifting Summing Point After the Block

The first term '$G(s)R(s)$' is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $G(s)$. It is having the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. This block diagram is shown in the following figure.



Shifting Summing Point Before the Block

Consider the block diagram shown in the following figure. Here, the summing point is present after the block.

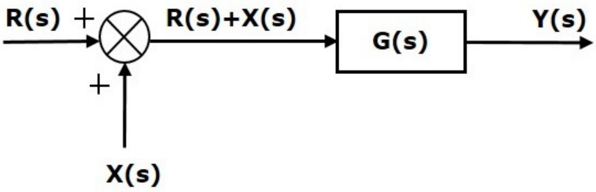


Output of this block diagram is -

$$Y(s) = G(s)R(s) + X(s) \qquad \textbf{(Equation 3)}$$

Shifting Summing Point Before the Block

Now, shift the summing point before the block. This block diagram is shown in the following figure.

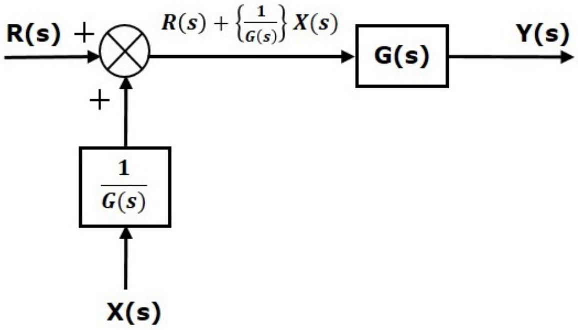


Output of this block diagram is -

$$Y(S) = G(s)R(s) + G(s)X(s) \qquad \textbf{(Equation 4)}$$

Shifting Summing Point Before the Block

The first term $'G(s)R(s)'$ is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block $\frac{1}{G(s)}$. It is having the input $X(s)$ and the output of this block is given as input to summing point instead of $X(s)$. This block diagram is shown in the following figure.



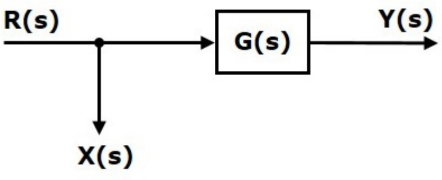
Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to blocks:

- Shifting take-off point after the block
- Shifting take-off point before the block

Shifting Take-off Point After the Block

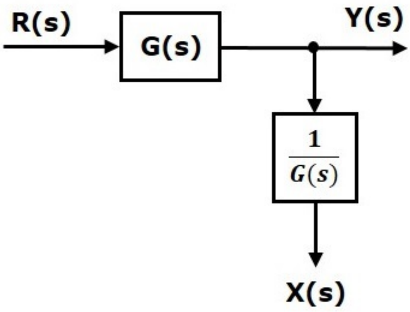
Consider the block diagram shown in the following figure. In this case, the take-off point is present before the block.



Here, $X(s) = R(s)$ and $Y(s) = G(s)R(s)$

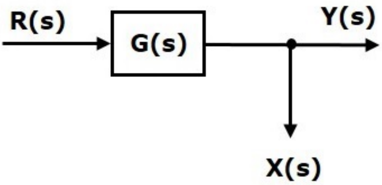
When you shift the take-off point after the block, the output $Y(s)$ will be same. But, there is difference in $X(s)$ value. So, in order to get the same $X(s)$ value, we require one more block $\frac{1}{G(s)}$. It is having the input $Y(s)$ and the output is $X(s)$. This block

Shifting Take-off Point After the Block



Shifting Take-off Point Before the Block

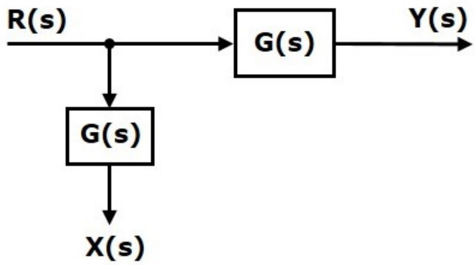
Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



Here, $X(s) = Y(s) = G(s)R(s)$

When you shift the take-off point before the block, the output $Y(s)$ will be same. But, there is difference in $X(s)$ value. So, in order to get same $X(s)$ value, we require one

Shifting Take-off Point Before the Block

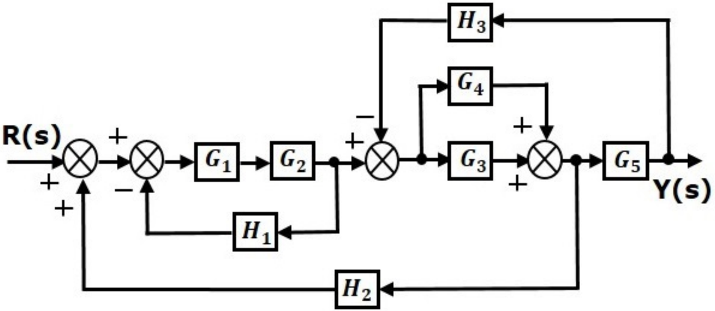


Block diagram Reduction

Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- Rule 1 Check for the blocks connected in series and simplify.
- Rule 2 Check for the blocks connected in parallel and simplify.
- Rule 3 Check for the blocks connected in feedback loop and simplify.
- Rule 4 If there is difficulty with take-off point while simplifying, shift it towards right.
- Rule 5 If there is difficulty with summing point while simplifying, shift it towards left.
- Rule 6 Repeat the above steps till you get the simplified form, i.e., single block.

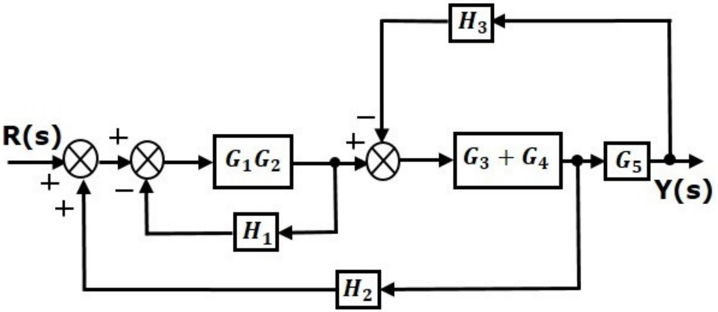
Example 1: Block diagram Reduction



Example 1: Block diagram Reduction

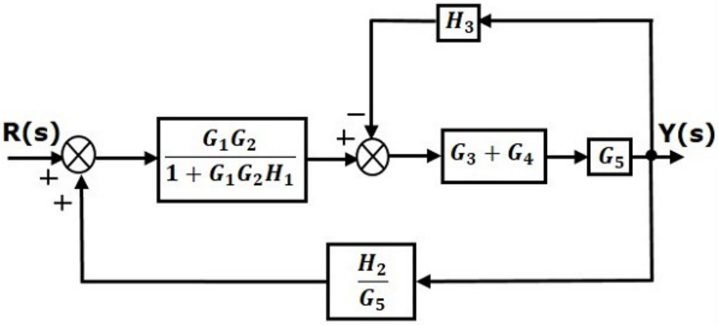
Step 1 – Use Rule 1 for blocks G_1 and G_2 . Use Rule 2 for blocks G_3 and G_4

The modified block diagram is shown in the following figure.



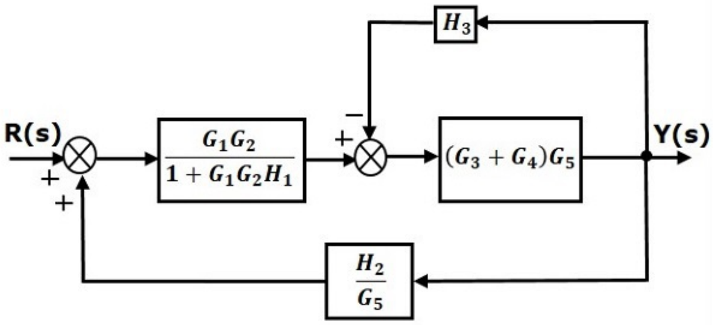
Example 1: Block diagram Reduction

Step 2 – Use Rule 3 for blocks G_1G_2 and H_1. Use Rule 4 for shifting take-off point after the block G_5. The modified block diagram is shown in the following figure.



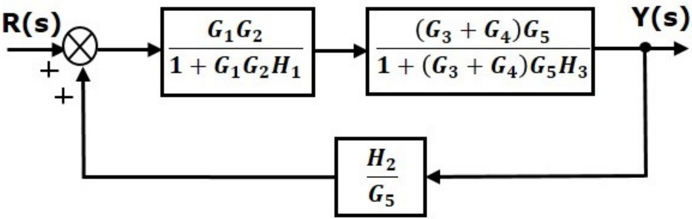
Example 1: Block diagram Reduction

Step 3 – Use Rule 1 for blocks $(G_3 + G_4)$ and G_5. The modified block diagram is shown in the following figure.



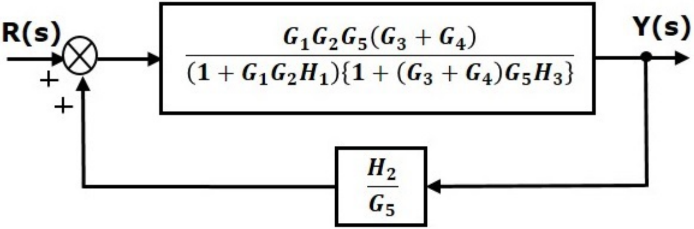
Example 1: Block diagram Reduction

Step 4 – Use Rule 3 for blocks $(G_3 + G_4)G_5$ and H_3. The modified block diagram is shown in the following figure.



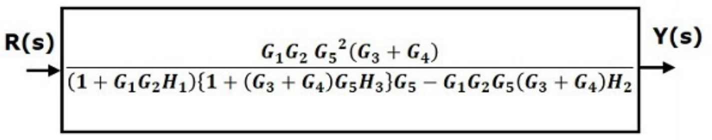
Example 1: Block diagram Reduction

Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Example 1: Block diagram Reduction

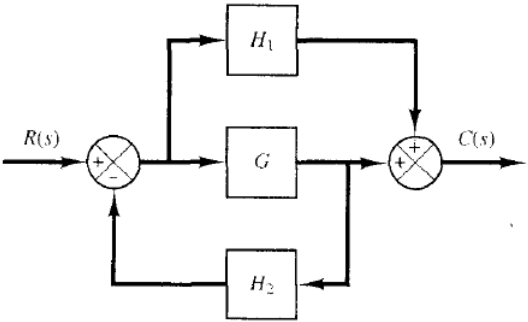
Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



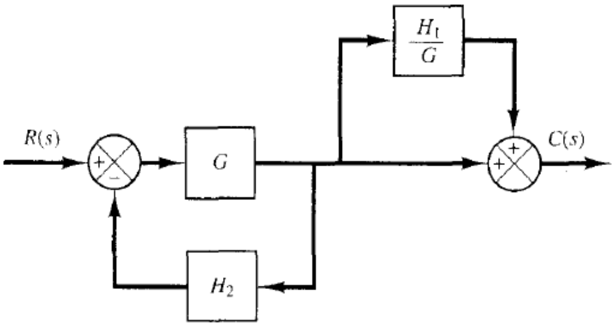
Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_5^2 (G_3 + G_4)}{(1 + G_1 G_2 H_1)\{1 + (G_3 + G_4) G_5 H_3\} G_5 - G_1 G_2 G_5 (G_3 + G_4) H_2}$$

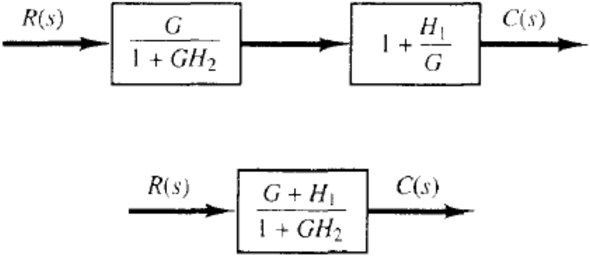
Example 2: Block diagram Reduction



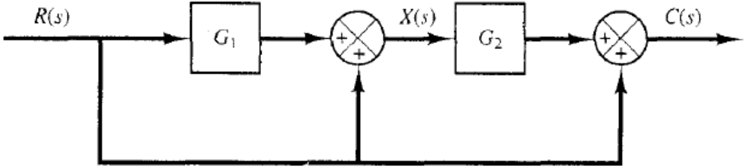
Example 2: Block diagram Reduction



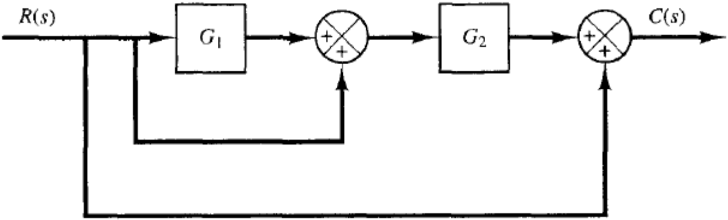
Example 2: Block diagram Reduction



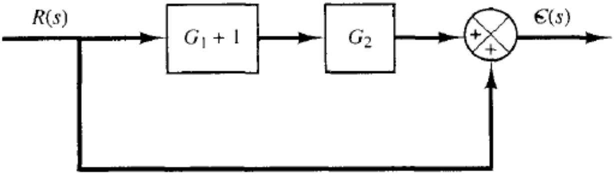
Example 3: Block diagram Reduction



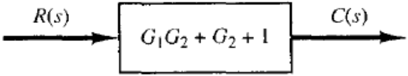
Example 3: Block diagram Reduction



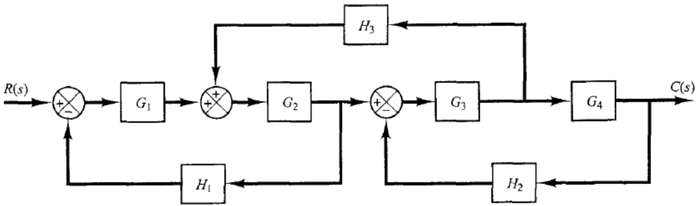
Example 3: Block diagram Reduction



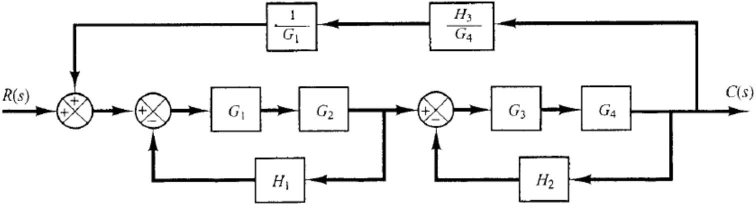
(b)



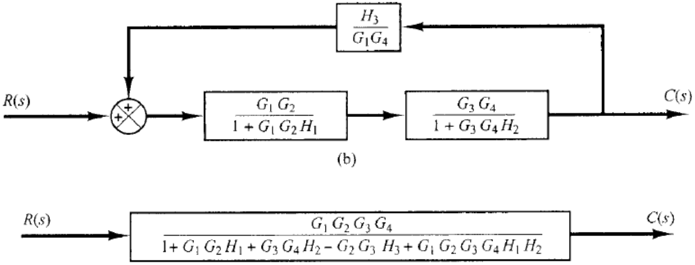
Example 4: Block diagram Reduction



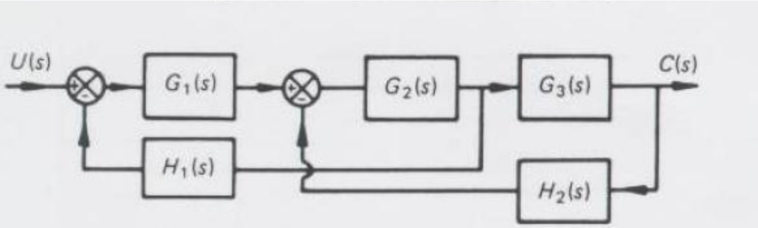
Example 4: Block diagram Reduction



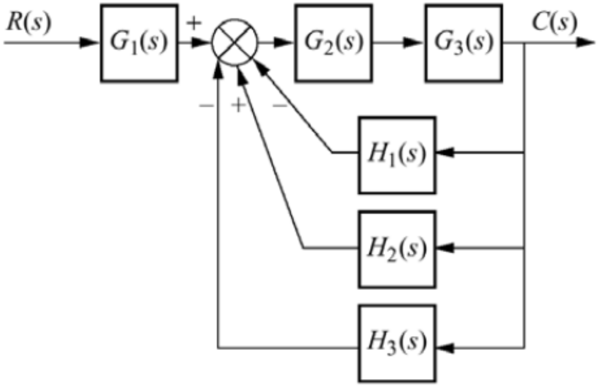
Example 4: Block diagram Reduction



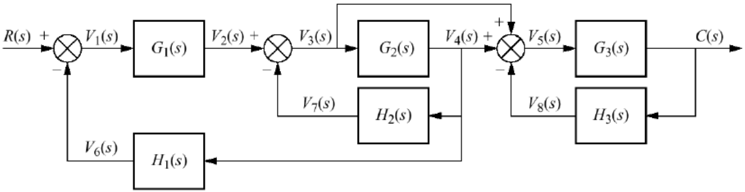
Group 1 Problem: Block diagram Reduction



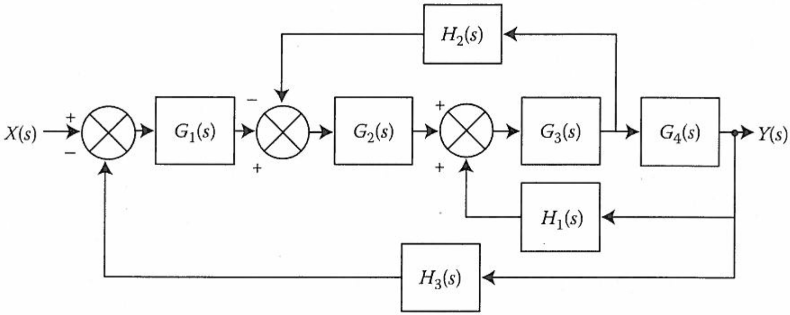
Group 2 Problem: Block diagram Reduction



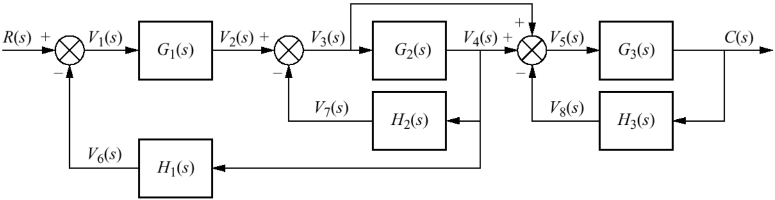
Group 3 Problem: Block diagram Reduction



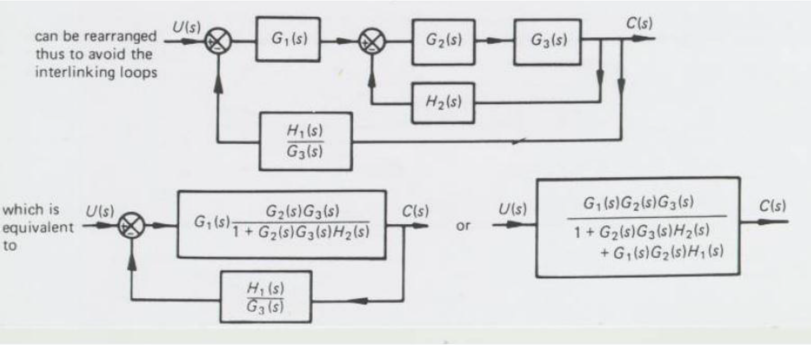
Group 4 Problem: Block diagram Reduction



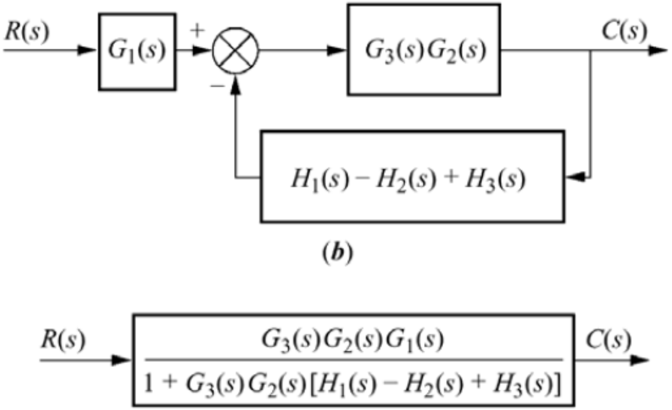
Group 5 Problem: Block diagram Reduction



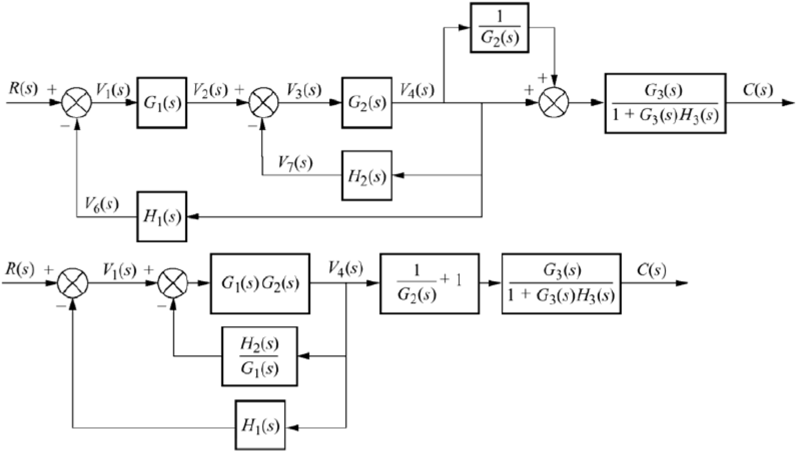
Group 1 Solution: Block diagram Reduction



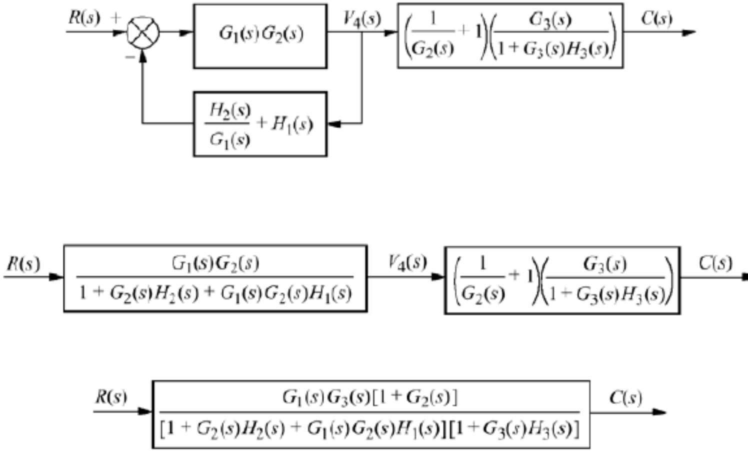
Group 2 Solution: Block diagram Reduction



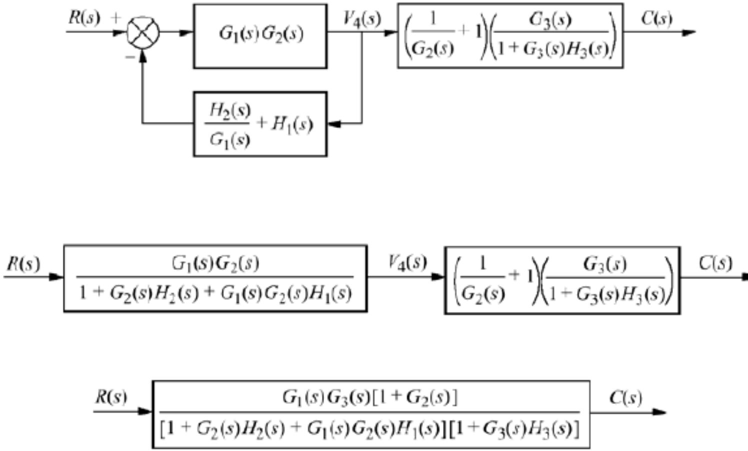
Group 3 Solution: Block diagram Reduction



Group 3 Solution: Block diagram Reduction

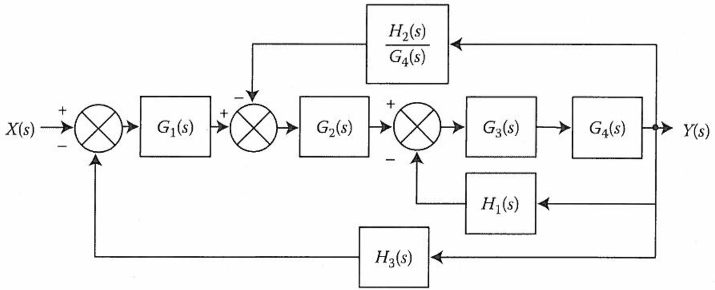


Group 3 Solution: Block diagram Reduction



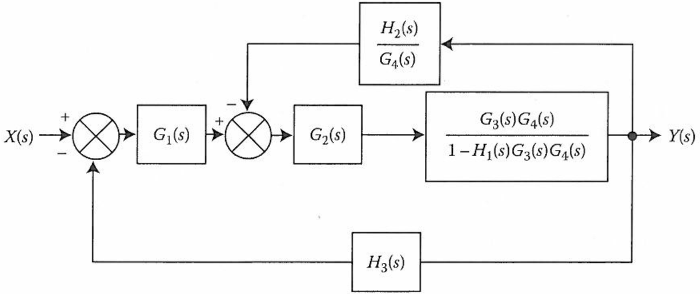
Group 4 Solution: Block diagram Reduction

By applying transformation 7 (Table F2.1), the branch point at the left of the block with transfer function $G_4(s)$ is moved at the right of $G_4(s)$. The equivalent block diagram is:



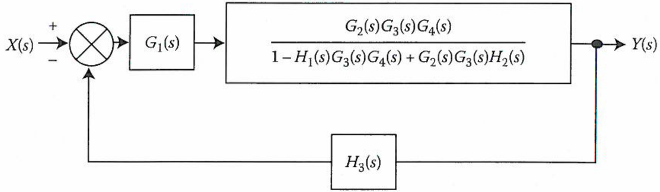
Group 4 Solution: Block diagram Reduction

The next block diagram emerges when transformations 1 and 3 are applied to the loop that contains the blocks with transfer functions $G_3(s)$, $G_4(s)$, and $H_1(s)$.

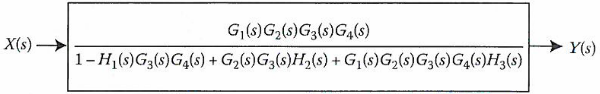


Group 4 Solution: Block diagram Reduction

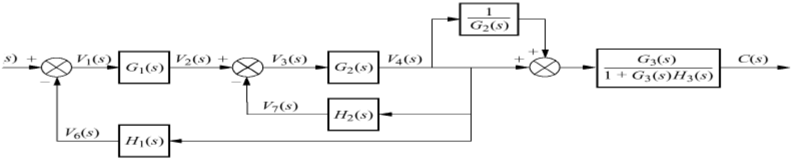
Next we apply transformations 1 and 3 to the loop that contains the transfer function $H_2(s)/G_4(s)$ as feedback and get the following block diagram:



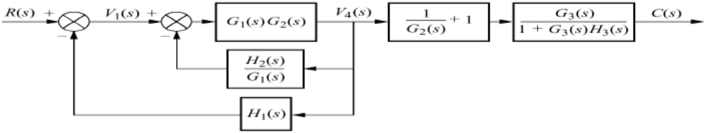
Similarly, by applying transforms 1 and 3 we obtain the simplified block diagram that represents the system's transfer function.



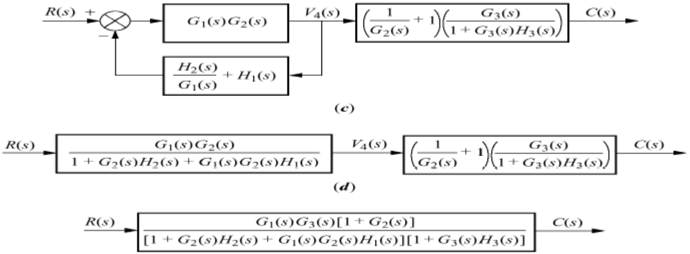
Group 5 Solution: Block diagram Reduction



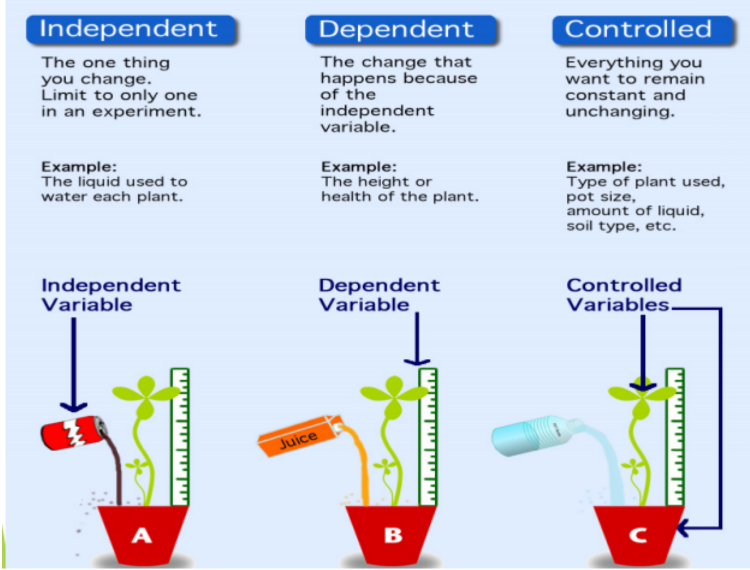
(a)



Group 5 Solution: Block diagram Reduction



Types of Variables



The End