Control Systems

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Role of Control System in Engineering & Science

- played a vital role
- Space-vehicle systems
- Missile-guidance systems
- Robotic systems
- Numerical control of machine tools in the manufacturing industries
- Design of autopilot systems in the aerospace industries
- Google's Driver-less car
- design of cars and trucks in the automobile industries

What does it control?

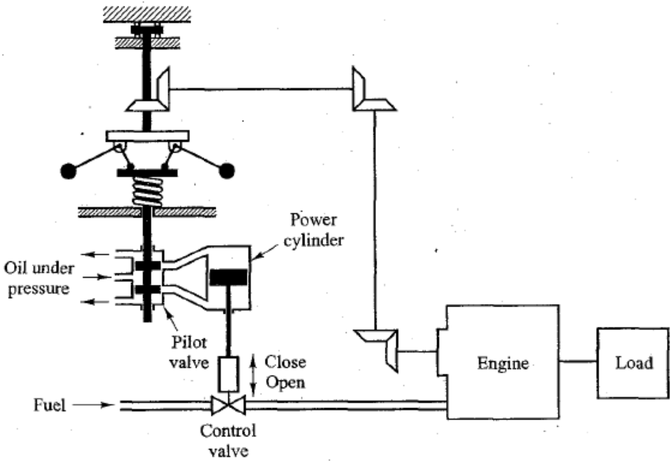
Pressure, Temperature, Humidity, Viscosity, and Flow etc.

Advantage of using Automatic Control System

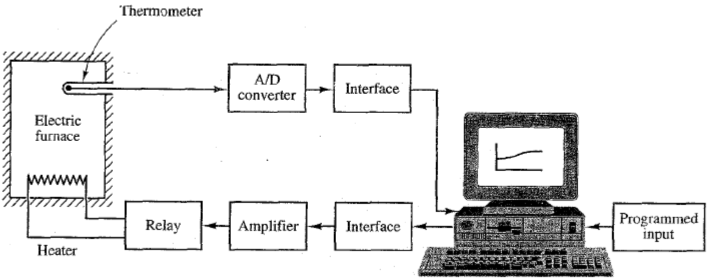
- Productivity
- Relieving the drudgery of many routine repetitive manual operations
- Reducing Manpower

James Watt's centrifugal governor for the speed control of a steam engine

Speed Control System



Temperature Control System



Definitions

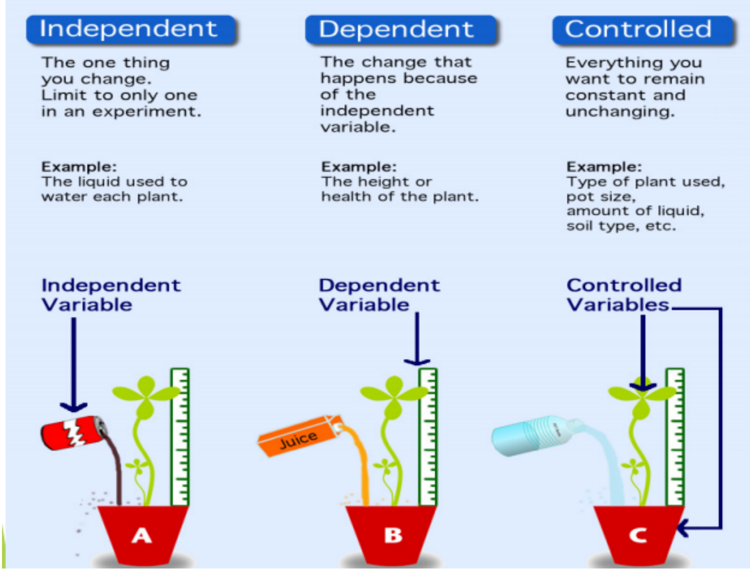
- Systems:
  A system is a combination of components that act together and perform a certain objective.
- Plants:
  A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.
- Processes:
  The Merriam-Webster Dictionary defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes
- Disturbances:
  A disturbance is a signal that tends to adversely affect the value of the output of a system

Definitions (Continue ...)

Controlled Variable and Manipulated Variable:

- The controlled variable is the quantity or condition that is measured and controlled.
- The manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable.
- Normally, the controlled variable is the output of the system.
- Control means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to correct or limit deviation of the measured value from a desired value.

Types of Variables



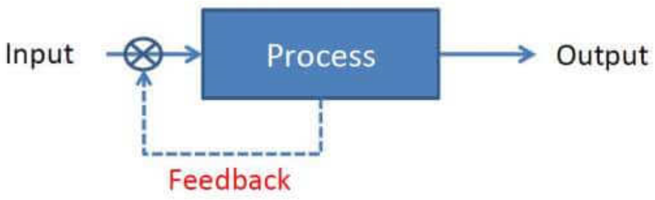
Definitions (Continue ...)

Feedback Control:

- Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference
- Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

Closed Loop Control System

- System that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system/Closed Loop Control System. Eg: Room temperature control system (Smart AC)



Advantage and Disadvantages

- Advantages:
  - Closed loop control systems are more accurate even in the presence of non-linearities
  - The sensitivity of the system may be made small to make the system more stable
  - The closed loop systems are less affected by noise.
- Disadvantages:
  - Closed loop control systems are costlier and complex
  - The feedback in the closed loop system may lead to oscillatory response
  - The feedback reduces the overall gain of the system
  - Stability is the major problem in the closed loop system and more care is needed to design a stable closed loop system.

Open Loop Control System

- Those systems in which the output has no effect on the control action are called open-loop control systems.
  Eg: Washing machine

Advantage and Disadvantages

Open -Loop Control system is called as Manual control system.

- Advantages:
  - Simplicity and stability: they are simpler in their layout and hence are economical and stable too due to their simplicity.
  - Construction: Since these are having a simple layout so are easier to construct.
- Disadvantages:
  - Accuracy and Reliability: since these systems do not have a feedback mechanism, so they are very inaccurate in terms of result output and hence they are unreliable too.
  - Due to the absence of a feedback mechanism, they are unable to remove the disturbances occurring from external sources.

Close Loop Vs Open Loop

Open Loop Control System	Closed-Loop Control System
In this system, the controlled action is free from the output	In this system, the output mainly depends on the controlled act of the system.
This control system is also called a Non feedback control system	This type of control system is also called a feedback control system
The components of this system include a controlled process and controller.	The components of this kind of system include an amplifier, controlled process, controller and feedback
The construction of this system is simple	The construction of this system is complex
The consistency is non-reliable	The consistency is reliable

Close Loop Vs Open Loop

Open Loop Control System	**Closed-Loop Control System**
The accuracy of this system mainly depends on the calibration	These are accurate due to feedback
The stability of these systems are stable	The stability of these systems are less stable
The optimization in this system is not possible	The optimization in this system is possible
The response is fast	The response is slow
The calibration of this system is difficult	The calibration of this system is easy
The disturbance of this system will be affected	The disturbance of this system will not be affected
These systems are non-linear	These systems are linear
The best examples of this control system are automatic washing machine, traffic light, TV remote,	Examples of this kind of control system are AC, control systems for temperature, pressure and

Effects of Negative Feedback

- Feedback reduce the time constant of the system and system becomes more fast.
- It reduces the effects of the disturbance.
- Feedback increases the bandwidth i.e. frequency range over which system performance is satisfactory.
- Negative feedback tends to reduce the effects of gain change giving what is generally called gain stability.

Requirement of Good Control System

There are some specifications for a system to be a good control system:

- Sensitivity
- Accuracy
- Stability
- Noise
- Bandwidth
- Oscillation
- Speed

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- Sensitivity:
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- Accuracy:
  The tolerance to errors of an instrument is known as Accuracy. It defines the limits of errors of an instrument at normal operating conditions. We can improve the accuracy by using feedback elements.

Requirement of Good Control System

- Stability:
  Stability of a system can be explained as, if the input of the system is zero or null then its output should also be zero value. If the input changes, the output also changed as per the system function. Then the system is considered as stable.

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- Stability:
  Stability of a system can be explained as, if the input of the system is zero or null then its output should also be zero value. If the input changes, the output also changed as per the system function. Then the system is considered as stable.
- Noise:
  The undesired signal input that is occurred or added to input signal by external resources is known as Noise. A good control system should have the high noise tolerance value. This means it should be able to reduce the noise level. If the noise occurrence increases, the system performance will decrease.

Requirement of Good Control System

- Bandwidth:
  Bandwidth is defined as the range of frequencies of a system. Band width is decided by the operating frequencies. The system having high band width is considered as a good control system.

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- Oscillation:
  Oscillation means the fluctuations of output of system. These oscillations will effect the stability. The increase in the number of fluctuations of a system will decrease the stability of the system.
- Speed:
  In a control system, the time taken by the output to be stable is known as Speed. High speed systems are considered as good control systems. The time taken by the output to reach its stable state is known as Transient state.

Mathematical Modeling of Control Systems

- The control systems can be represented with a set of mathematical equations known as mathematical model.
- These models are useful for analysis and design of control systems.
- Analysis of control system means finding the output when we know the input and mathematical model.
- Design of control system means finding the mathematical model when we know the input and the output.

Types of Mathematical Models of Control Systems

The following mathematical models are mostly used:

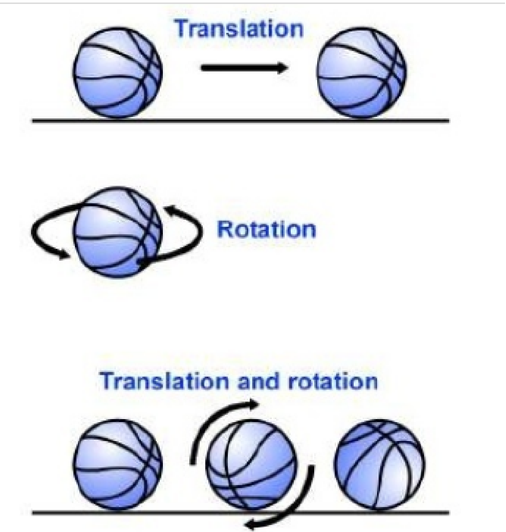
- Differential equation model
- Transfer function model
- State space model

Types of mechanical Control Systems

There are two types of mechanical systems based on the type of motion:

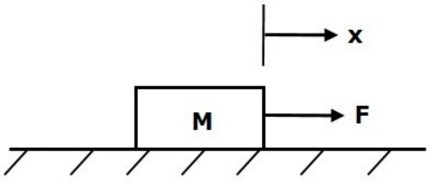
- Translational mechanical systems:
  Translational mechanical systems move along a straight line. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.
- Rotational mechanical systems:
  Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are moment of inertia, torsional spring and dashpot.

Types of mechanical Control Systems



Mass

Mass is the property of a body, which stores kinetic energy. If a force is applied on a body having mass M, then it is opposed by an opposing force due to mass.



$$F_m \propto a$$

$$\Rightarrow F_m = Ma = M\frac{d^2x}{dt^2}$$

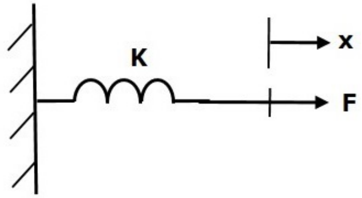
$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,

- **F** is the applied force
- $\mathbf{F_m}$ is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Spring

Spring is an element, which stores potential energy. If a force is applied on spring K, then it is opposed by an opposing force due to elasticity of spring.



$$F \propto x$$

$$\Rightarrow F_k = Kx$$

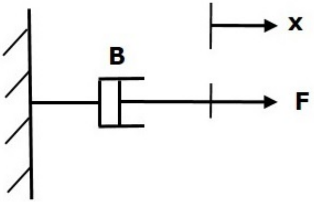
$$F = F_k = Kx$$

Where,

- **F** is the applied force
- **F$_k$** is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement

Dashpot

If a force is applied on dashpot B, then it is opposed by an opposing force due to friction of the dashpot. This opposing force is proportional to the velocity of the body.



$$F_b \propto \nu$$

Where,

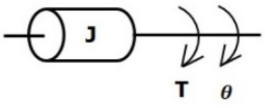
$$\Rightarrow F_b = B\nu = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

- F_b is the opposing force due to friction of dashpot
- **B** is the frictional coefficient
- **v** is velocity
- **x** is displacement

$$F = F_b = B\frac{\mathrm{d}x}{\mathrm{d}t}$$

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores kinetic energy.



$$T_j \propto \alpha$$

Where,

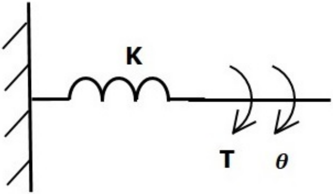
- **T** is the applied torque
- $\mathbf{T_j}$ is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement

$$\Rightarrow T_j = J\alpha = J\frac{d^2\theta}{dt^2}$$

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Torsional Spring

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores potential energy.



$T_k \propto \theta$ Where,

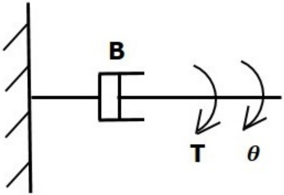
- **T** is the applied torque
- $\mathbf{T_k}$ is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

$\Rightarrow T_k = K\theta$

$T = T_k = K\theta$

Dashpot

If a torque is applied on dashpot B, then it is opposed by an opposing torque due to the rotational friction of the dashpot.



$T_b \propto \omega$

$$\Rightarrow T_b = B\omega = B\frac{d\theta}{dt}$$

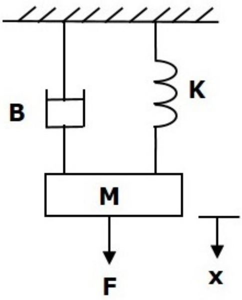
$$T = T_b = B\frac{d\theta}{dt}$$

Where,

- $\mathbf{T_b}$ is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- $\boldsymbol{\omega}$ is the angular velocity
- $\boldsymbol{\theta}$ is the angular displacement

Example

Derive the transfer function for following mechanical system



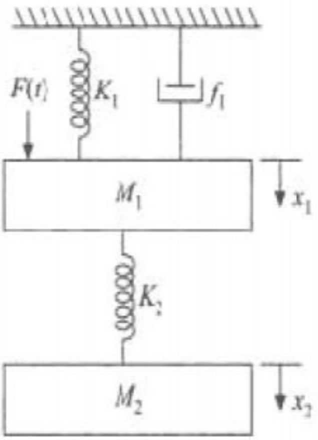
Solution: Example

The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Example 2

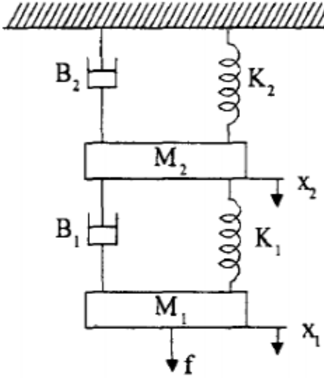


Solution: Example 2

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + f_1 \frac{dx_1}{dt} + K_1 x_1 + K_2 (x_1 - x_2)$$

$$M_2 \frac{d^2 x_2}{dt^2} + K_2 (x_2 - x_1) = 0$$

Example 2



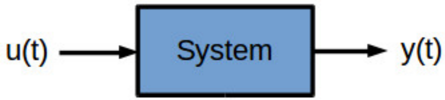
Solution: Example 2

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2) = f$$

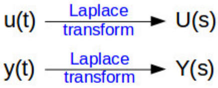
$$M_2 \frac{d^2 x_2}{dt} + B_2 \frac{dx_2}{dt} + B_1 \frac{d(x_2 - x_1)}{dt} + K_2 x_2 + K_1 (x_2 - x_1) = 0$$

Transfer Function

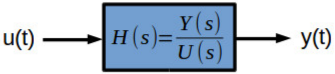
A transfer function is determined using Laplace transform and plays a vital role in the development of the automatic control systems theory.
A system can be defined as a mathematical relationship between the input, output and the states of a system. In control theory, a system is represented a a rectangle with an input and output.



Transfer Function



The **transfer function** defines the relation between the **output** and the **input** of a dynamic system, written in complex form (***s*** variable). For a dynamic system with an input *u(t)* and an output *y(t)*, the transfer function *H(s)* is the ratio between the complex representation (***s*** variable) of the output *Y(s)* and input *U(s)*.

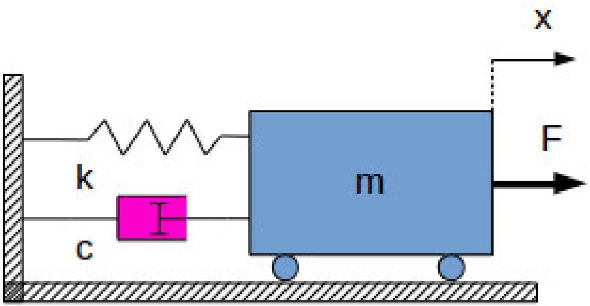


Laplace Transform of Derivative

For a given continuous and differentiable function f(t), the following Laplace transforms properties applies:

$$\mathcal{L}[f(t)] = F(s)$$
$$\mathcal{L}[f'(t)] = sF(s) - f(0)$$
$$\mathcal{L}[f''(t)] = s^2F(s) - sf(0) - f'(0)$$

Transfer Function for Mechanical system



Transfer Function

$$F(t) = m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t)$$

$$\mathcal{L}\left[\frac{d^2x(t)}{dt^2}\right] = s^2X(s) - sx(0) - \frac{dx(0)}{dt}$$

$$\mathcal{L}\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0)$$

$$\mathcal{L}\left[x(t)\right] = X(s)$$

$$\mathcal{L}[F(t)] = F(s)$$

Deriving Transfer Function

$$x(0) = 0$$
$$\frac{dx(0)}{dt} = 0$$

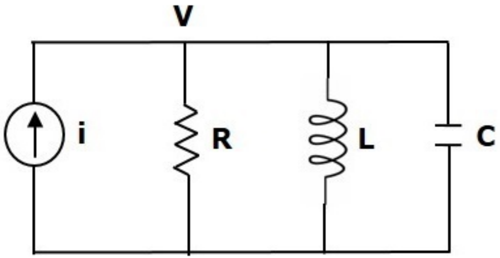
Replacing the Laplace transforms and initial conditions in the equation (1) gives:

$$F(s) = ms^2X(s) + csX(s) + kX(s)$$
$$F(s) = X(s)\left(ms^2 + cs + k\right)$$
$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

We have now found the **transfer function** of the translational mass system with spring and damper:

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

Deferential equation for Electronics Circuit



Deferential equation for Electronics Circuit

The nodal equation is

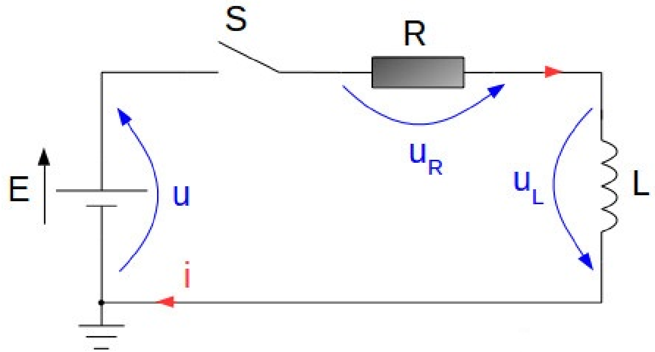
$$i = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} \qquad \textbf{(Equation 5)}$$

Substitute, $V = \frac{d\Psi}{dt}$ in Equation 5.

$$i = \frac{1}{R}\frac{d\Psi}{dt} + \left(\frac{1}{L}\right)\Psi + C\frac{d^2\Psi}{dt^2}$$

$$\Rightarrow i = C\frac{d^2\Psi}{dt^2} + \left(\frac{1}{R}\right)\frac{d\Psi}{dt} + \left(\frac{1}{L}\right)\Psi \qquad \textbf{(Equation 6)}$$

Transfer Function for Electronics Circuit



Transfer Function for Electronics Circuit

The ordinary differential equation describing the dynamics of the RL circuit is:

$$u(t) = L\frac{di(t)}{dt} + Ri(t)$$

where:

R [Ω] – resistance

L [H] – inductance

u [V] – voltage drop across the circuit

i [A] – electrical current through the circuit

Deriving Transfer Function for Electronics Circuit

$$\mathcal{L}\left[\frac{di(t)}{dt}\right] = sI(s) - i(0)$$
$$\mathcal{L}\left[i(t)\right] = I(s)$$
$$\mathcal{L}[u(t)] = U(s)$$

The initial condition of the electrical current is:

$$i(0) = 0$$

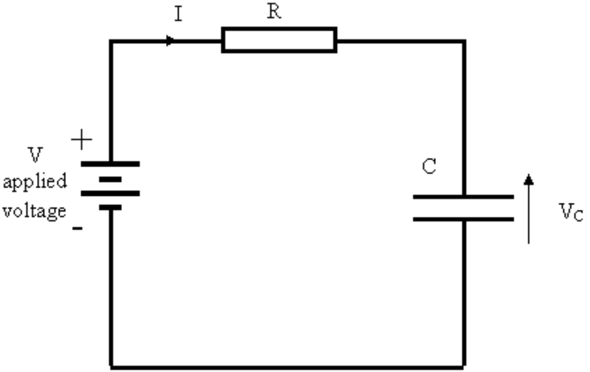
Replacing the Laplace transforms and initial conditions in the equation (2) gives:

$$U(s) = LsI(s) + RI(s)$$
$$U(s) = I(s)\,(Ls + R)$$
$$\frac{I(s)}{U(s)} = \frac{1}{Ls + R}$$

We have now found the **transfer function** of the **series RL circuit**:

$$H(s) = \frac{I(s)}{U(s)} = \frac{1}{Ls + R}$$

Deriving Transfer Function for RC Circuit



Deriving Transfer Function for RC Circuit

$$V = V_R + V_C$$

Since V_R=IR,

$$V = IR + V_C$$

Now, the current I, is the rate at which charge moves to or from the capacitor plates, i.e. $I = dq/dt$. Also, the charge, $q = V_C x C$.

$$V = R\frac{dq}{dt} + V_C$$

$$\Rightarrow V = RC\frac{dV_C}{dt} + V_C$$

$$\Rightarrow RC\frac{dV_C}{dt} + V_C = V$$

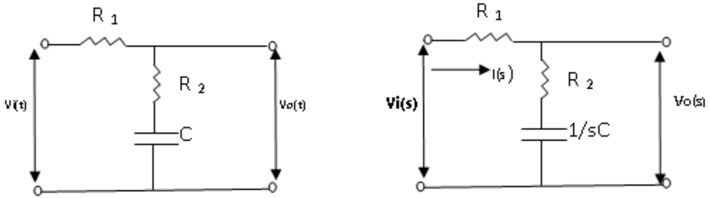
$$RC\frac{dV_C}{dt} + V_C = V$$

$$\Rightarrow RCsV_C(s) + V_C(s) = V(s)$$

$$RCsV_C(s)(RCs+1) = V(s)$$

$$\Rightarrow \frac{V_C(s)}{V(s)} = \frac{1}{(RCs+1)}$$

Example: Deriving Transfer Function for Electrical Circuit



Example: Deriving Transfer Function for Electrical Circuit

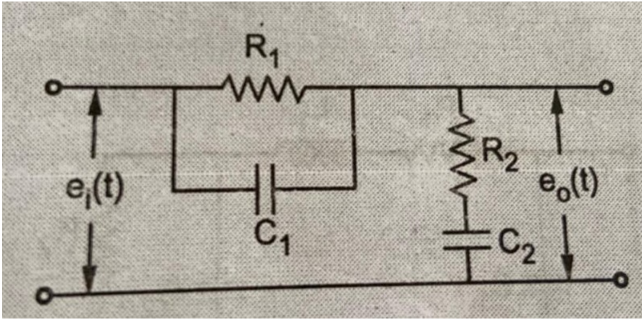
By KVL in the left hand- mesh,

$V_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{sC} I(s)$By KVL in the right-hand- mesh

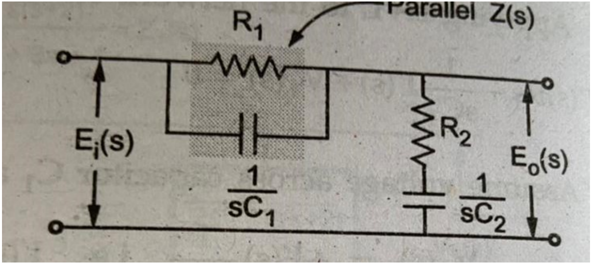
$V_o(s) = R_2 I(s) + \frac{1}{sC} I(s)$The transfer function from the above two equations is given b

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 sC+1}{(R_1+R_2)sC+1}$$

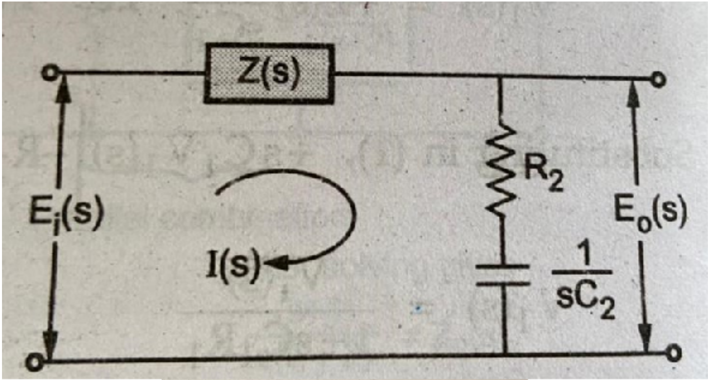
Example: Deriving Transfer Function for Electrical Circuit



Example: Deriving Transfer Function for Electrical Circuit



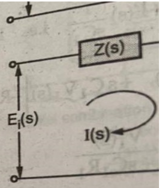
Example: Deriving Transfer Function for Electrical Circuit



$$Z(s) = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1+sC_1R_1}$$

Example: Deriving Transfer Function for Electrical Circuit

Replacing parallel combination by impedance Z as shown,



Apply KVL,

$$E_i(s) = Z\,I(s) + R_2 I(s) + \frac{I}{sC_2} I(s)$$

$$E_o(s) = I(s)\left[R_2 + \frac{1}{sC_2}\right]$$

$$\therefore \quad I(s) = \frac{sC_2\,E_o(s)}{1+sR_2\,C_2}$$

$$\therefore \quad E_i(s) = \frac{sC_2\,E_o(s)}{(1+sR_2\,C_2)}\left[Z(s)+R_2+\frac{1}{sC_2}\right]$$

$$E_i(s) = \frac{sC_2\,E_o(s)}{(1+sR_2\,C_2)}\left[\frac{Z(s)\,s\,C_2+1+sC_2\,R_2}{sC_2}\right]$$

$$\frac{E_o(s)}{E_i(s)} = \frac{(1+sR_2\,C_2)}{(1+sC_2\,R_2+Z(s)\,C_2)}$$ and using value of Z(s

$$\frac{E_o(s)}{E_i(s)} = \frac{(1+sC_1\,R_1)\,(1+sR_2\,C_2)}{(1+sC_2\,R_2)\,(1+sC_1\,R_1)+sR_1\,C_2}$$

Analogous Systems

Two systems are said to be analogous to each other if the following two conditions are satisfied.

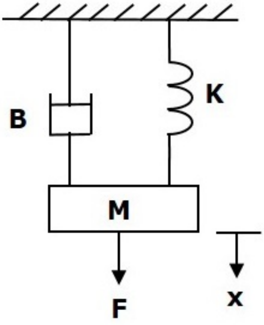
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of translational mechanical system are compared with mesh equations of the electrical system.

Force Voltage Analogy

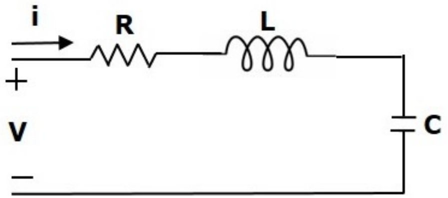


The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Force Voltage Analogy



Mesh equation for this circuit is

$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i dt$$

Substitute, $i = \frac{dq}{dt}$ in Equation 2.

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \left(\frac{1}{c}\right)q$$

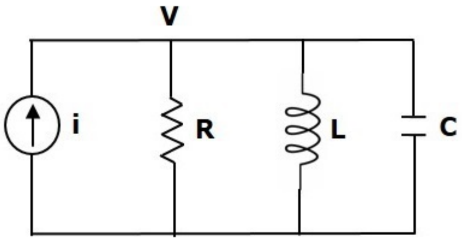
Force Voltage Analogy

Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the translational mechanical system are compared with the nodal equations of the electrical system.

Force Current Analogy



The nodal equation is

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dV}{dt}$$

Substitute, $V = \frac{d\Psi}{dt}$ in Equation 5.

$$i = \frac{1}{R}\frac{d\Psi}{dt} + \left(\frac{1}{L}\right)\Psi + C\frac{d^2\Psi}{dt^2}$$

$$\Rightarrow i = C\frac{d^2\Psi}{dt^2} + \left(\frac{1}{R}\right)\frac{d\Psi}{dt} + \left(\frac{1}{L}\right)\Psi$$

Force Current Analogy

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance $(\frac{1}{R})$
Spring constant(K)	Reciprocal of Inductance $(\frac{1}{L})$
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

The End