Network Theory(19EC33)

2020-21

Class-1: Overview of Syllabus and Introduction to Network Theory

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Overview of Syllabus

- Subject Title : **Network Theory**
- Subject code : **19EC33**
- Credits: **04**
- Total number of Contact hours : **52 hours**
- Number of teaching hours per week: **04 hours**
- **3**- CIE's, **25** marks each
- Final CIE=Sum of two best CIE marks and reduced it to **40** marks + **5** marks Assignment + **5** marks Group Activity.
- Assignments – **Problems**
- Group Activity – **PSPICE Simulation**

Overview of Syllabus…

Pre-requisites:

- Engineering Mathematics
- Basic Electrical Engineering

Objectives:

- Different types of Electrical Elements and their characteristics.
- Circuit Analysis Techniques such as Circuit simplification, loop analysis and node analysis.
- Different Network Theorems and its applications,.
- Concepts of Resonance and its importance.
- Study of dynamic behavior(Transient and steady state response) of electrical systems using initial conditions
- Applications of Laplace Transforms to electrical systems.
- Two port networks and its importance in the analysis of electrical circuits.

Overview of Syllabus…

Contents:

Unit-I:

Ch-1: Basic Circuit Concepts

Unit-II:

Ch-1: Network Theorems

Ch-2: Resonant Circuits

Unit-III:

Ch-1: Transient Behaviour and Initial Conditions

Unit-IV:

Ch-1: Laplace Transforms

Unit-V:

Ch-1: Two Port Network Parameters

Overview of Syllabus…

Outcomes:

- Apply the network reduction techniques to simplify the electrical circuits and analyze electrical circuits using loop and nodal analysis.
- Apply the network theorems to find the load quantities, explain the resonant parameters and the analyze the circuit.
- Explain and find the transient behavior of electrical circuits with initial conditions.
- Apply the Laplace Transforms for the analysis of electrical circuits.
- Define, explain and find the two port network parameters of electrical circuits and derive the relationship between one parameter to other parameter.

Pre-Requisite for:

- Electronic Circuits
- Communications
- Power Electronics

Overview of Syllabus…

- **Text Books:**
  - Charles K Alexander and Mathew N O Sadiku, "**Fundamentals of Electric Circuits**", 3rd edition, Tata McGraw-Hill, 2009.

Introduction

Definition: Network Theory

Theory: A set of Principles or Ideas are used to perform an activity (In this context activity is to study and analysis of a Networks).

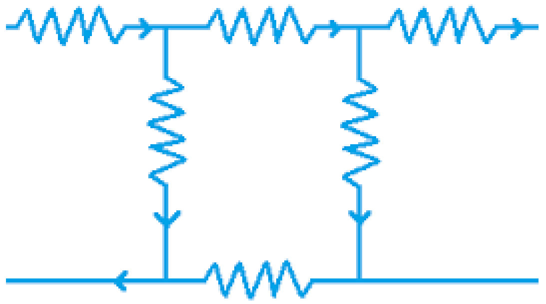
Network (Electrical): Interconnection or combination of electrical elements is called an electrical network, generally network.

Network Theory: Set of principles or ideas are used to study the behaviour of electrical networks.

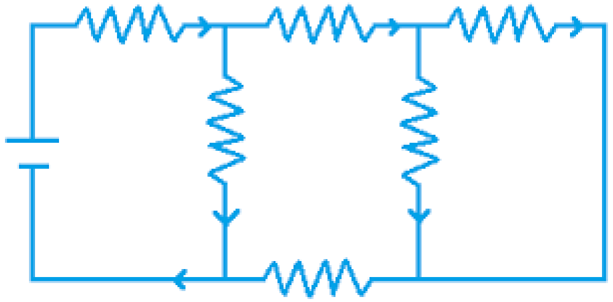
Circuit and Network:

Network- Open loop or Closed Loop

Circuit- Closed Loop



Network



Network/Circuit

All circuits are networks but all networks are not circuits

Introduction…

Electrical Elements

- Electrical Elements
  - Passive Elements
    - Resistors
    - Capacitors
    - Inductors
  - Active Elements
    - Independent sources
      - Ideal
        - Voltage
        - Current
      - Practical
        - voltage
        - current
    - Dependent sources
      - VCVS
      - VCIS
      - ICVS
      - ICIS

Introduction…

Basic principles

1. **Current:** Rate of change of Charge is called current, it is denoted as "I" and unit is Amperes.

$$I = \frac{dQ}{dt}$$

1. **Voltage:** Rate of change of Flux is called Voltage, it is denoted as "V" and unit is volts.

$$V = \frac{d\O}{dt}$$

1. **Power:** Product of Voltage and Current is called Power, it is denoted as "P" and unit is watts.

$$P = V \times I$$

Branch: A Path of element is connecting between two nodes is called Branch.

Node: Two or more elements connected at a Point/Junction is called as a Node.

Ohm's law: Ohm's law states that the voltage across an element is directly proportional to the current flowing through that element.

$$i.e.,\ V \alpha\ I$$

Introduction…

Passive Elements

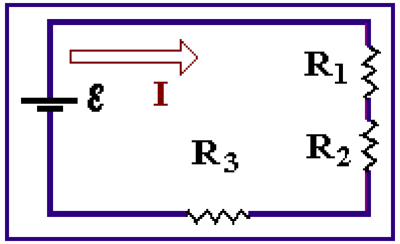
1. Resistor[R]:

Circuit Symbol

- A resistor opposes the flow of electric current.
- Resistors dissipate energy in the form of heat.
- Resistors exhibit negative temperature effects.
- Obeys ohm's Law

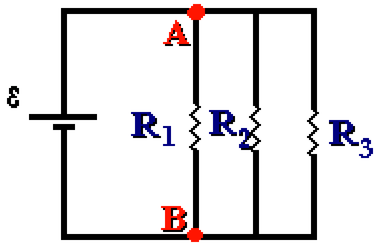
$$V \alpha I,\ V = R\ I \text{ ohms.}$$

Where, R is the Proportionality constant called Resistance measured in Ohms, V is the Voltage and I is the Current.



Resistors are in Series

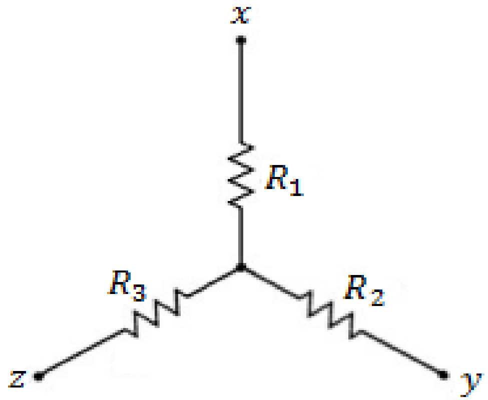
- If Resistors are connected in Series. $R_{eq} = \sum_{i=1}^{n} R_i$
- If Resistors are connected in Parallel. $\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$
- Power $P = VxI \Rightarrow \frac{V^2}{R} \Rightarrow I^2 R\ Watts$



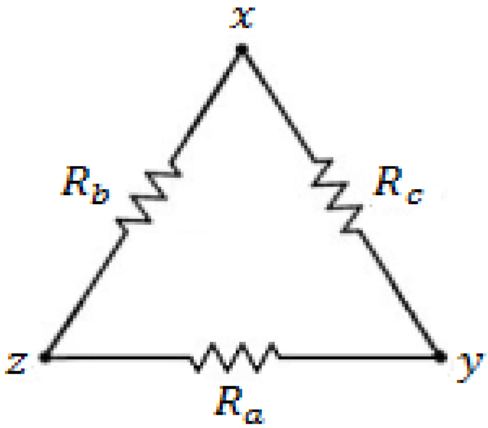
Resistors are in Parallel

Introduction…

Resistors are connected in Star form.



Resistors are connected in Delta form.



- **Star to Delta conversion**

$$R_a = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1};\ R_b = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2};\ R_c = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3}.$$

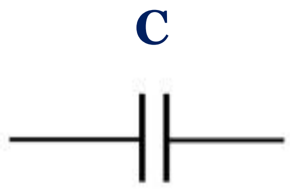
- **Delta to Star conversion**

$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c};\ R_2 = \frac{R_aR_c}{R_a+R_b+R_c};\ R_3 = \frac{R_aR_b}{R_a+R_b+R_c}.$$

Introduction…

2. Capacitor [C]:

- Capacitors stores energy in the form of Electrostatic Field.
- $Q = CV$
- v(t)= $\frac{1}{C}\int i(t).dt$; i(t) = C$\frac{dv(t)}{dt}$.
- $E = \frac{1}{2}CV^2$ Joules.
- $C_{eq} = \frac{1}{\sum_{i=1}^{n} 1/C_i}$; If the Capacitors are connected in Series.
- $C_{eq} = \sum_{i=1}^{n} C_i$; If the Capacitors are connected in Parallel.
- Star to delta and delta to star conversion is applicable to capacitors, only if it is in Reactance form.
- Capacitive Reactance $X_C = \frac{1}{2\pi f C}$ Ohms.

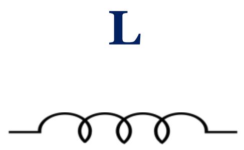


Circuit Symbol

Introduction…

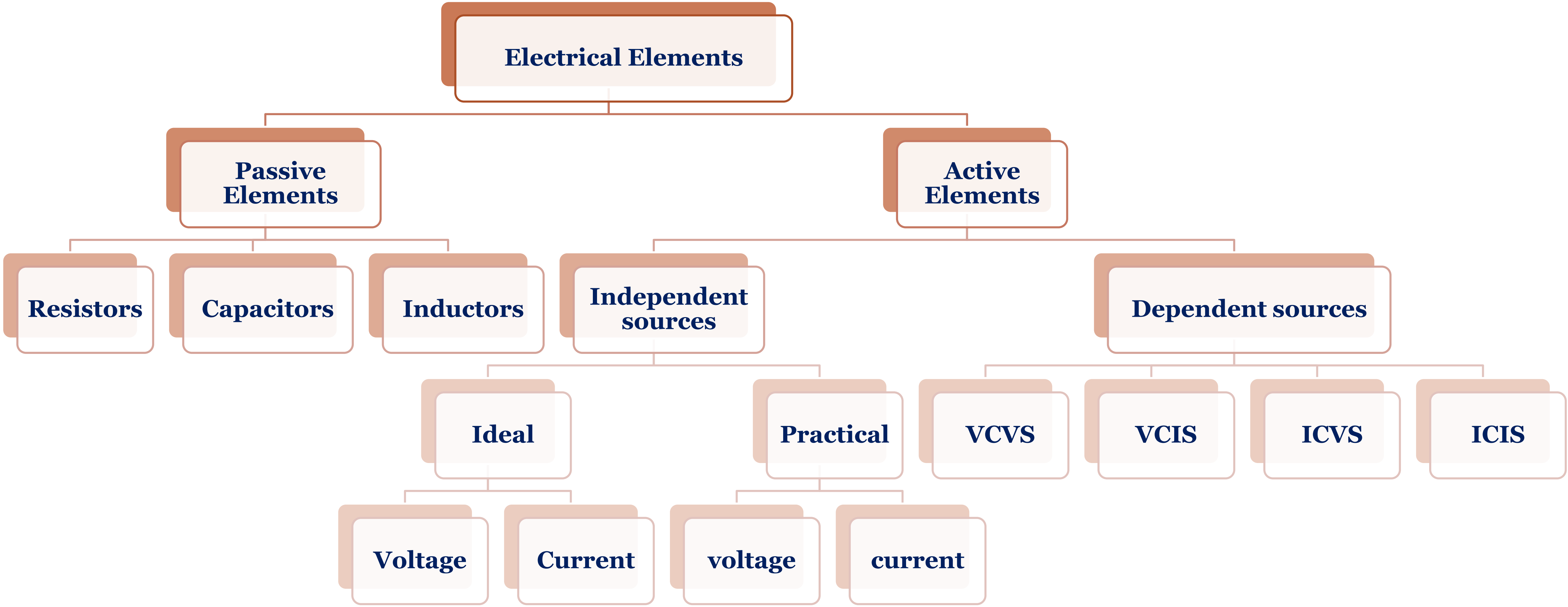
3. Inductor [L]:

- Inductors stores energy in the form of Electro Magnetic Field.
- $\varphi = LI$
- $v(t) = L\frac{di(t)}{dt}$; $i(t) = \frac{1}{L}\int v(t).dt.$
- $E = \frac{1}{2}LI^2$ Joules.
- $L_{eq} = \sum_{i=1}^{n} L_i$; If the Inductors are connected in Series.
- $L_{eq} = \frac{1}{\sum_{i=1}^{n} 1/L_i}$; If the Inductors are connected in Parallel.
- Star to delta and delta to star conversion applicable to Inductors also, only if it is in Reactance form.
- Inductive Reactance, $X_L = 2\pi f L$ Ohms.



Circuit Symbol

Basic Concepts



Basic Concepts……

Independent Ideal Sources

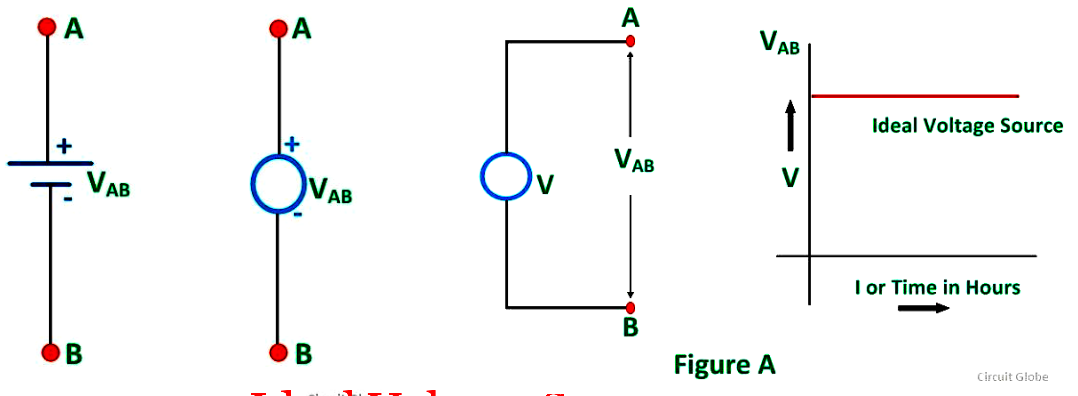
Sources which maintains a constant value and does not affected by any other quantity

1. Ideal Voltage Source

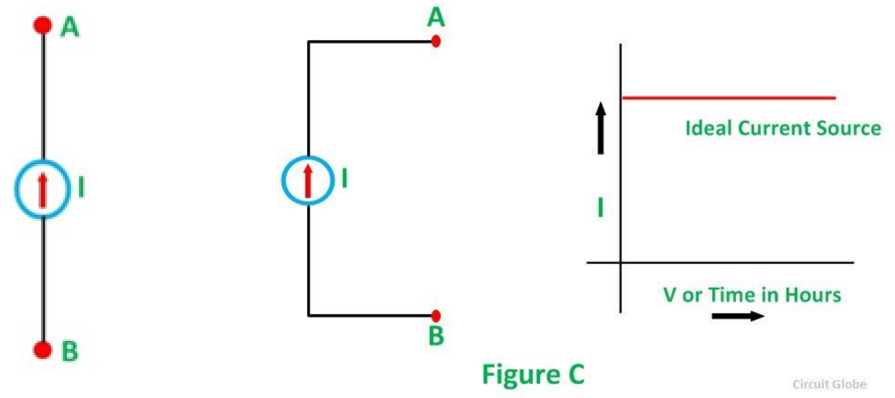
Source which maintains a constant voltage, and its is independent of the current drawn from it. These sources are having zero internal Impedance/Resistance.

2. Ideal Current Source

Source which maintains a constant current, and its is independent of the terminal voltage. These sources are having Infinite internal Impedance/Resistance.



Ideal Voltage Source



Ideal Current Source

Basic Concepts…

Independent Practical Sources

Sources having some internal resistance or impedance are called practical sources.

1. Practical Voltage Source

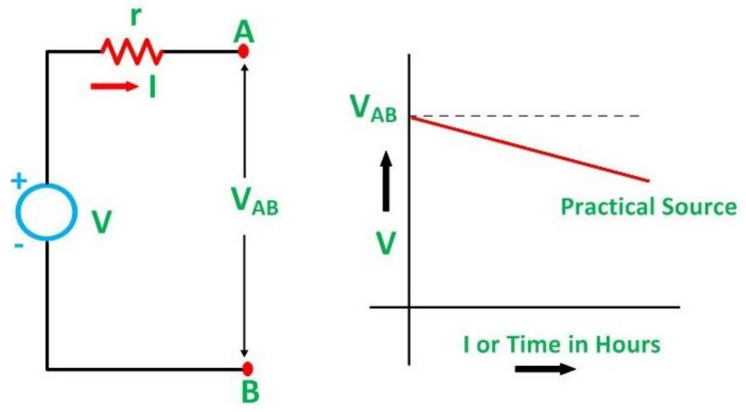
Due to internal impedance or resistance voltage drop takes place and it causes terminal voltage to reduce.

$$V_{AB} = V - Ir$$

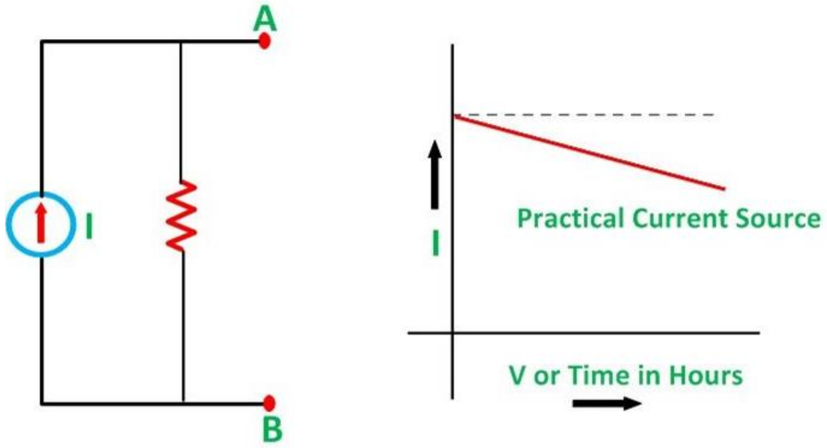
2. Practical Current Source

Current drop takes place.

$$I_{AB} = I - \frac{V_{AB}}{r}$$



Practical Voltage Source

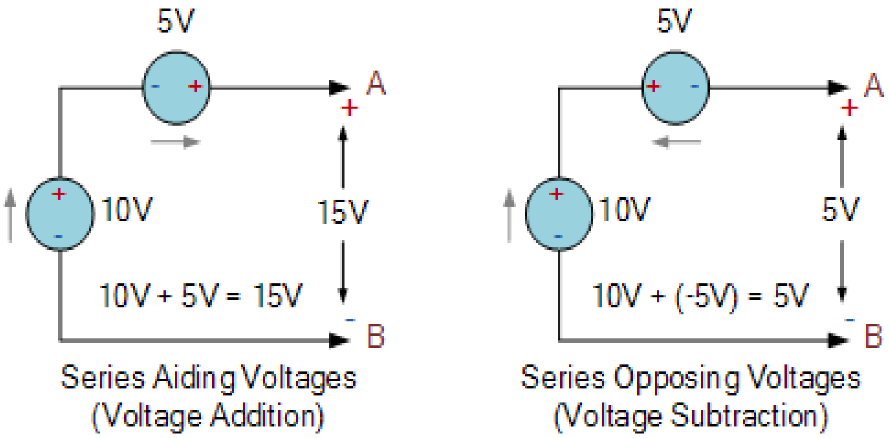


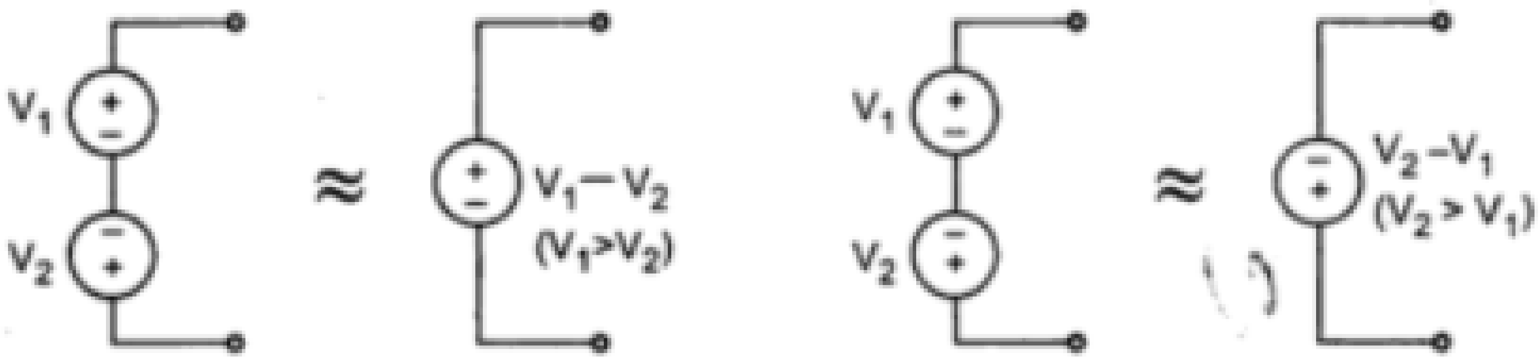
Practical Current Source

Circuit Simplification

Simplification

- Ideal Voltage sources in series- Can be replaced by a single voltage source.
- Equivalent voltage is the sum or difference of individual voltages source values.

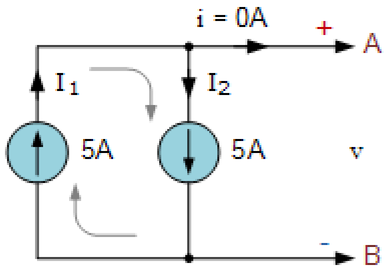


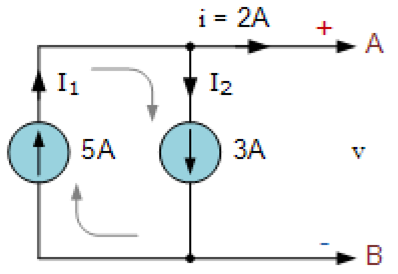


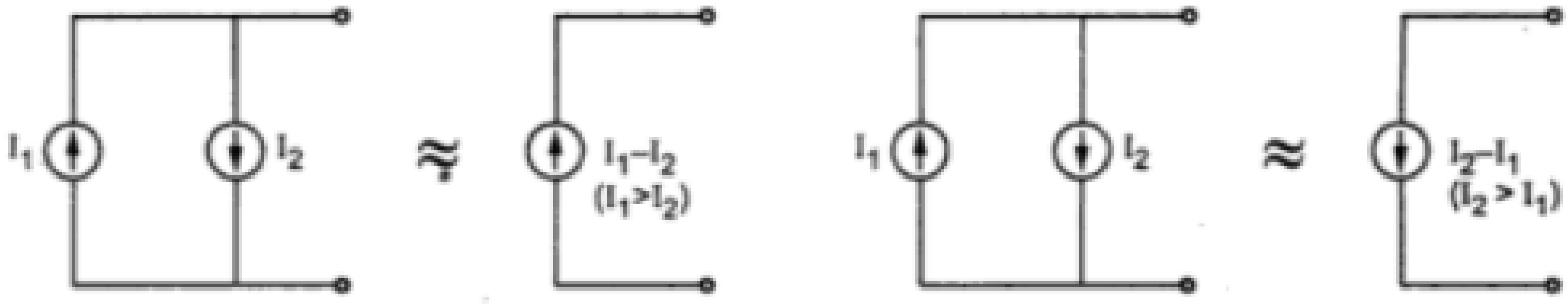
Circuit Simplification

Simplification

- Ideal Current sources in parallel – Can be replaced by a single current source.
- Equivalent current is the sum or difference of individual current source values.



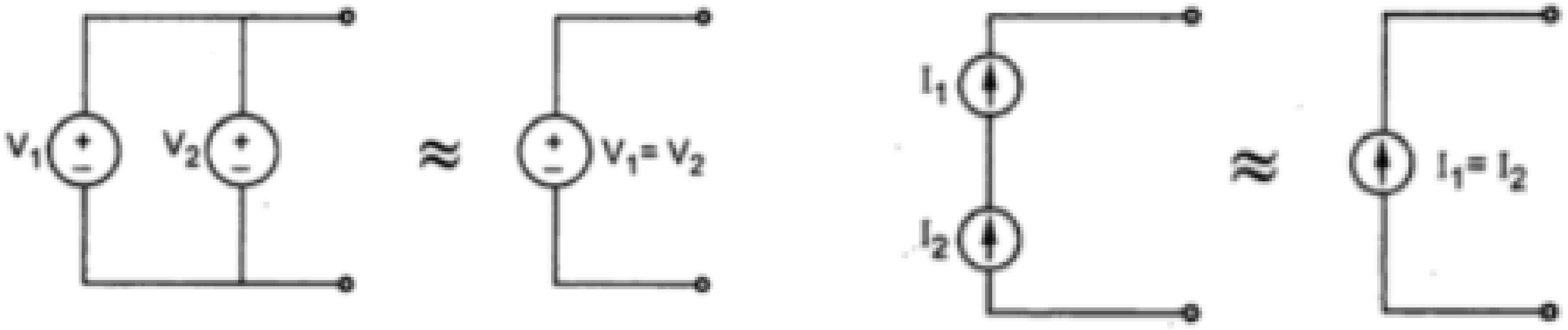




Circuit Simplification

Simplification

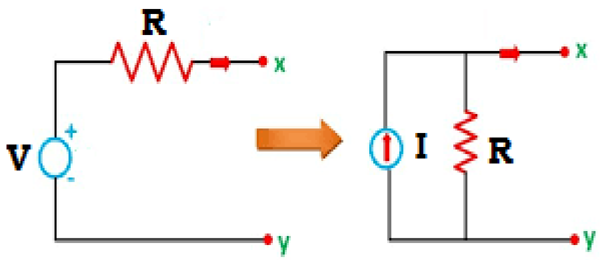
- Ideal Voltage Sources in Parallel and Ideal current sources in Series



Source Transformation

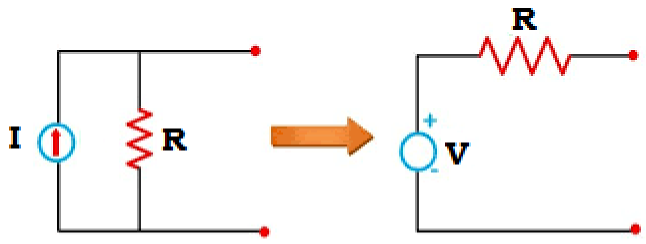
Source Transformation

- It is a process of converting practical voltage source into practical current source.
- Used for circuit/ network simplifications
- Not applicable to ideal sources.



Voltage source to current source

$$I = \frac{V}{R}$$



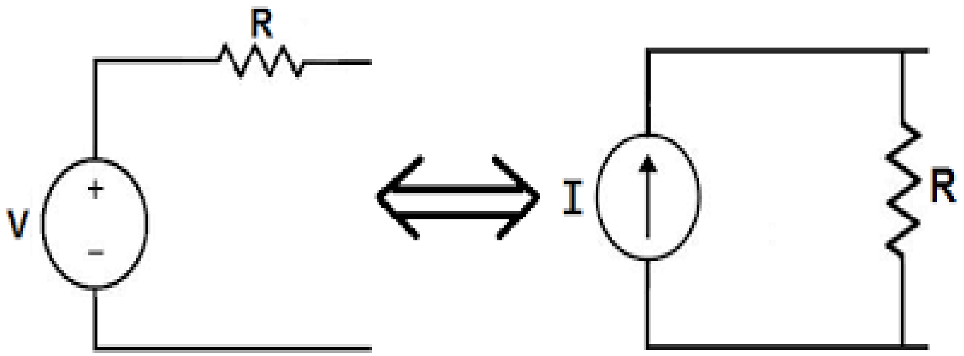
Current source to Voltage source

$$V = IR$$

Source Transformation

Key Points

1. Source Transformation is applicable to Practical sources only.
2. Ignore the resistors that are connected across ideal voltage sources.
3. Ignore the resistors that are connected in series with ideal Current sources.
4. While converting practical current source into practical voltage source, polarity of voltage source is always positive terminal at the arrow head and negative terminal at the other side.
5. While converting practical voltage source into practical current source, polarity of current source i.e., arrow head of current source must be indicated at the positive terminal of the voltage source.



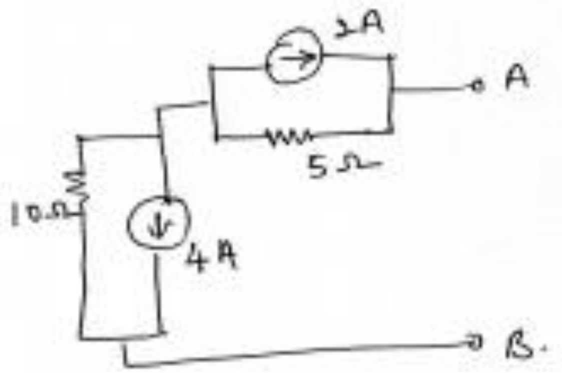
Source Transformation

Procedure to simplify the electrical networks/Circuits

1. Identify the load element, remove the load element and name the load terminals as A and B or X and Y etc.
2. Reduce the ideal voltage sources, that are connected in series.
3. Reduce the ideal current sources, that are connected in parallel.
4. Apply Source transformation.
5. Apply Source shifting.
6. Repeat the steps 2 to 5 until simplified form is obtained between the load terminals.
7. Connect the load element and find the load current or load voltage or power delivered or absorbed by the load element.

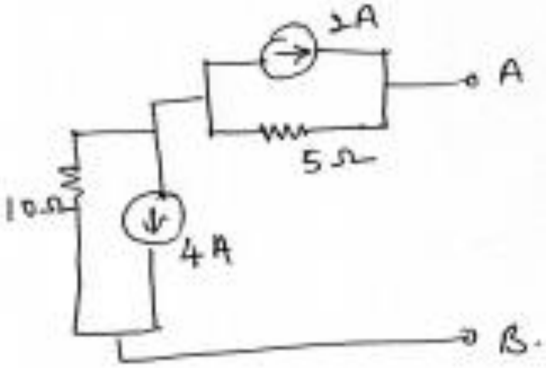
Examples

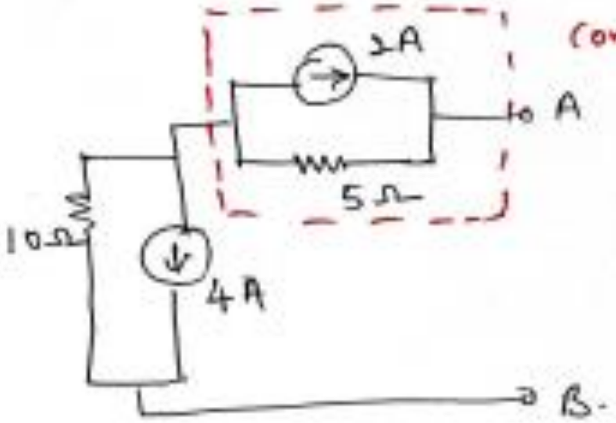
1) Obtain Single practical voltage source b/n A & B for the electrical network shown in figure.



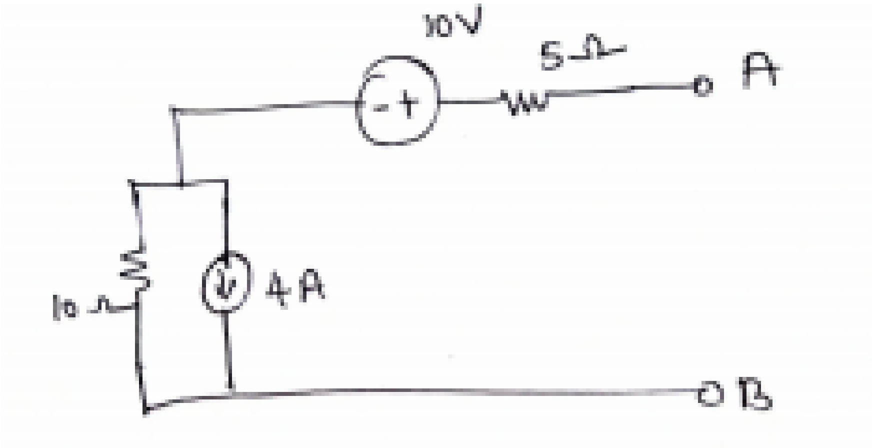
Examples

1) Obtain single practical voltage source b/n A & B for the electrical network shown in figure.

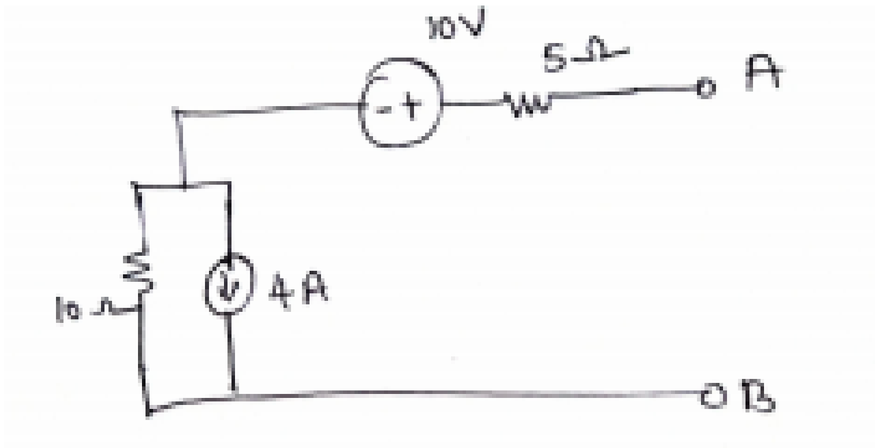


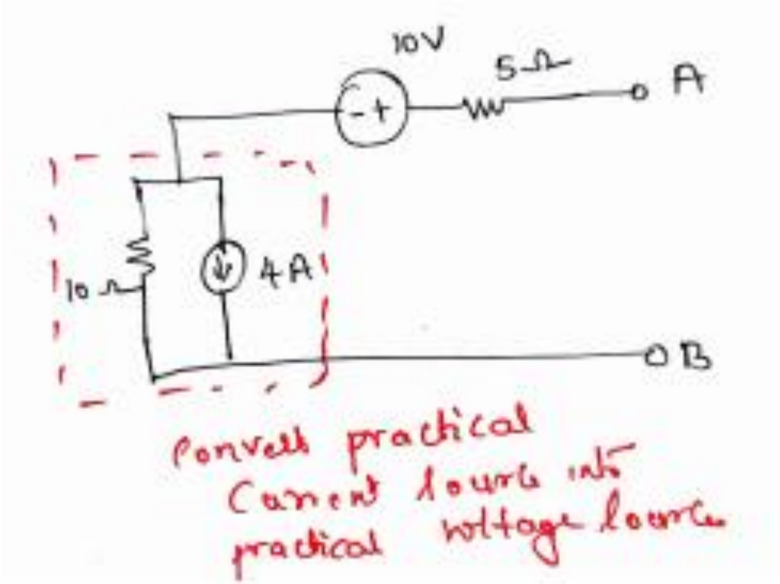


Examples

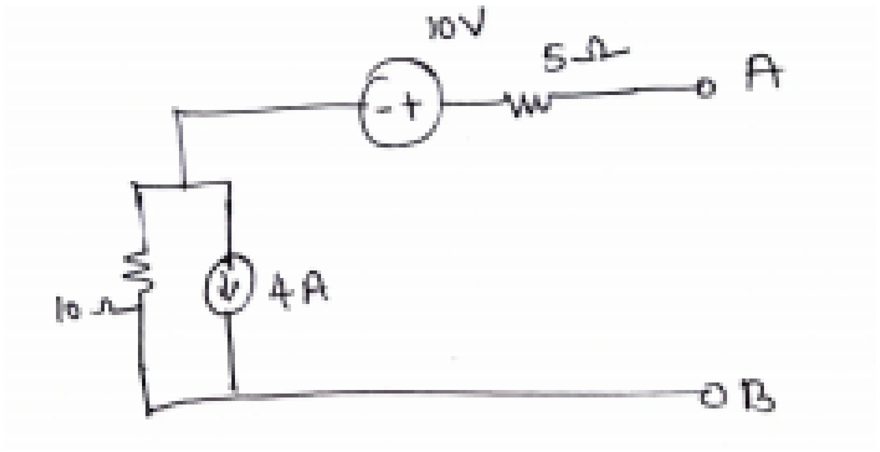


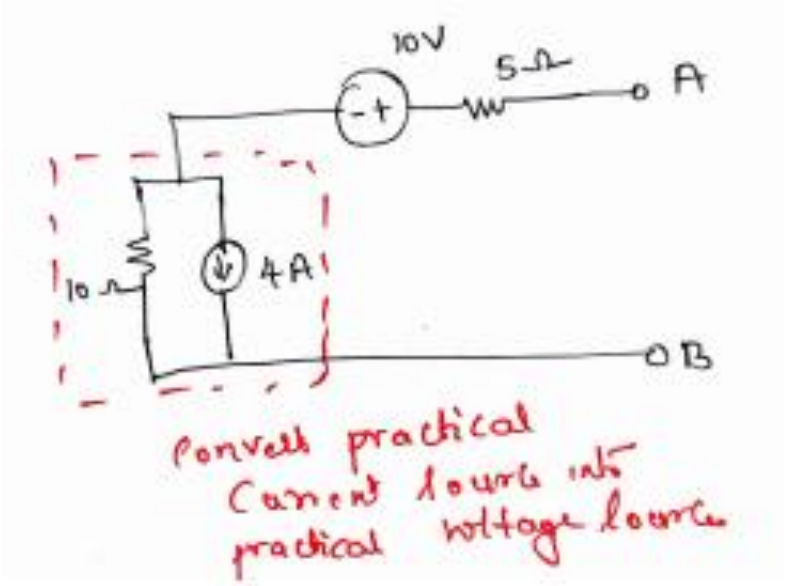
Examples

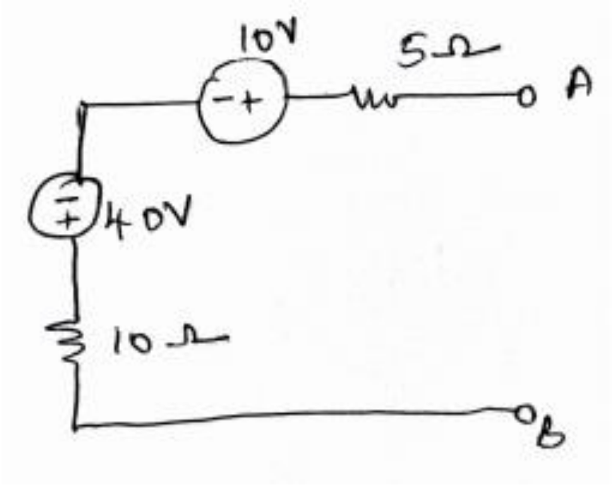

10V
5Ω
A
10Ω
4A
B


10V
5Ω
A
10Ω
4A
B
Convert practical
Current source into
practical voltage source

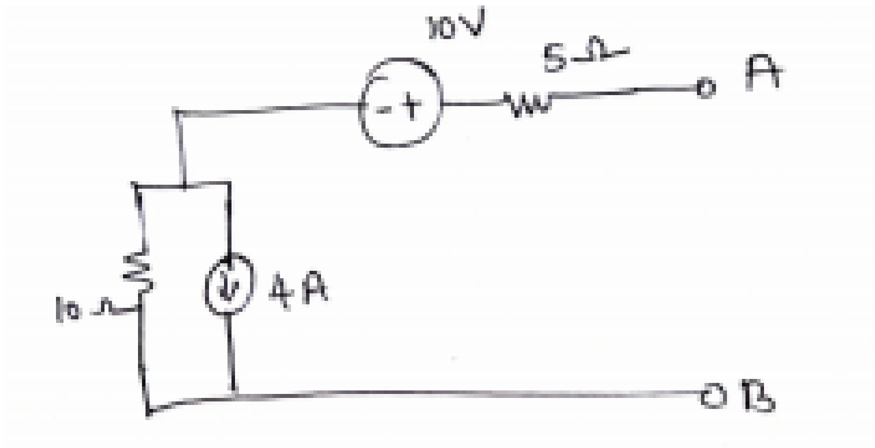
Examples

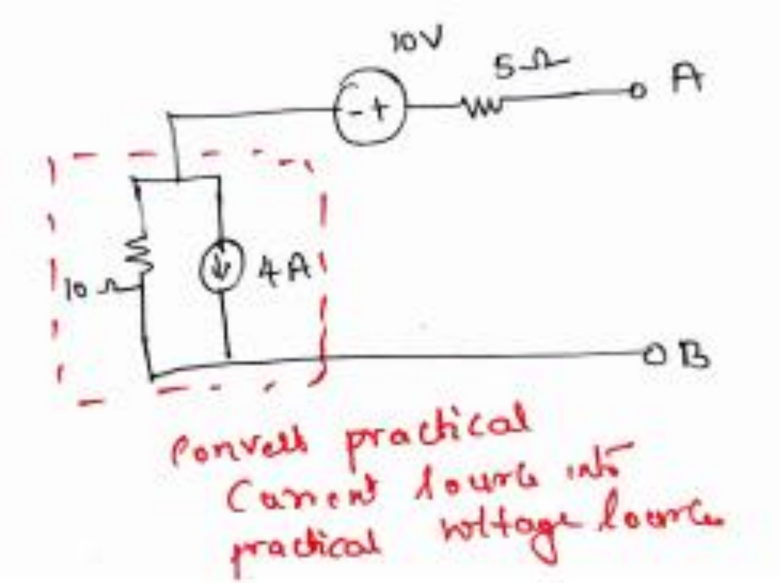

10V
5Ω
A
10Ω
4A
B

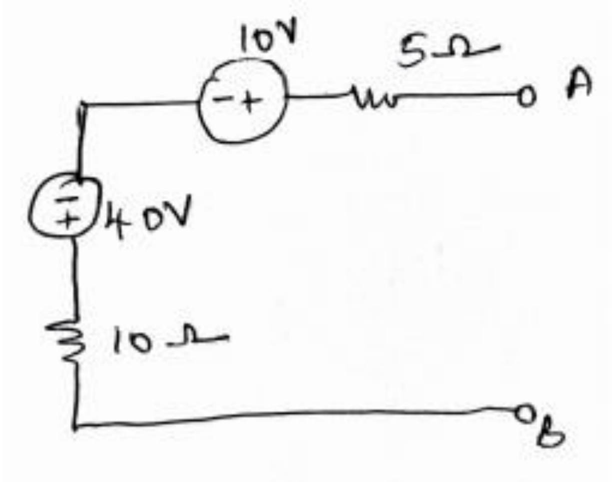

10V
5Ω
A
10Ω
4A
B
Convert practical
Current source into
practical voltage source

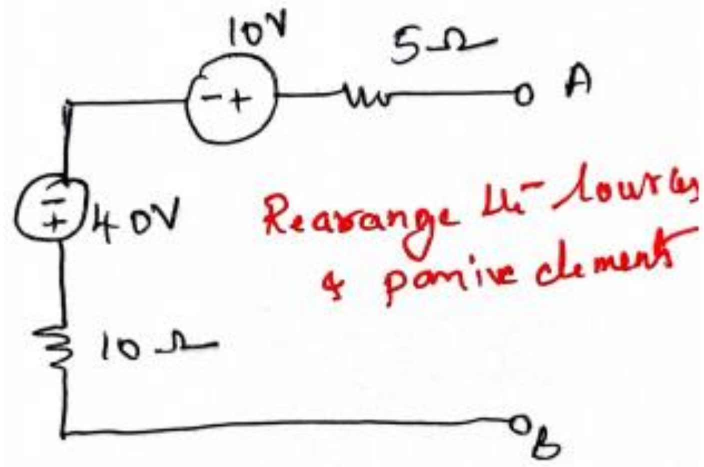

10V
5Ω
A
40V
10Ω
B

Examples

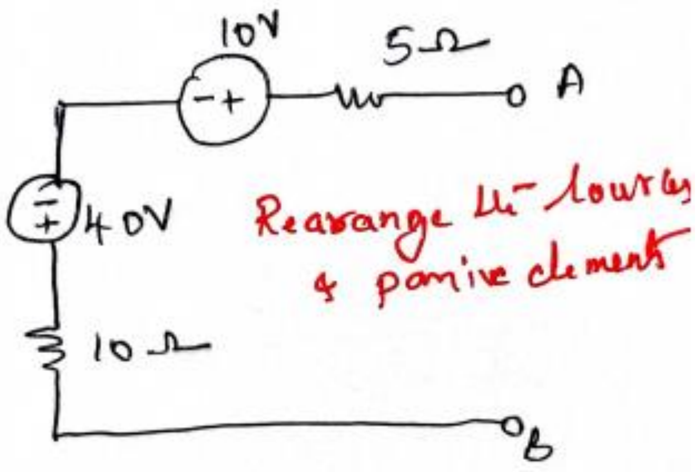

10V
5Ω
A
10Ω
4A
B

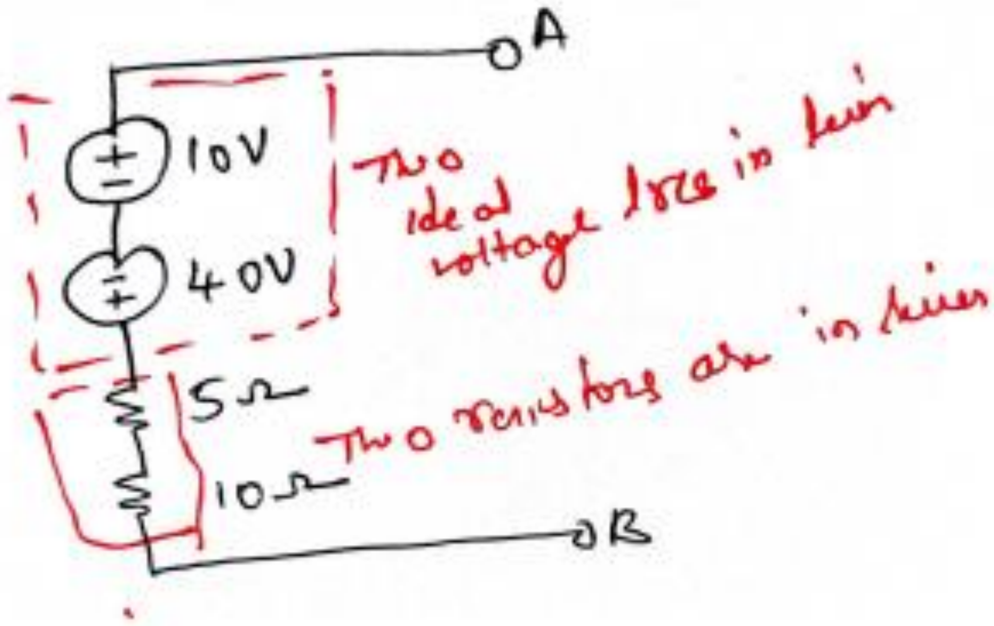

10V
5Ω
A
10Ω
4A
B
Convert practical Current source into practical voltage source


10V
5Ω
A
40V
10Ω
B

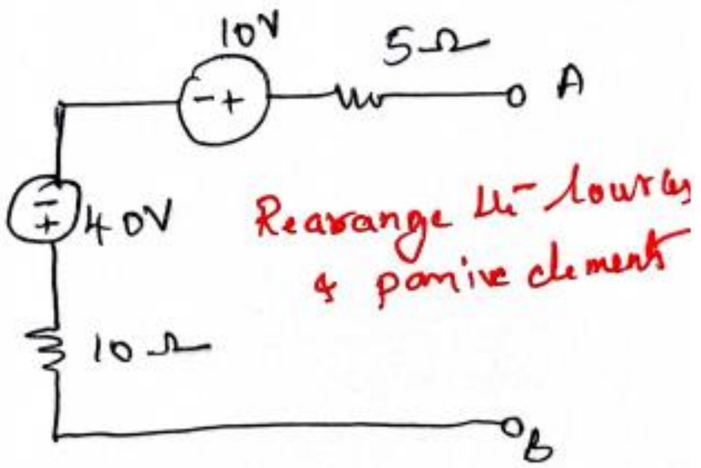

10V
5Ω
A
40V
10Ω
B
Rearrange the sources & passive elements

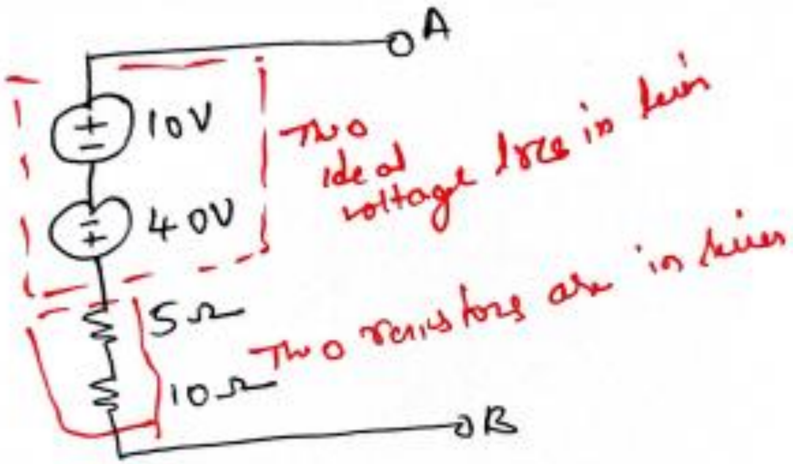
Examples


10V
5Ω
A
40V
Rearrange the sources
& passive element
10Ω
B

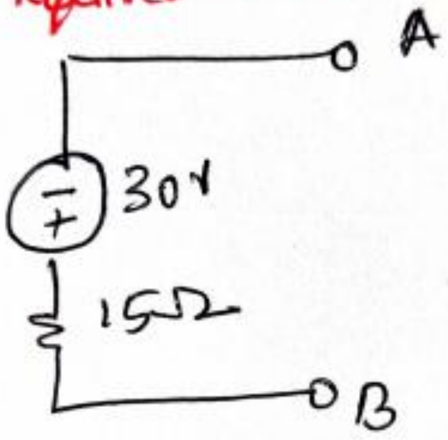

A
10V
40V
Two ideal voltage source in series
5Ω
10Ω
Two resistors are in series
B

Examples



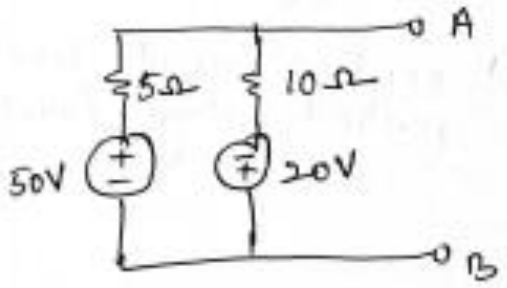


Required Solution:



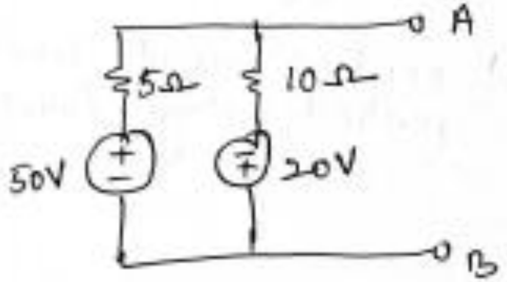
Examples

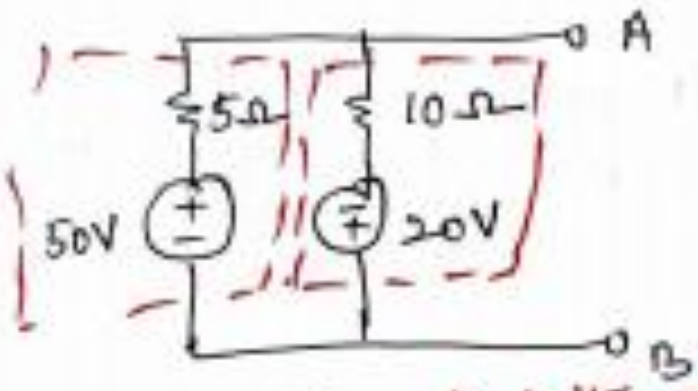
2) obtain Single practical voltage source btwn A & B for n/w shown in figure.



Examples

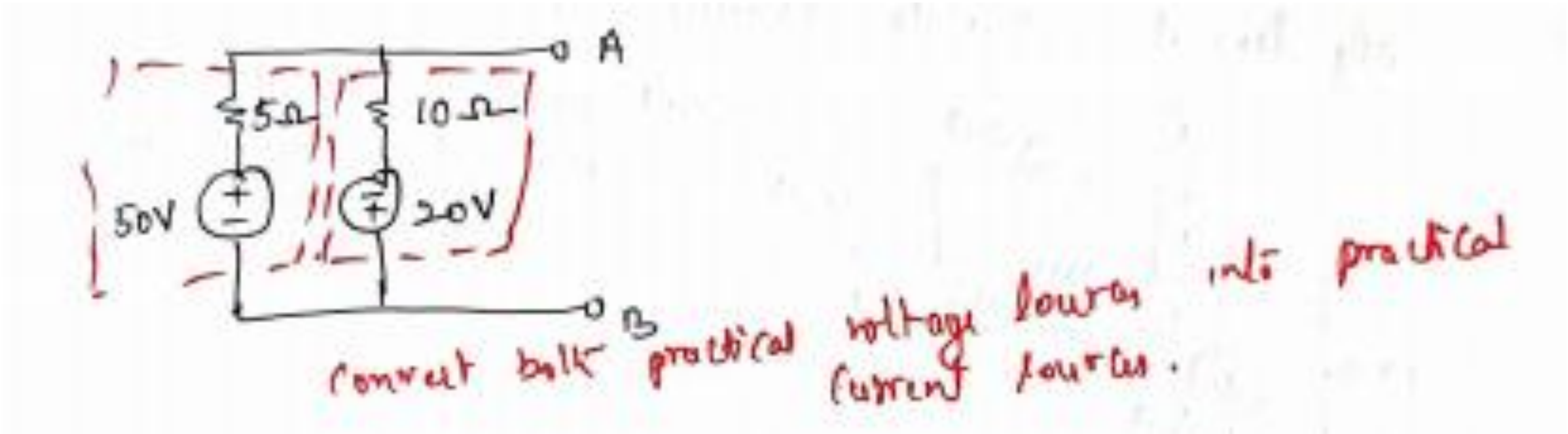
2) Obtain single practical voltage source between A & B for the n/w shown in figure.

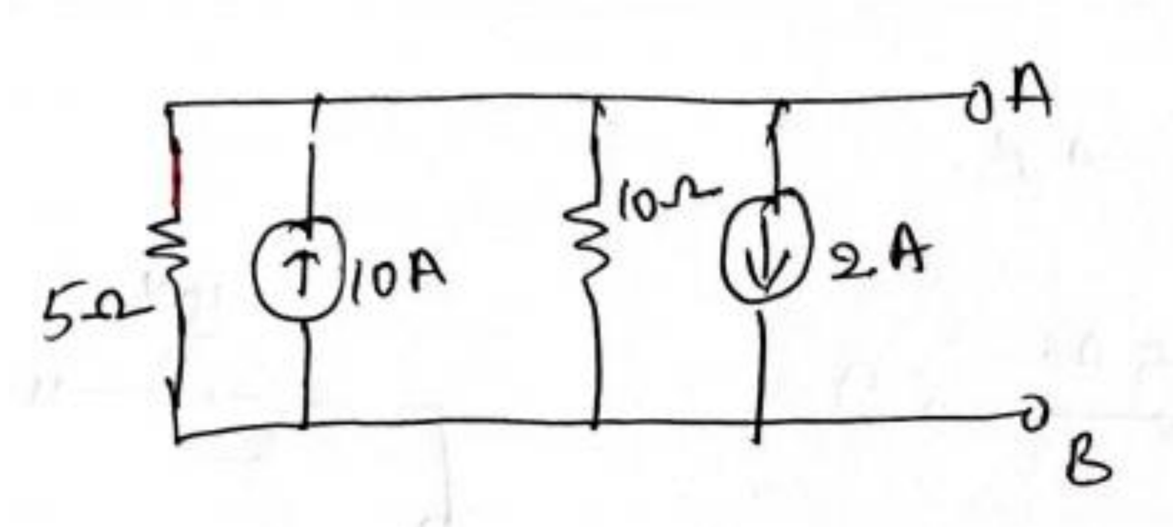




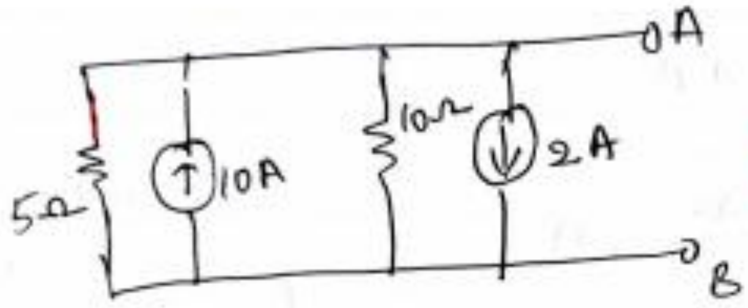
Convert both practical voltage sources into practical current sources.

Examples

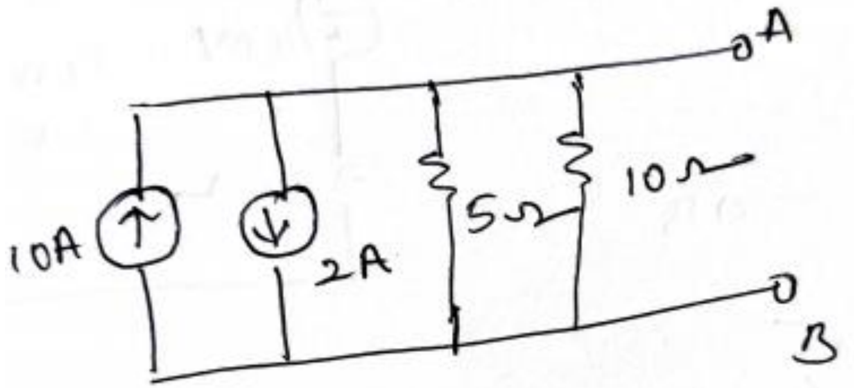

A
5Ω
10Ω
50V
20V
B
convert both practical voltage sources into practical current sources.


A
10Ω
5Ω
10A
2A
B

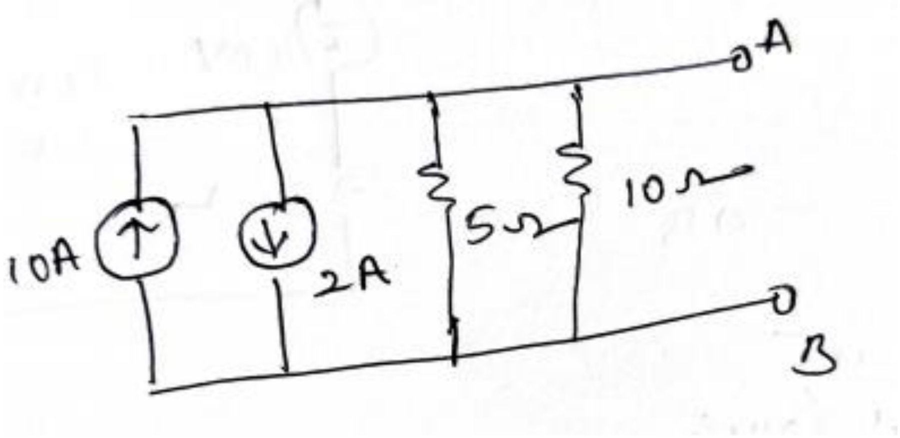
Examples

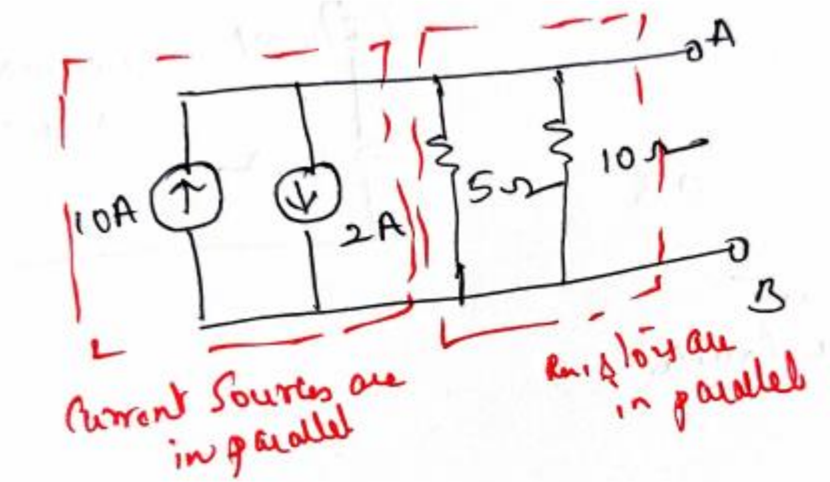


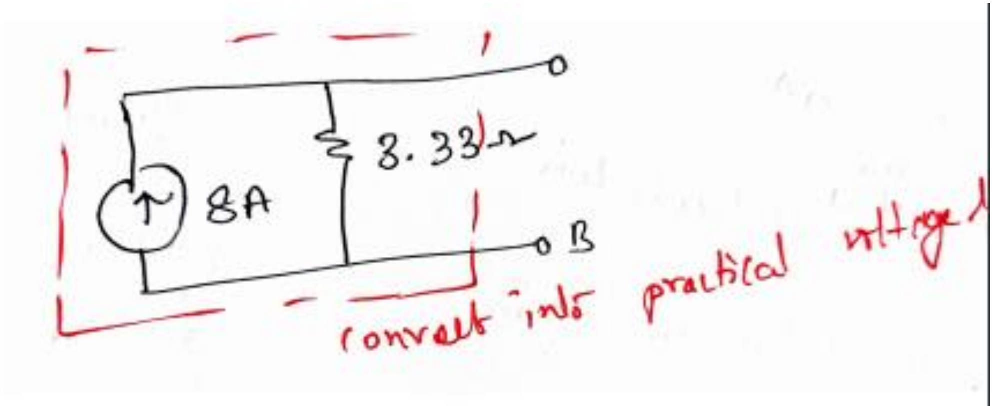
Re-arrange the elements

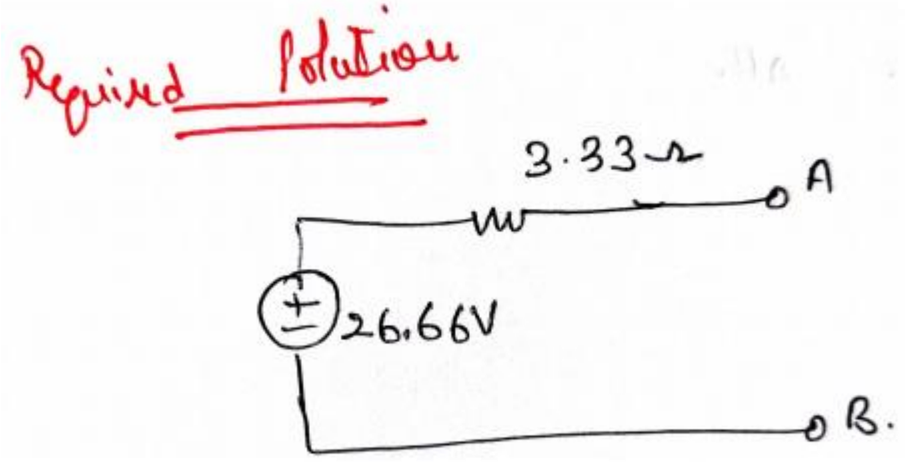


Examples


A
10A
2A
5Ω
10Ω
B

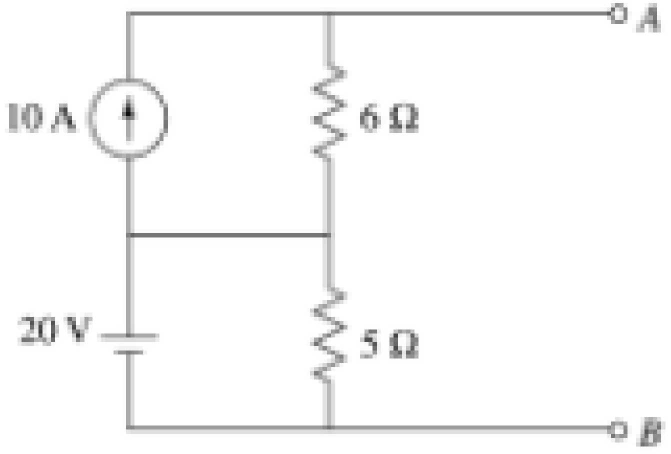

A
10A
2A
5Ω
10Ω
B
Current Sources are in parallel
Resistors are in parallel


8A
8.33Ω
B
convert into practical voltage


Required Solution
3.33Ω
A
26.66V
B.

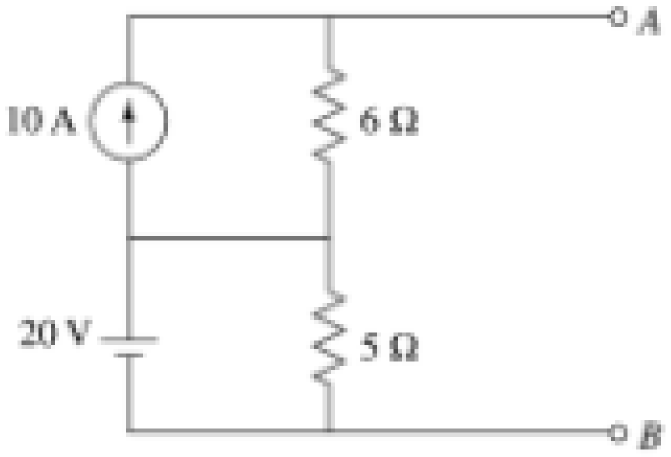
Examples

3. Obtain the single practical current source between the terminals A and B



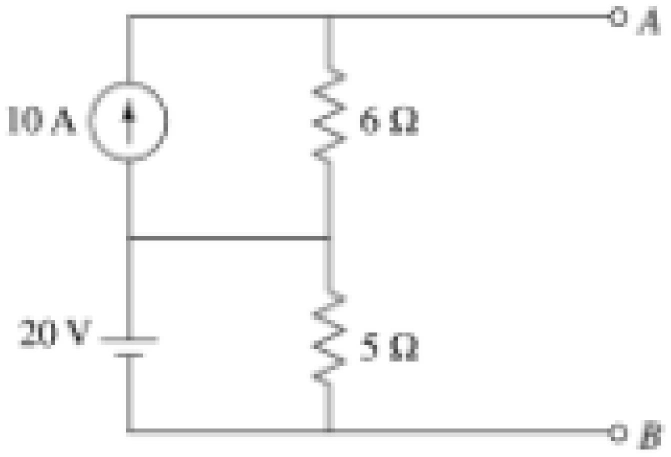
Examples

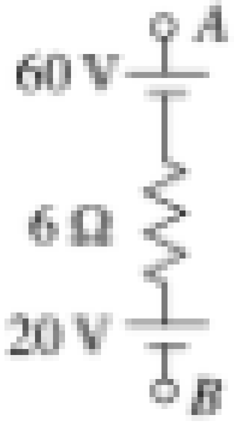
3. Obtain the single practical current source between the terminals A and B

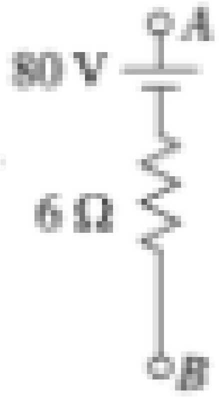


Examples

3. Obtain the single practical current source between the terminals A and B

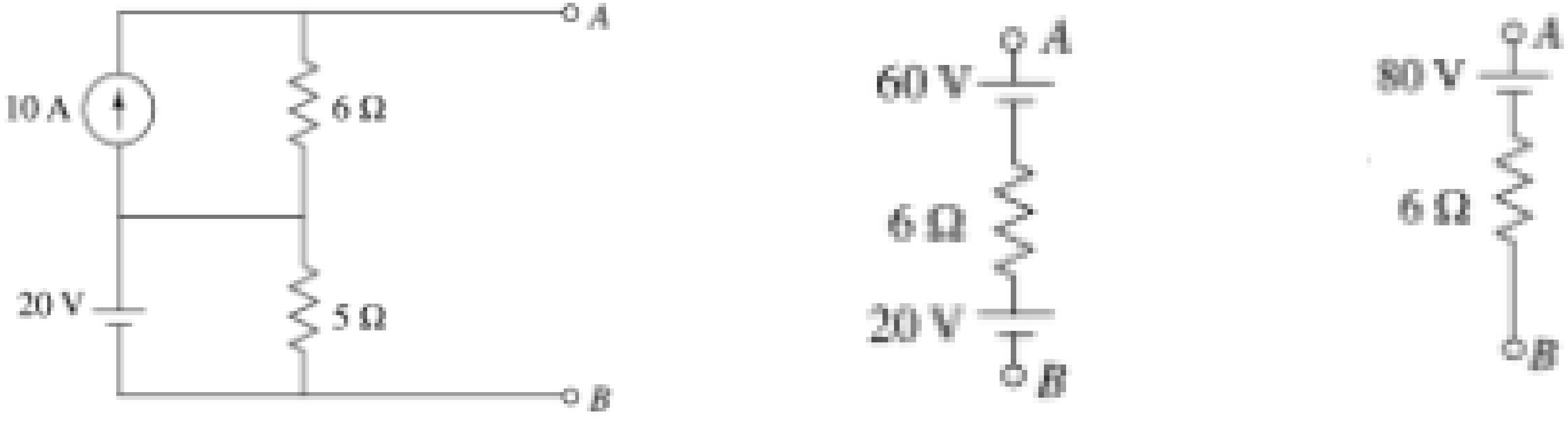


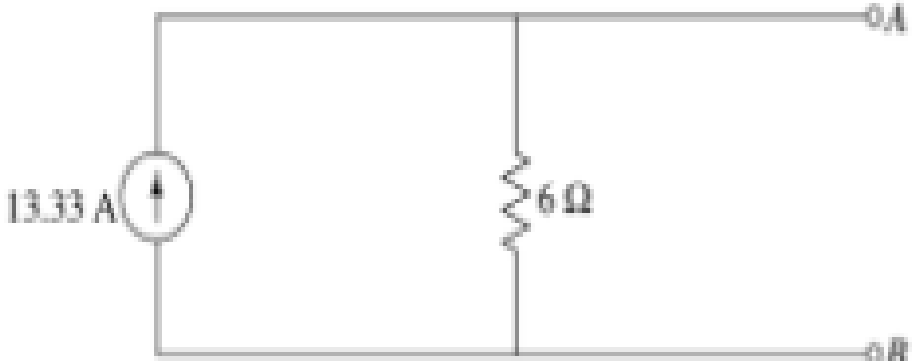




Examples

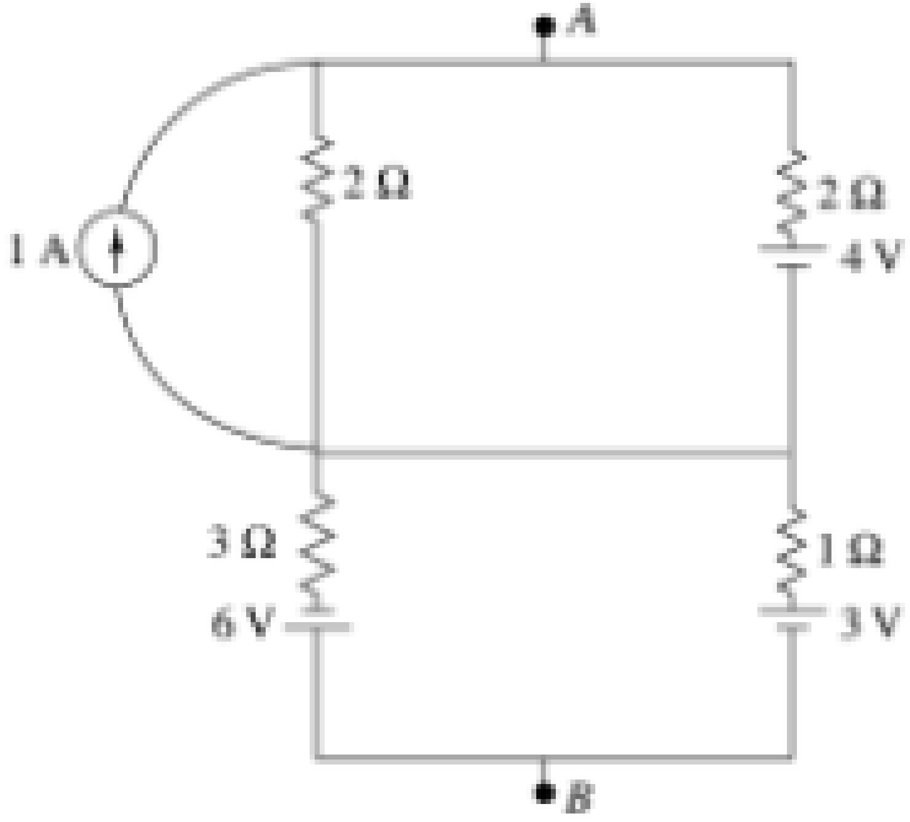
3. Obtain the single practical current source between the terminals A and B





Examples

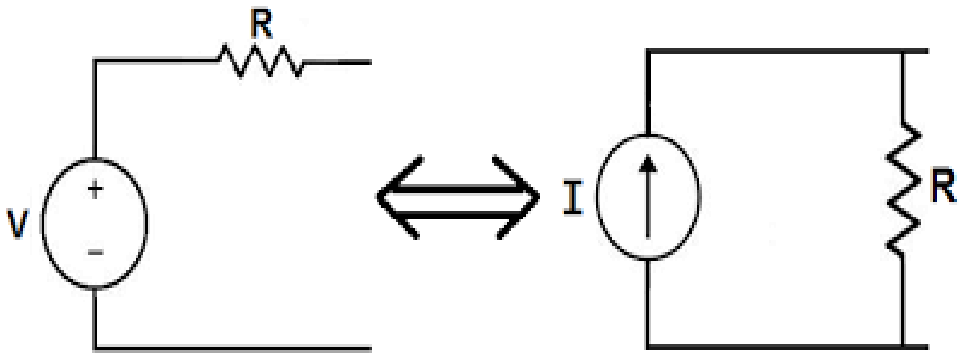
4. Practice Problem



Source Transformation

Key Points

1. Source Transformation is applicable to Practical sources only.
2. Ignore the resistors that are connected across ideal voltage sources.
3. Ignore the resistors that are connected in series with ideal Current sources.
4. While converting practical current source into practical voltage source, polarity of voltage source is always positive terminal at the arrow head and negative terminal at the other side.
5. While converting practical voltage source into practical current source, polarity of current source i.e., arrow head of current source must be indicated at the positive terminal of the voltage source.



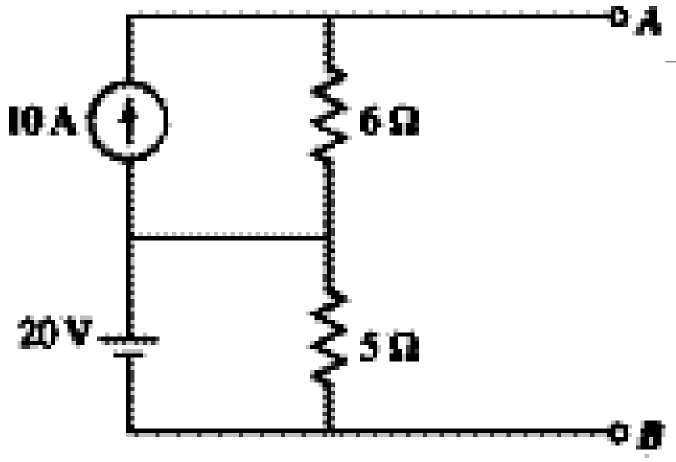
Source Transformation

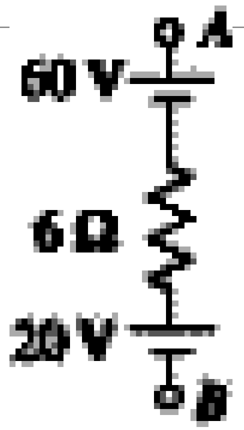
Procedure to simplify the electrical networks/Circuits

1. Identify the load element, remove the load element and name the load terminals as A and B or X and Y etc.
2. Reduce the ideal voltage sources, that are connected in series.
3. Reduce the ideal current sources, that are connected in parallel.
4. Apply Source transformation.
5. Apply Source shifting.
6. Repeat the steps 2 to 5 until simplified form is obtained between the load terminals.
7. Connect the load element and find the load current or load voltage or power delivered or absorbed by the load element.

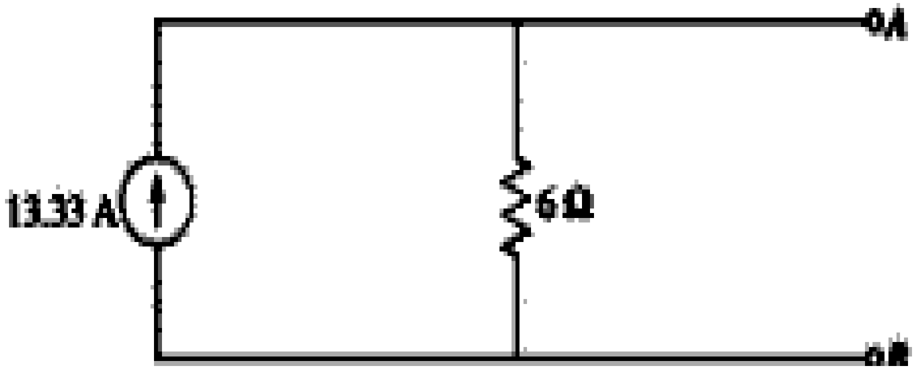
Examples

3. Obtain the single practical current source between the terminals A and B



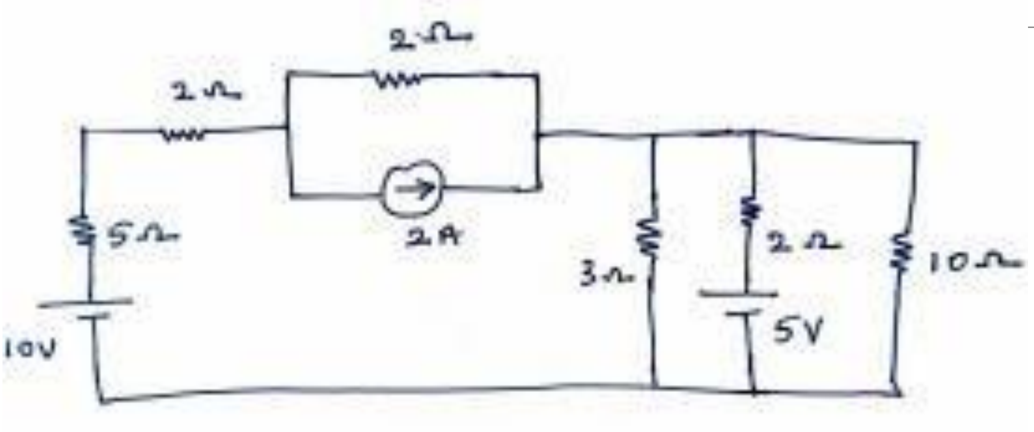


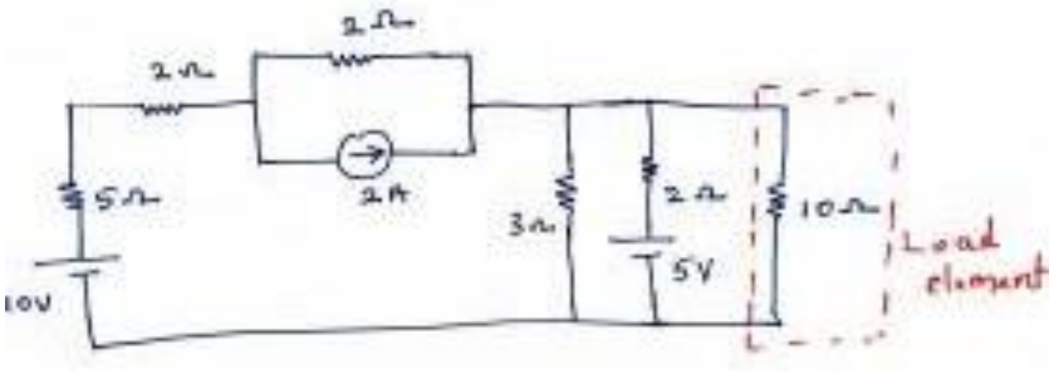
NOTE: 5 Ohms resistor is connected across the voltage source, hence 5 Ohm resistor is redundant. We can ignore for the analysis.

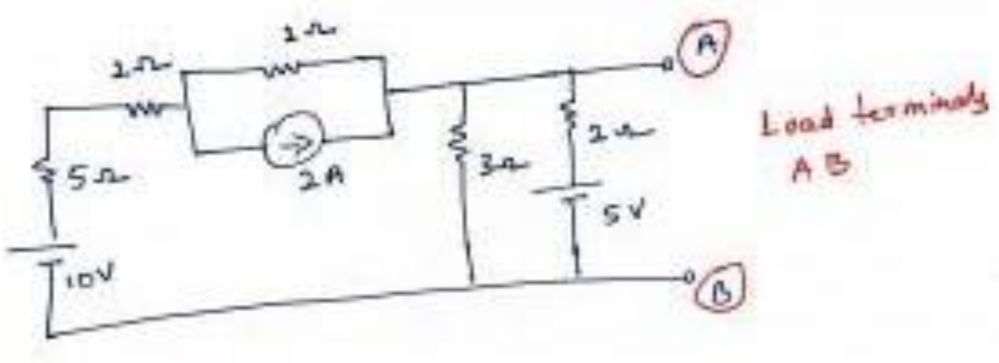


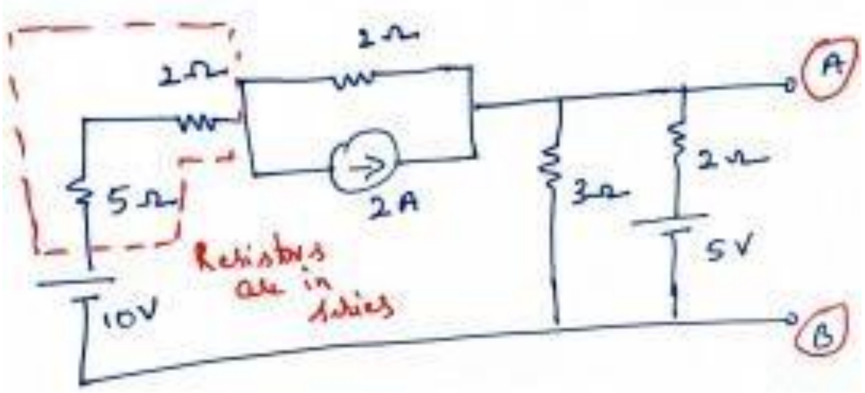
Examples

4. Find the current through 10 Ohm resistor for the circuit shown in figure using source transformation.

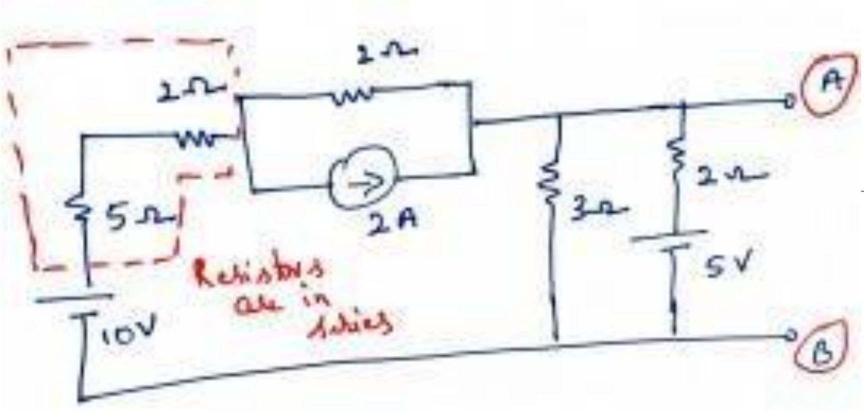


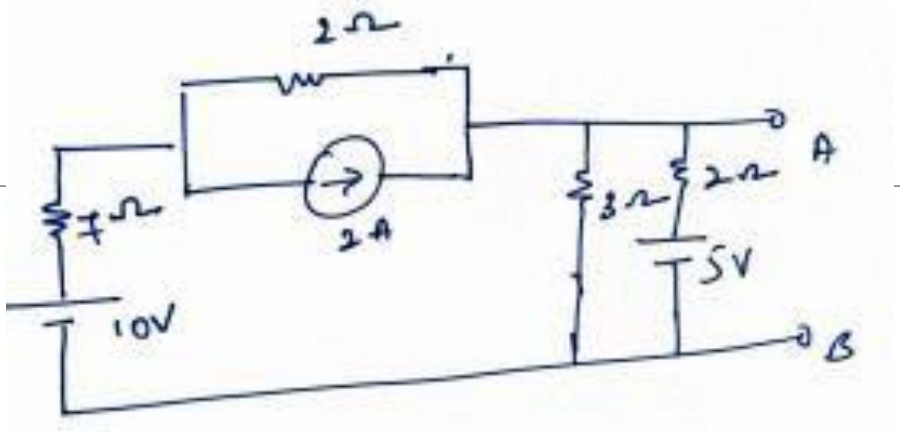


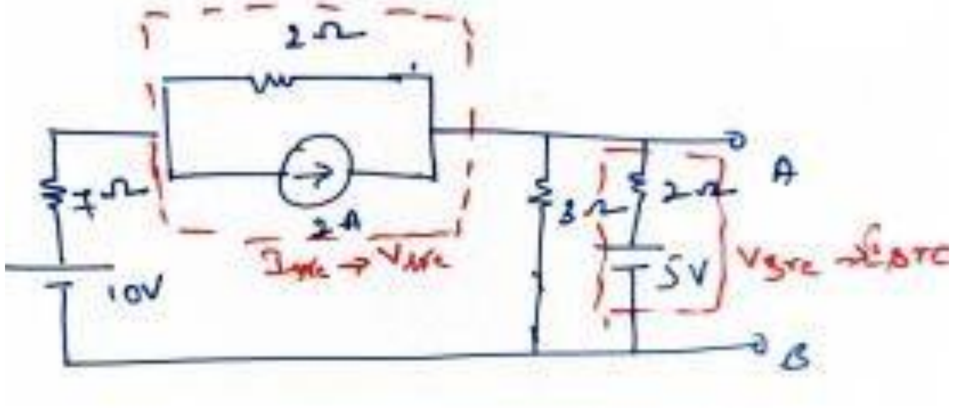


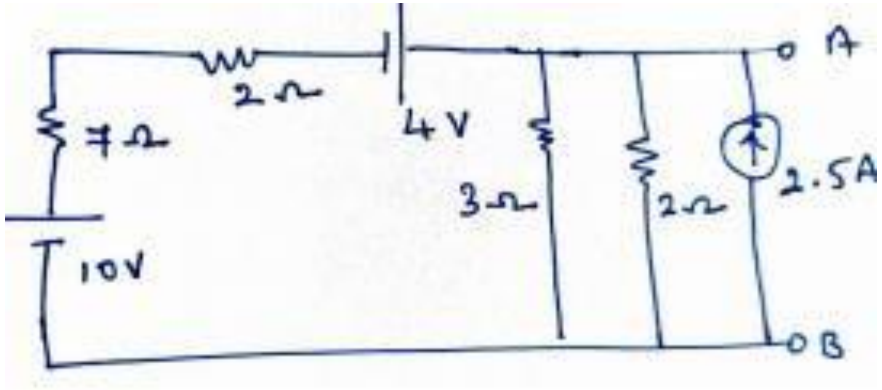


Examples

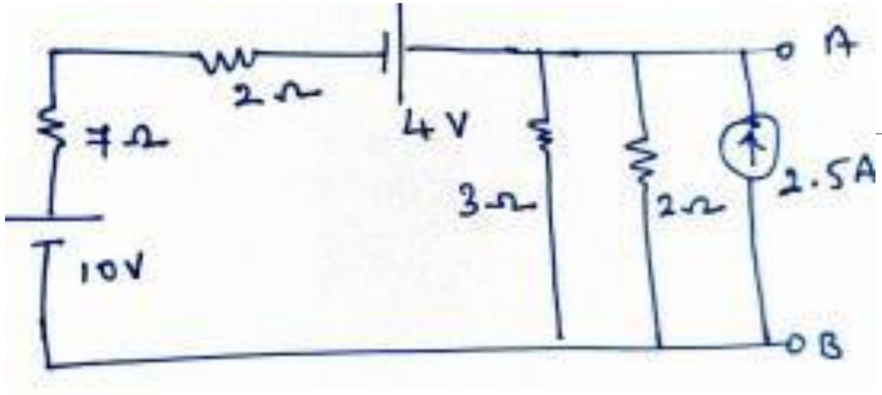

2Ω
2Ω
5Ω
2A
3Ω
2Ω
5V
10V
A
B
Resistors are in series

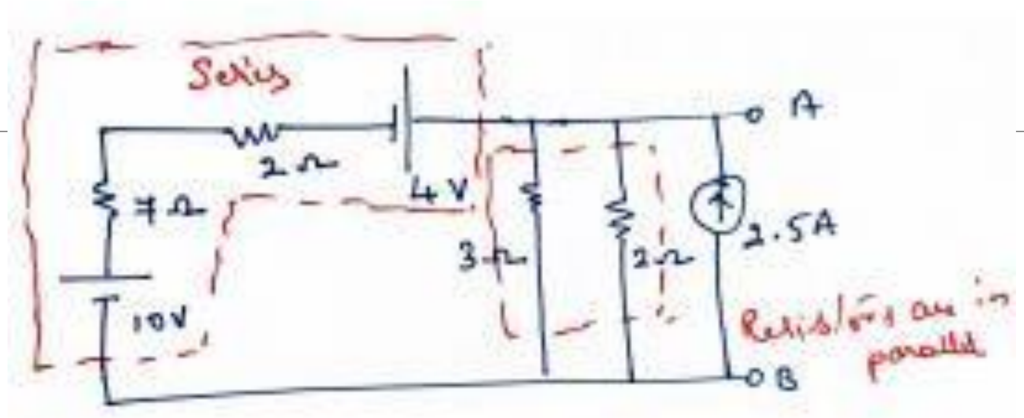

2Ω
7Ω
2A
3Ω
2Ω
5V
10V
A
B

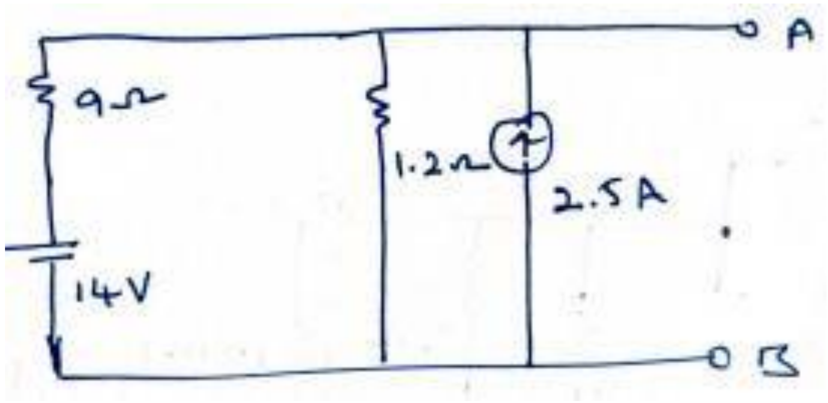

2Ω
7Ω
2A
3Ω
2Ω
5V
10V
A
B
Isrc → Vsrc

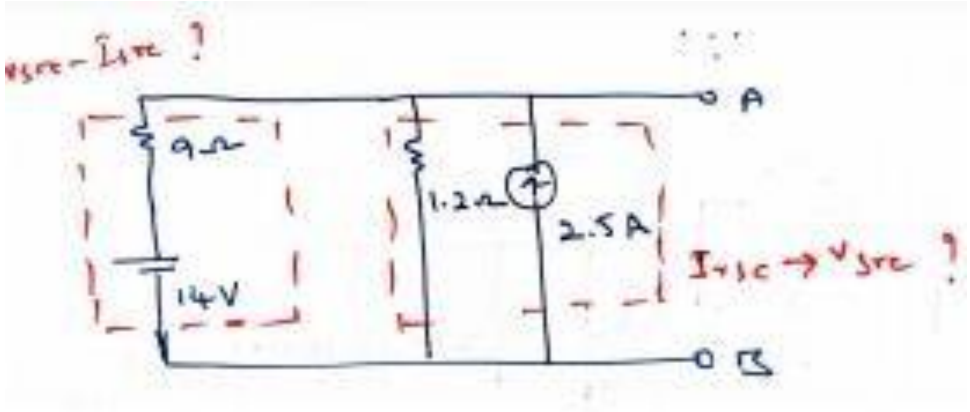

2Ω
4V
7Ω
3Ω
2Ω
2.5A
10V
A
B

Examples

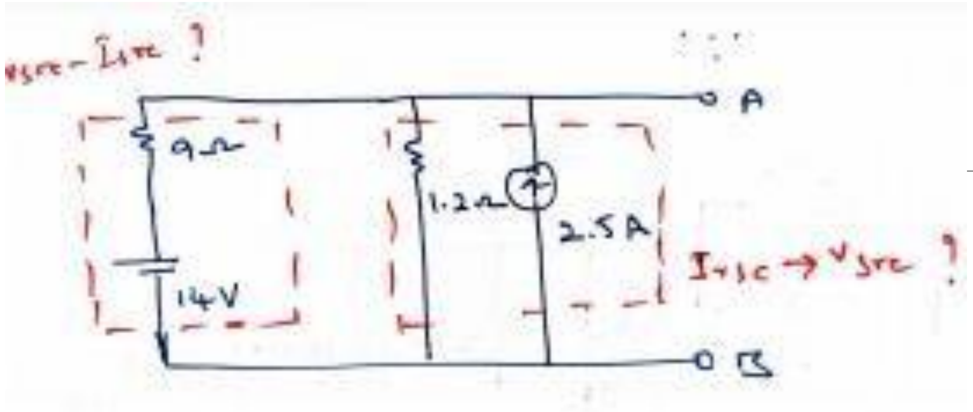

2Ω
4V
A
7Ω
3Ω
2Ω
2.5A
10V
B

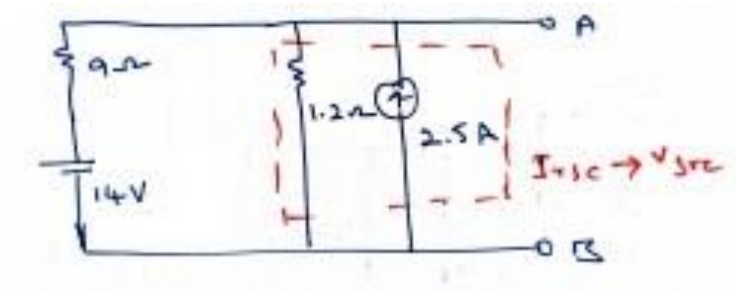

Series
2Ω
4V
A
7Ω
3Ω
2Ω
2.5A
10V
B

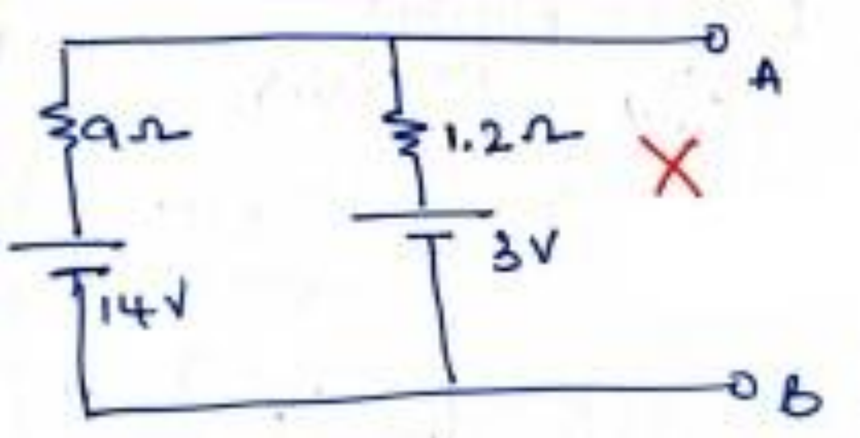

A
9Ω
1.2Ω
2.5A
14V
B


A
9Ω
1.2Ω
2.5A
14V
B

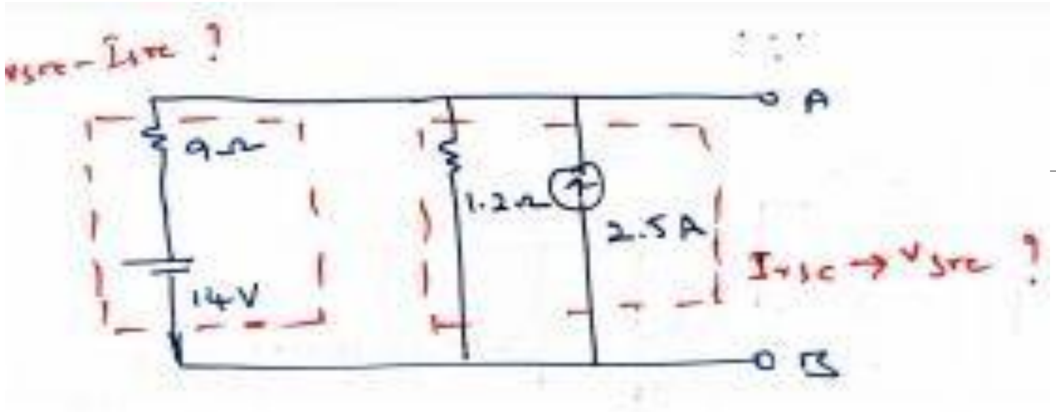
Examples

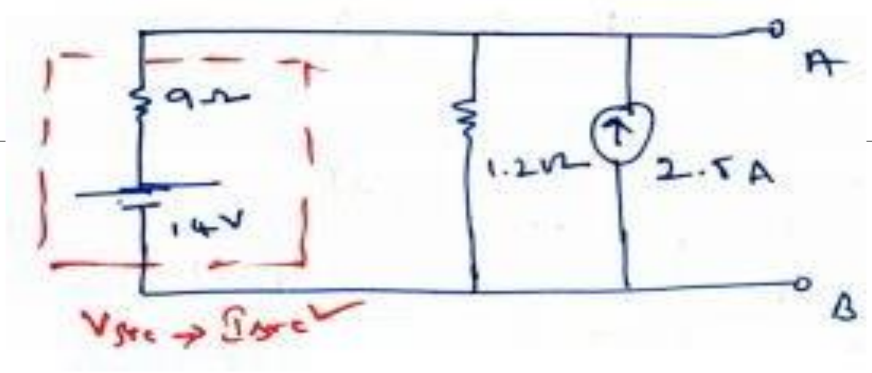

Vsrc – Isrc ?
A
9Ω
1.2Ω
2.5A
14V
Isrc → Vsrc ?
B

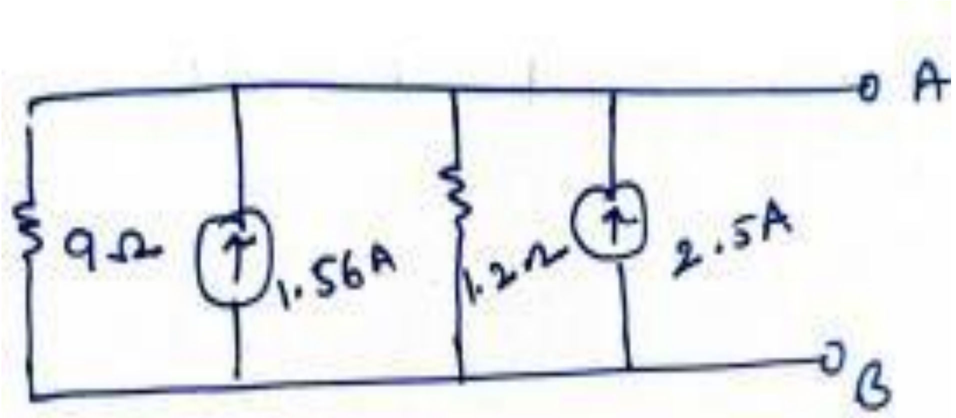

A
9Ω
1.2Ω
2.5A
14V
Isrc → Vsrc
B

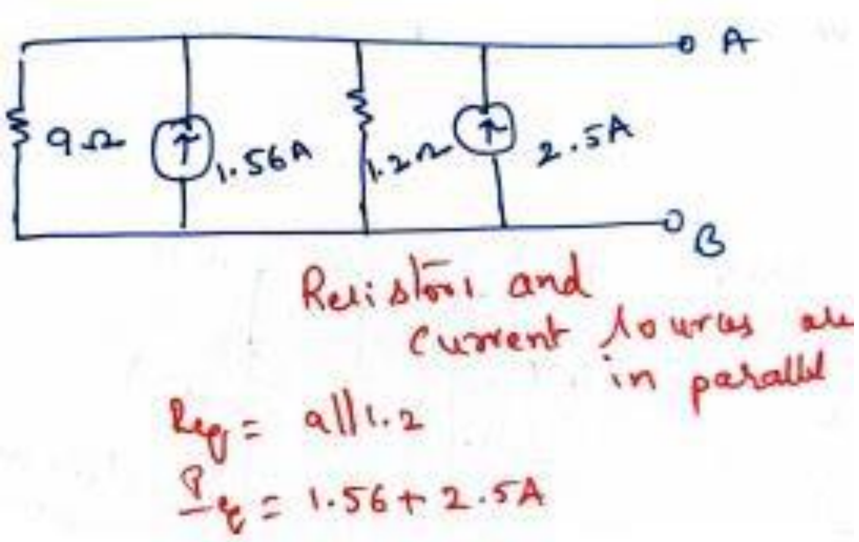

A
9Ω
1.2Ω
14V
3V
B

Examples

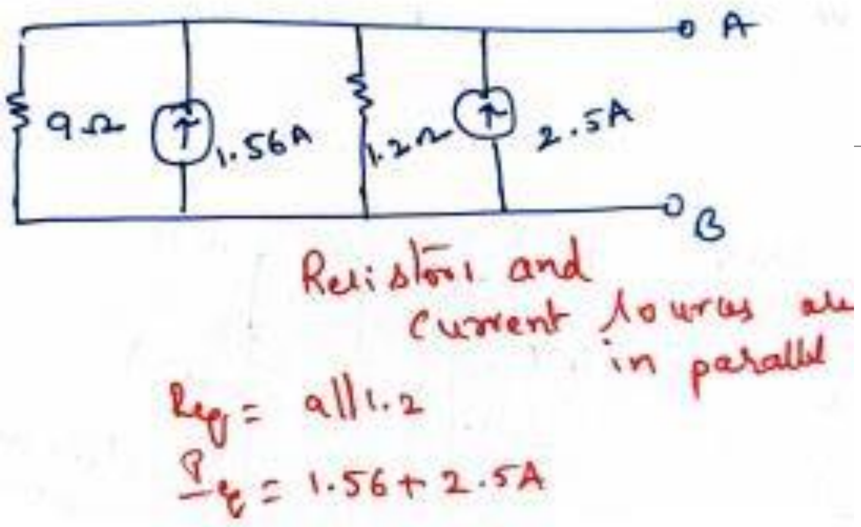

9Ω
14V
1.2Ω
2.5A
A
B

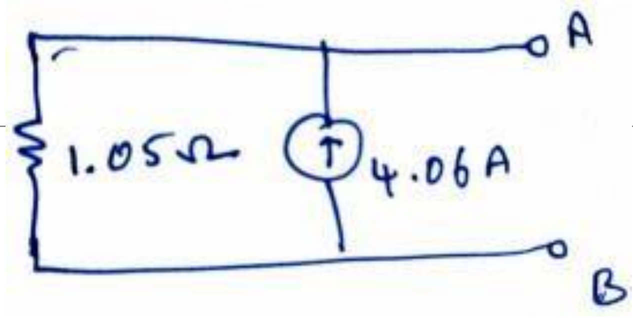

9Ω
14V
1.2Ω
2.5A
A
B

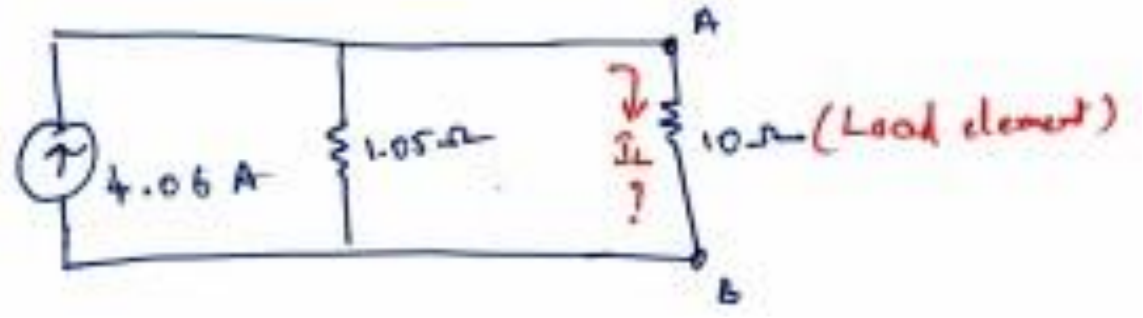

9Ω
1.56A
1.2Ω
2.5A
A
B

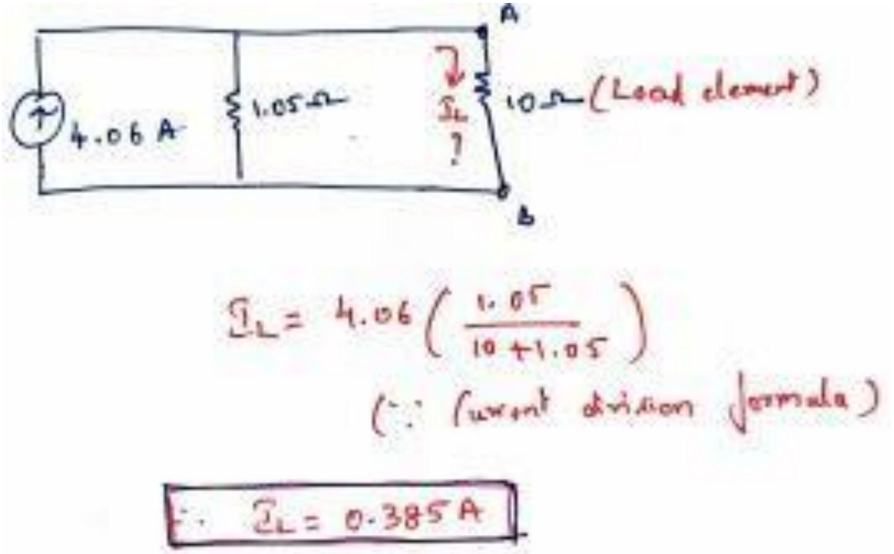

9Ω
1.56A
1.2Ω
2.5A
A
B

Examples


A
B
9Ω
1.56A
1.2Ω
2.5A
Resistors and current sources are in parallel
Req = 9||1.2
Ieq = 1.56 + 2.5A

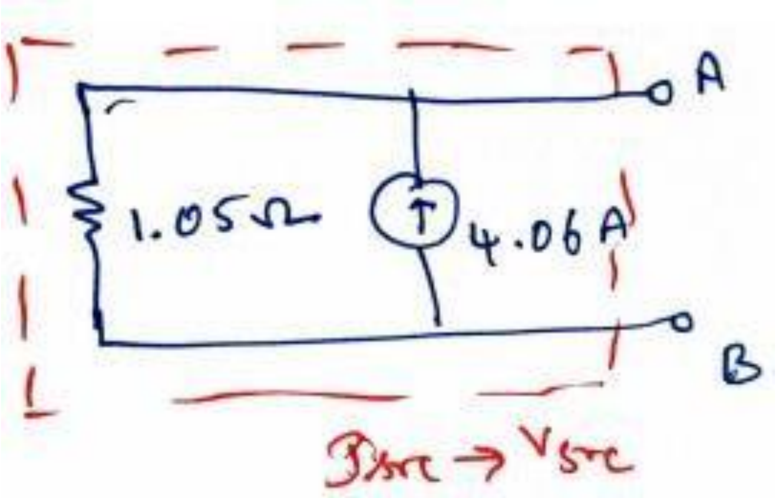

A
B
1.05Ω
4.06A

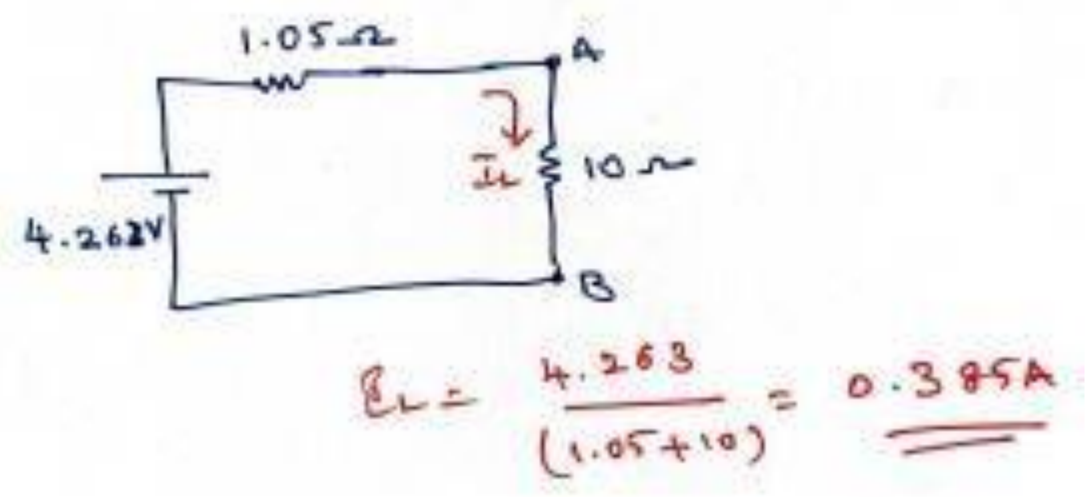

A
B
4.06 A
1.05Ω
IL ?
10Ω (Load element)


A
B
4.06 A
1.05Ω
IL ?
10Ω (Load element)
IL = 4.06 (1.05 / 10 + 1.05)
(∵ Current division formula)
∴ IL = 0.385 A

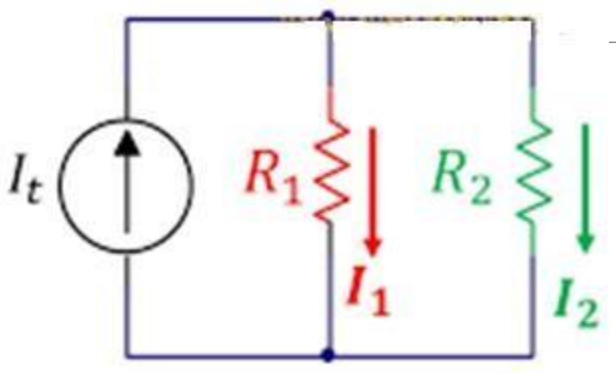
Examples

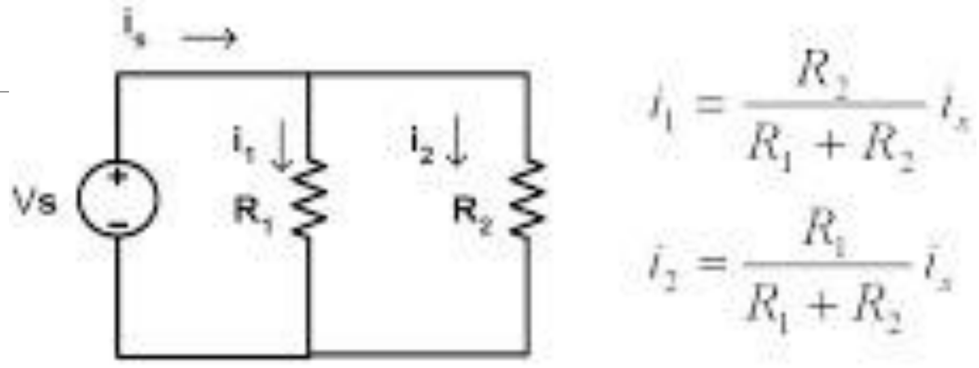
OR


1.05Ω
4.06A
A
B
Isrc → Vsrc


1.05Ω
A
B
10Ω
IL
4.263V
IL = 4.263 / (1.05+10) = 0.385A

Current Division Formula

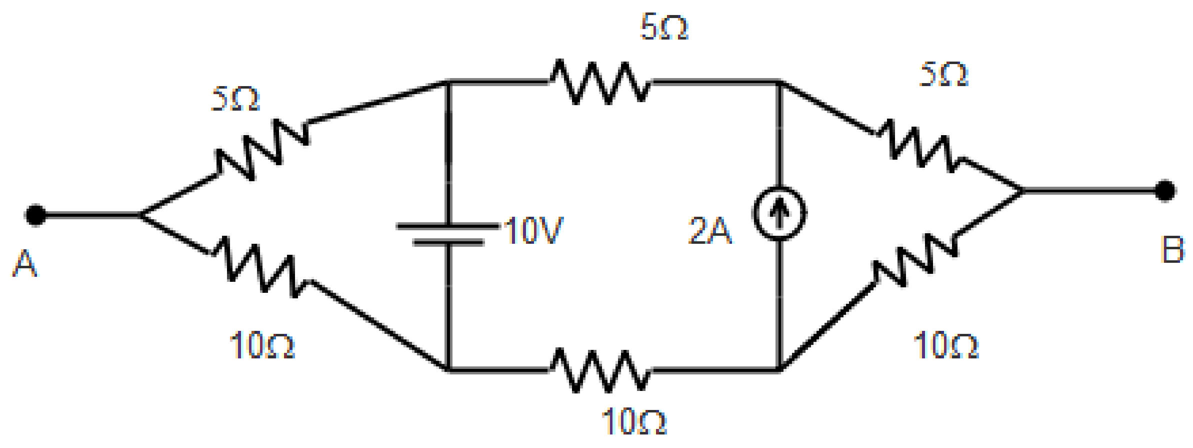




$$I_1 = \frac{R_2}{R_1 + R_2} I_t \,, \qquad I_2 = \frac{R_1}{R_1 + R_2} I_t$$

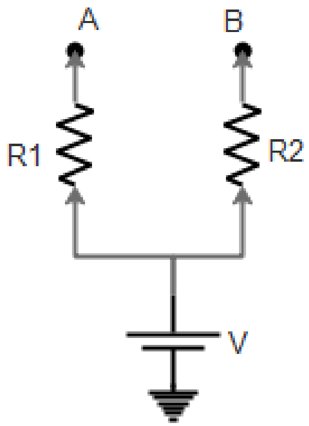
Examples

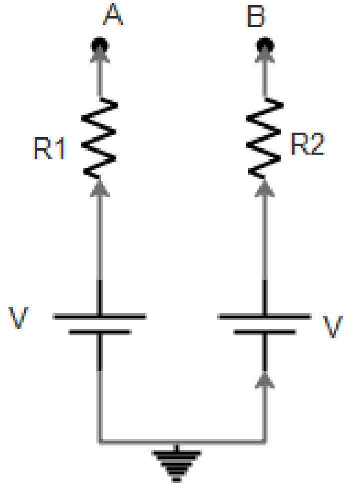
5. Simplify the network shown in figure.

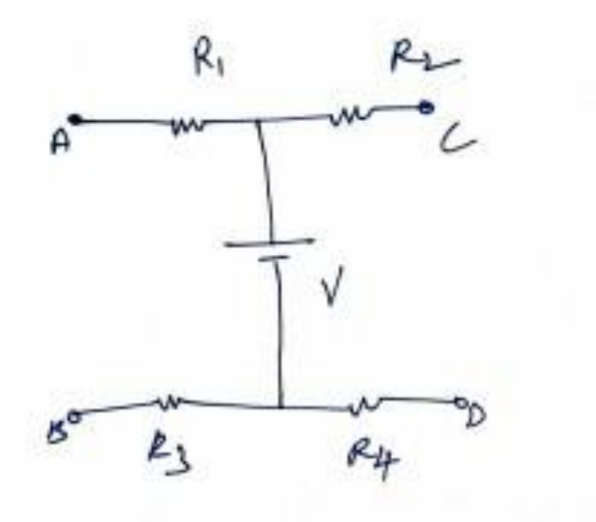


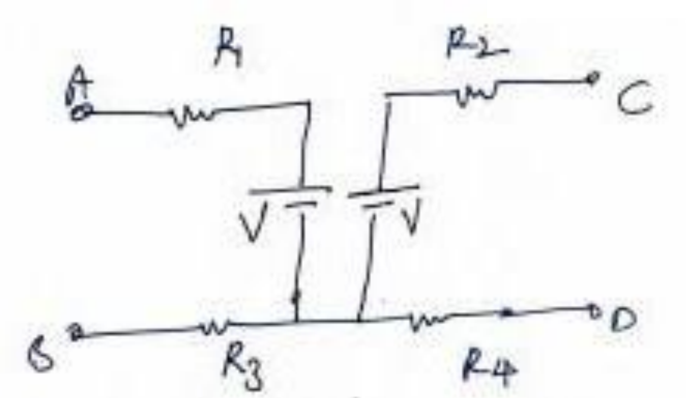
Source Shifting

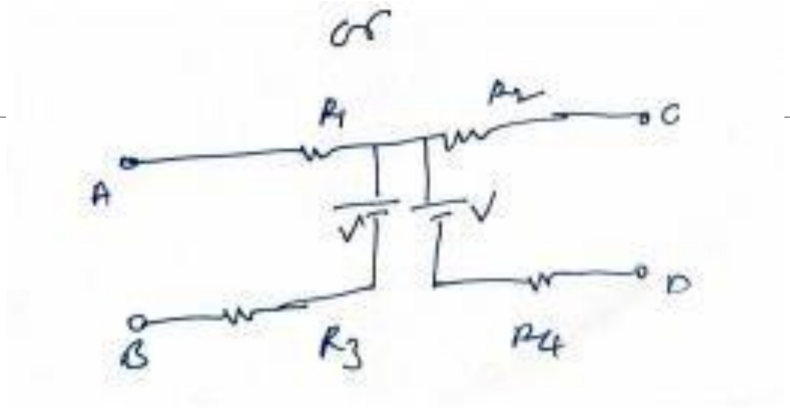
Voltage Source Shifting





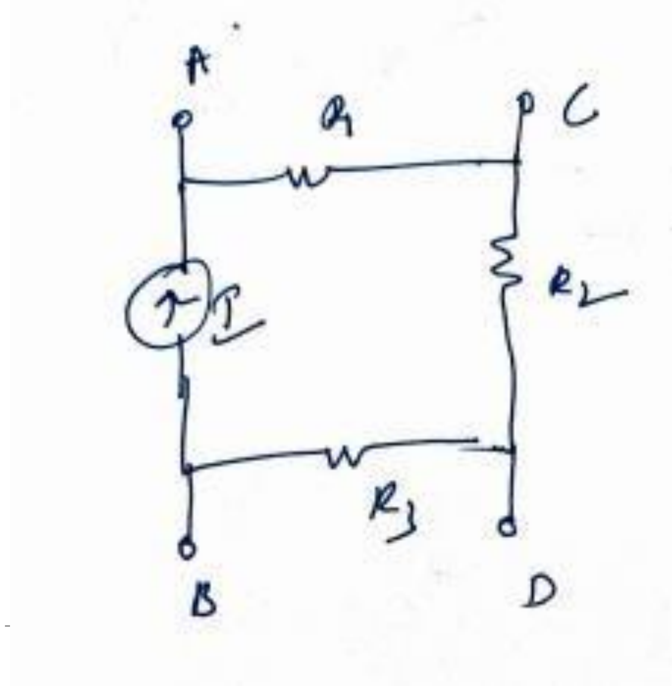


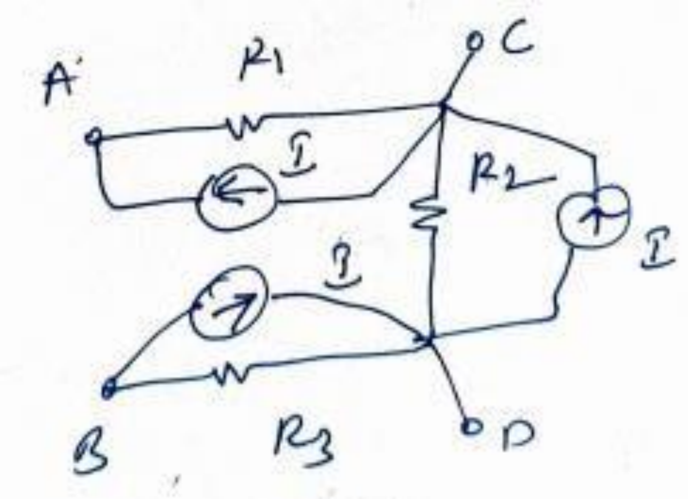




Source Shifting

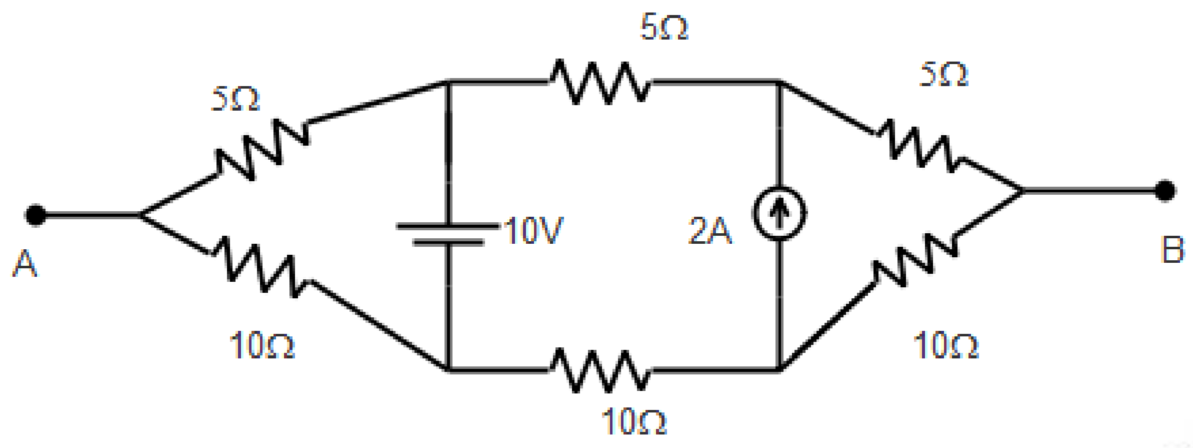
Current Source Shifting

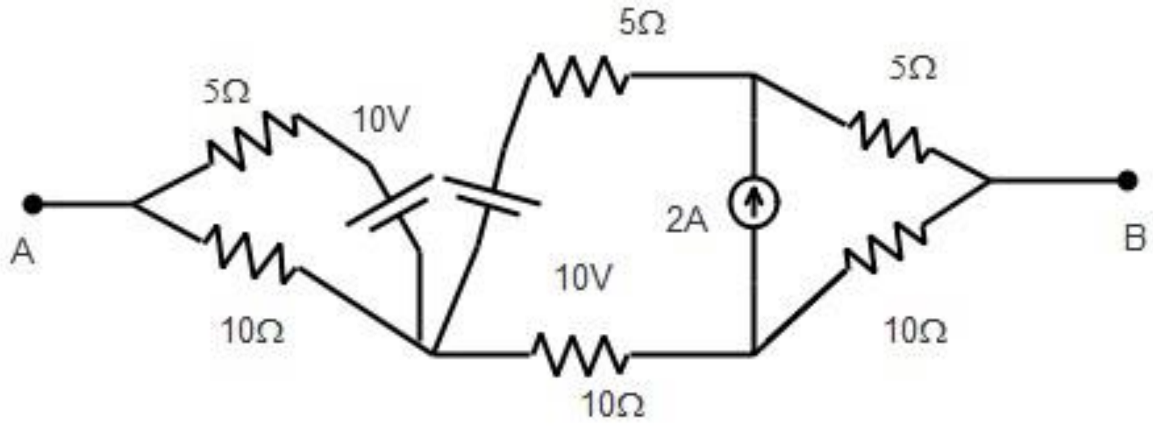




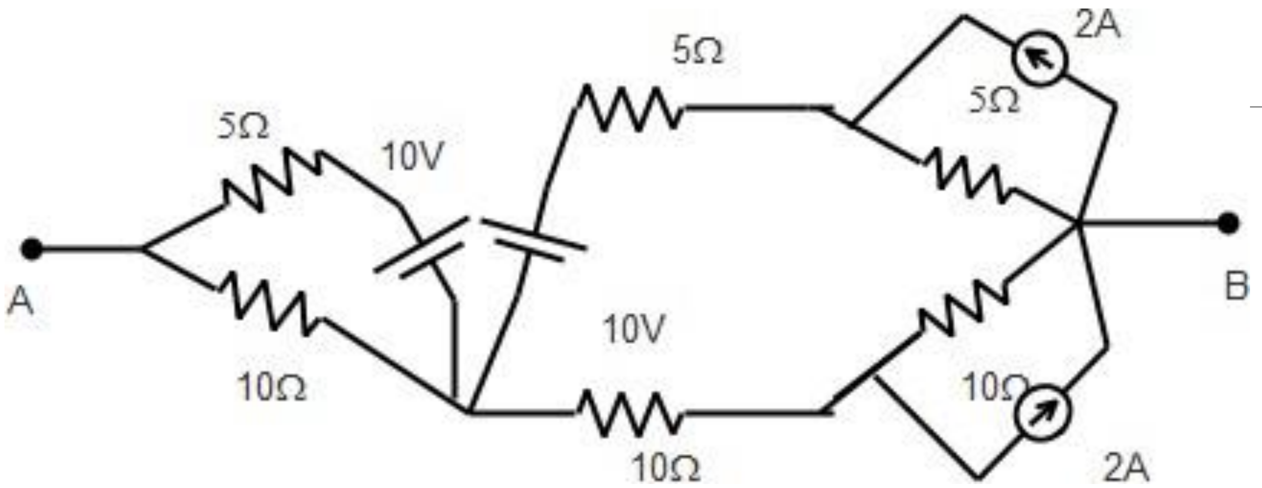
Examples

5. Simplify the network shown in figure.

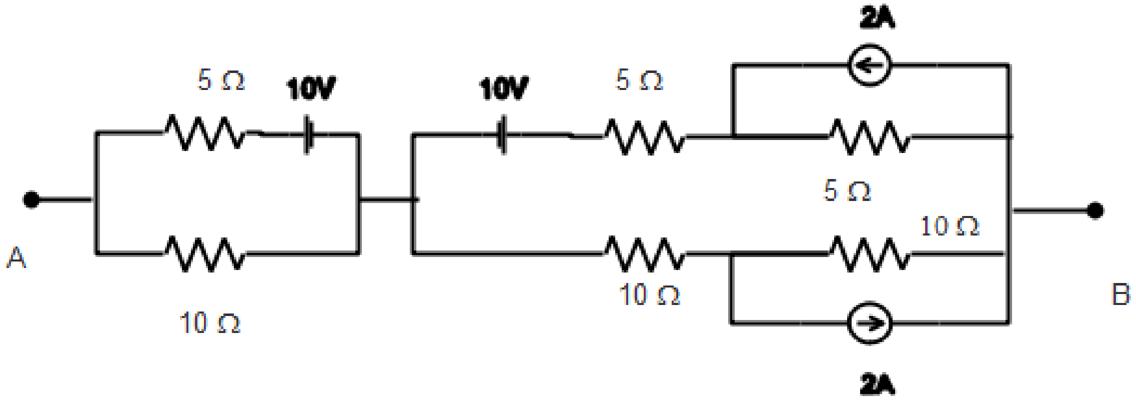




Examples

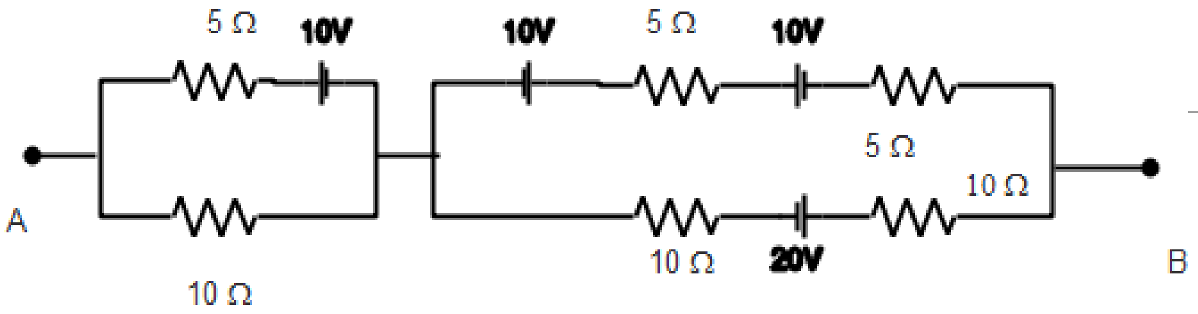


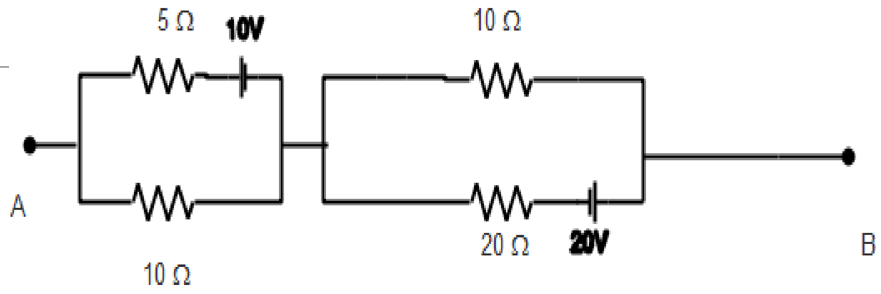
5Ω
5Ω
10V
2A
5Ω
A
B
10V
10Ω
10Ω
10Ω
2A

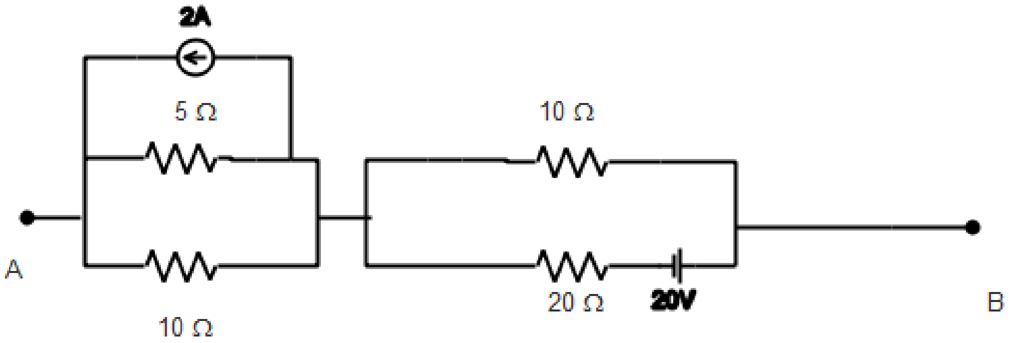


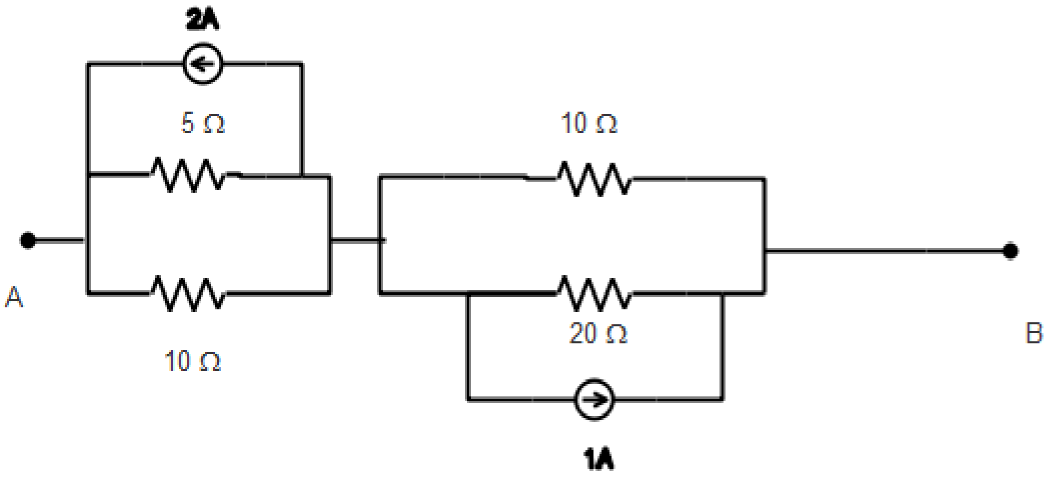
2A
5 Ω
10V
10V
5 Ω
5 Ω
10 Ω
A
10 Ω
B
10 Ω
2A

Examples

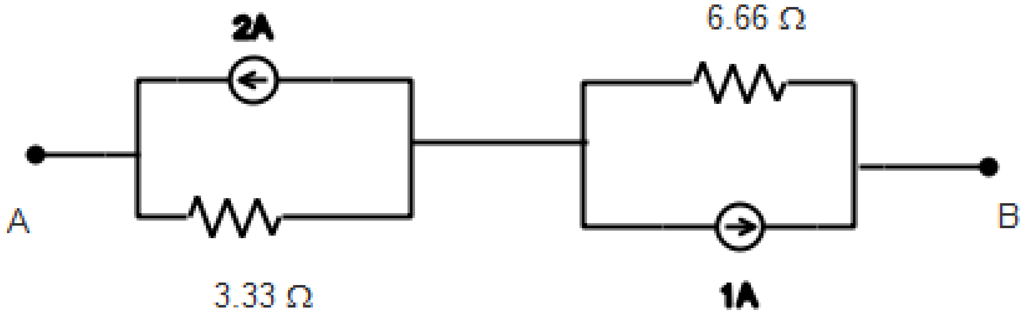

5 Ω
10V
10V
5 Ω
10V
5 Ω
10 Ω
A
10 Ω
20V
10 Ω
B

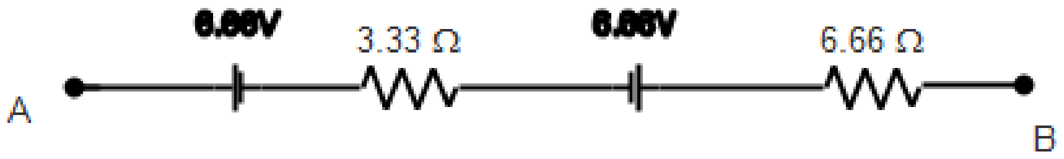

5 Ω
10V
10 Ω
A
20 Ω
20V
B
10 Ω


2A
5 Ω
10 Ω
A
20 Ω
20V
B
10 Ω


2A
5 Ω
10 Ω
A
20 Ω
B
10 Ω
1A

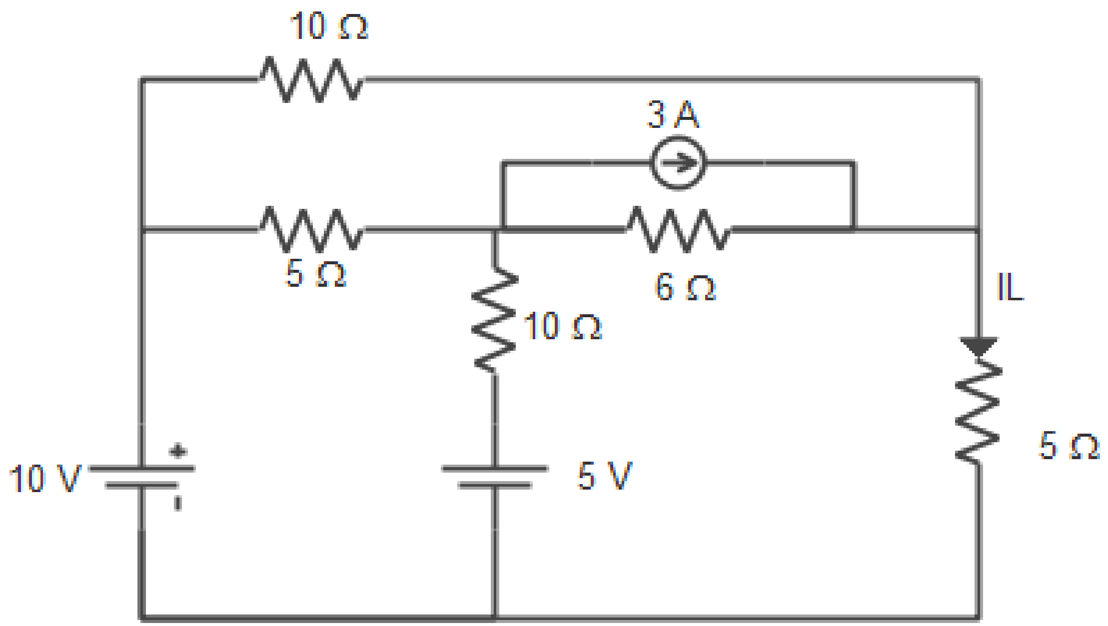
Examples



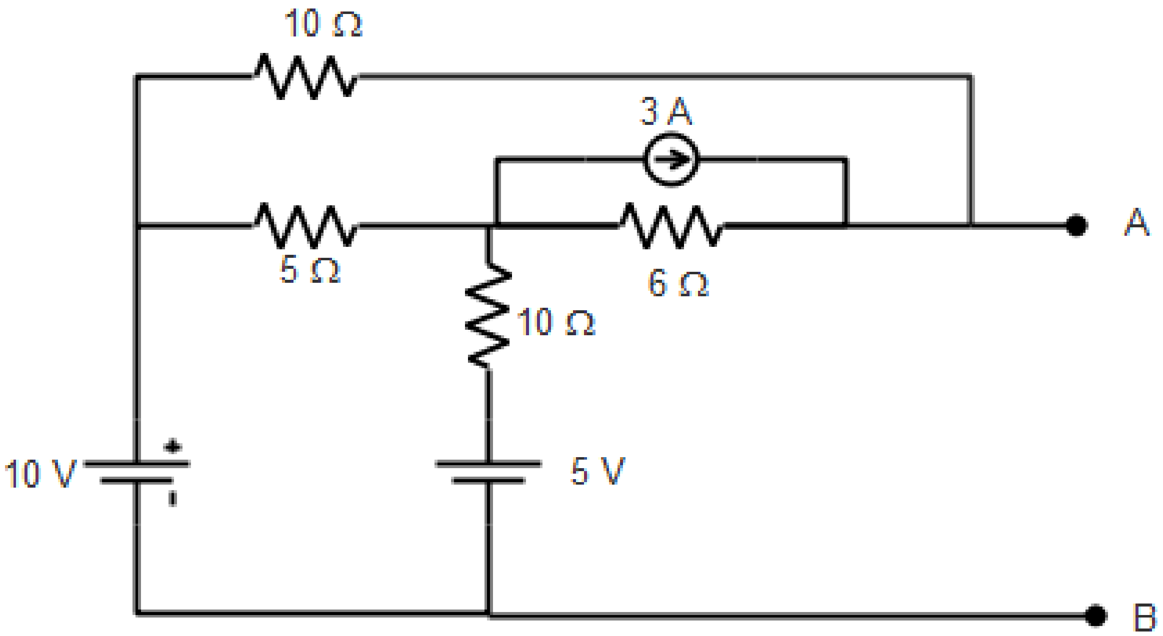


Examples

6. Find the Current IL in the circuit shown in figure.

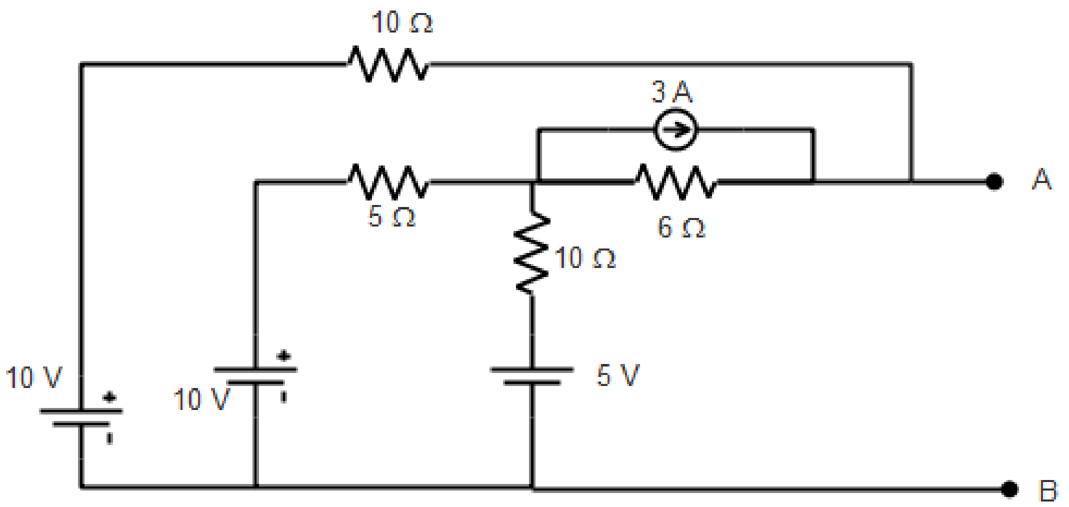


Identify the load element and remove it, then name the load terminals

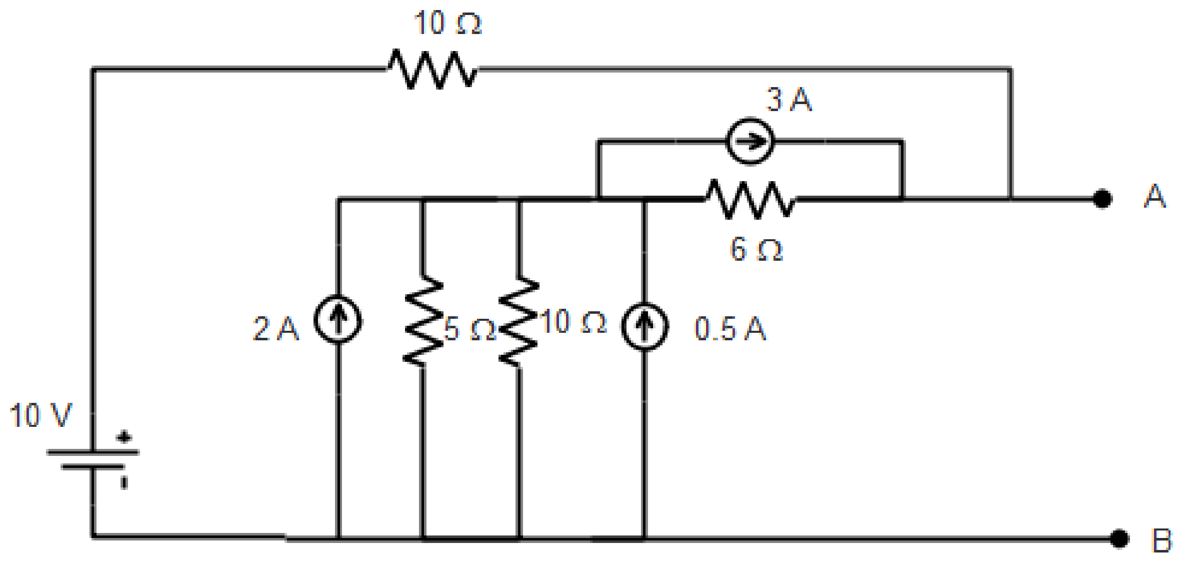


Examples

Apply Voltage source shifting for 10 V source

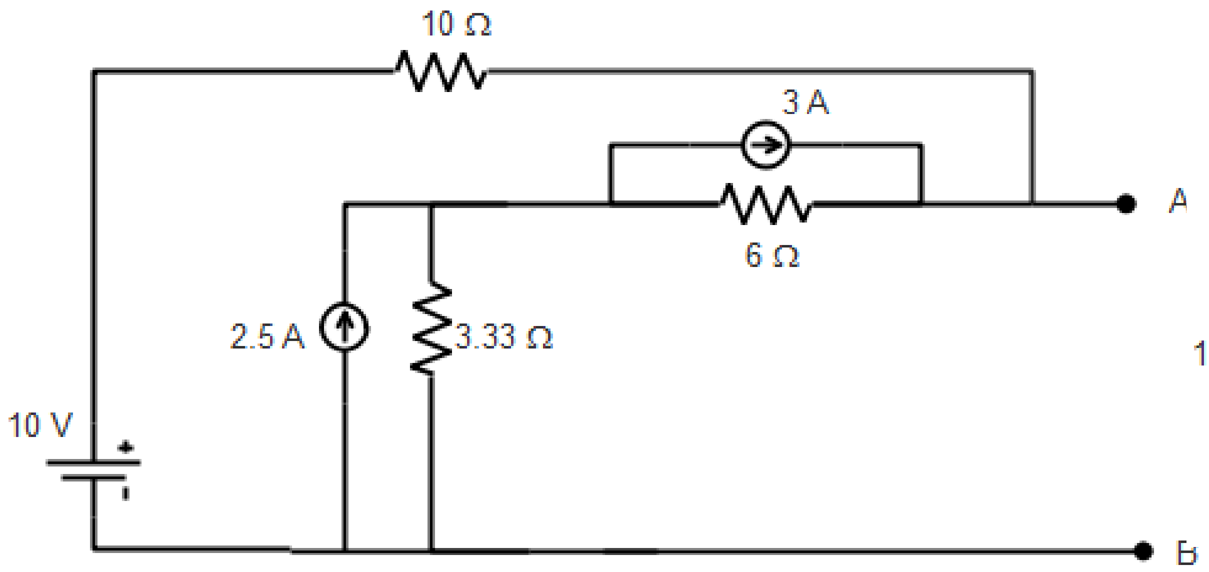


Convert voltage sources[(10 V, 5 Ω) , (5 V, 10 Ω)] into current sources

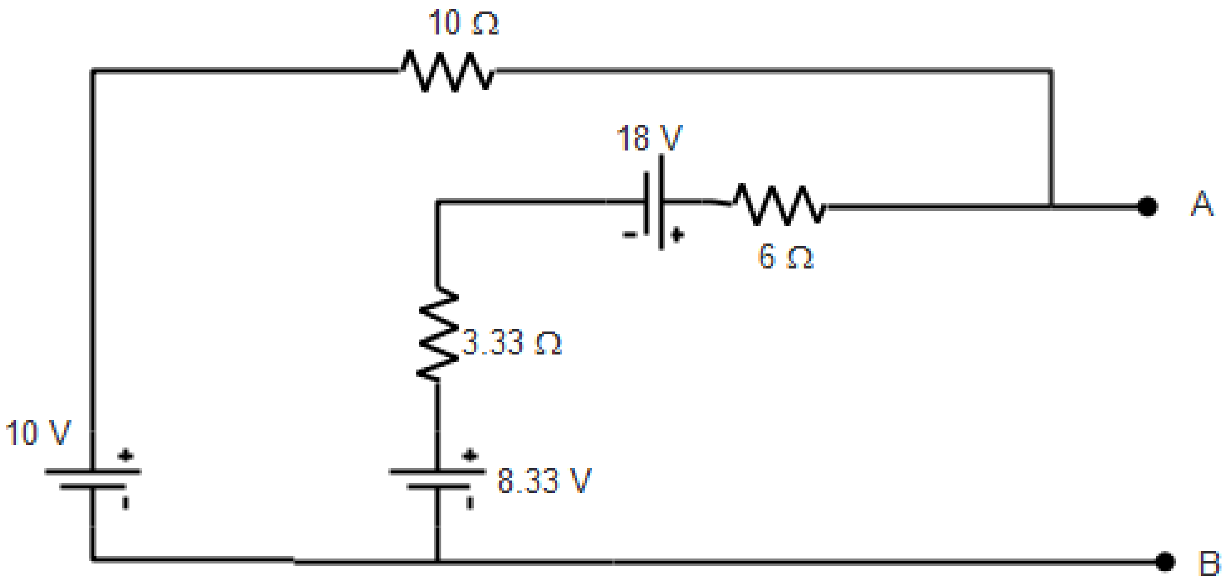


Examples

Combine Current sources (1A , 0.5A) and parallel Resistors (5 Ω, 10 Ω)

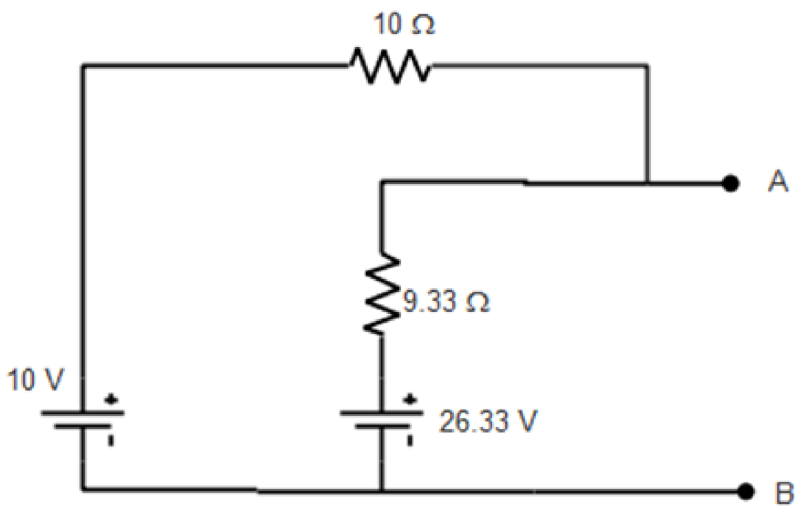


Combine Voltage sources (8.33 V , 18 V) and Series Resistors (6 Ω, 3.33Ω)

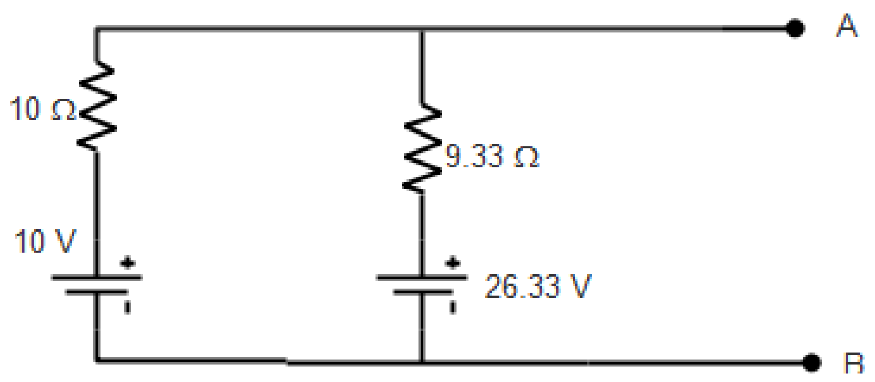


Examples

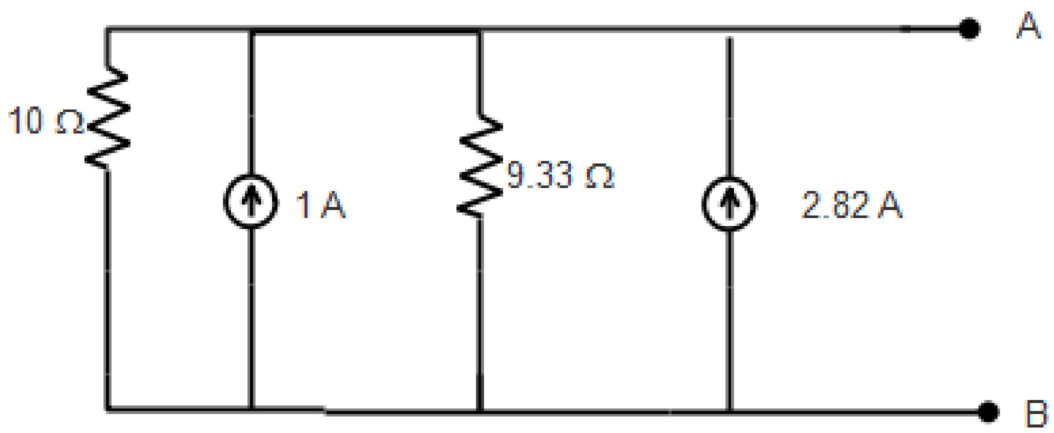
Re-arranging the elements



Convert voltage sources[(10 V, 10 Ω) , (26.33 V, 9.33 Ω)] into current sources

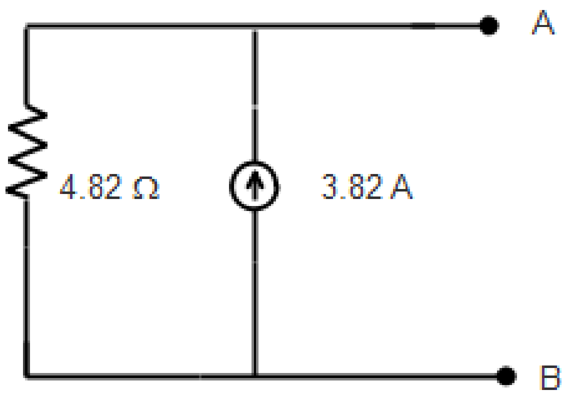


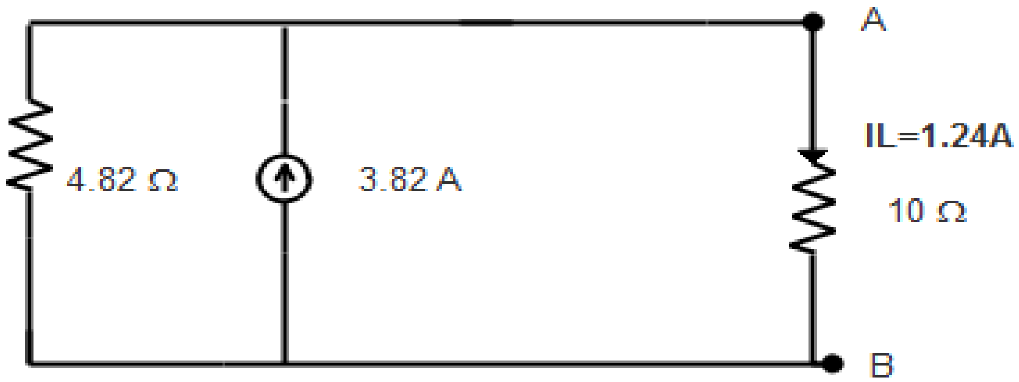
Combine Current sources (1A , 2.82 A) and parallel Resistors (9.33 Ω, 10 Ω)



Examples

Connect the load element across AB



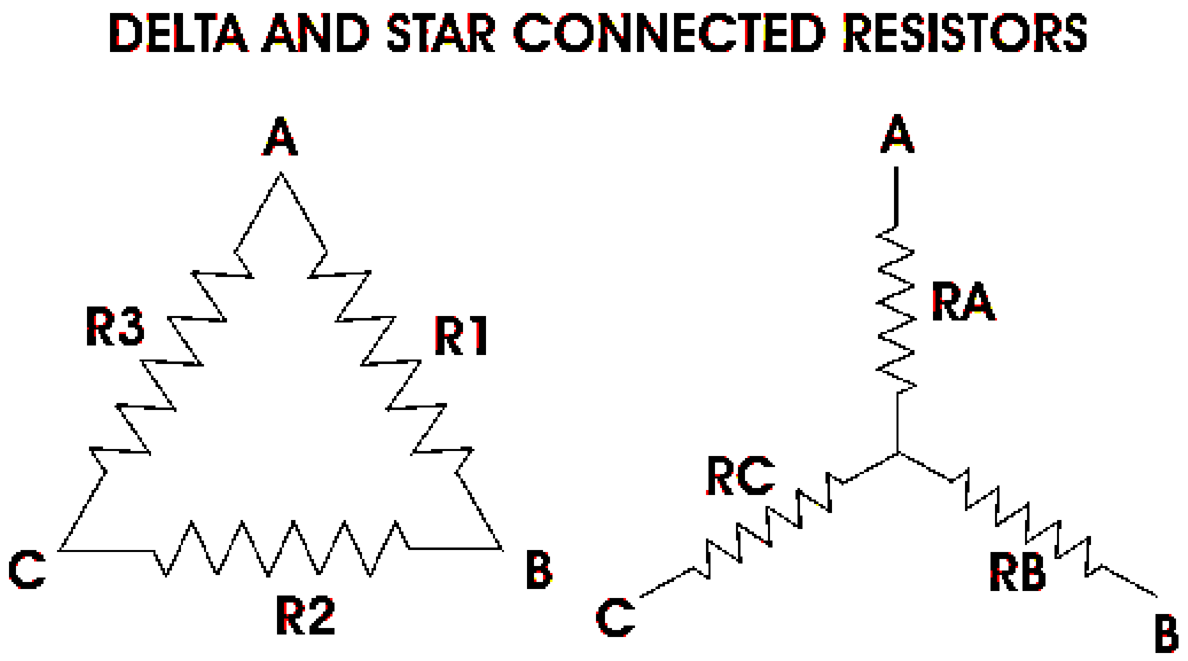


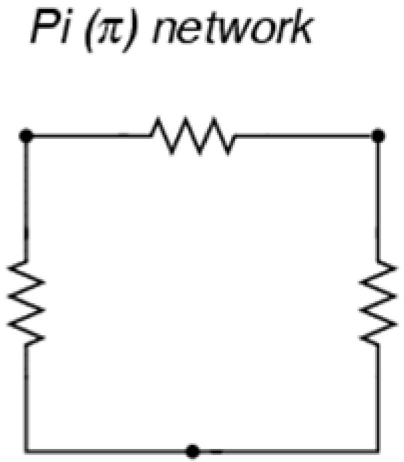
Apply current division formula

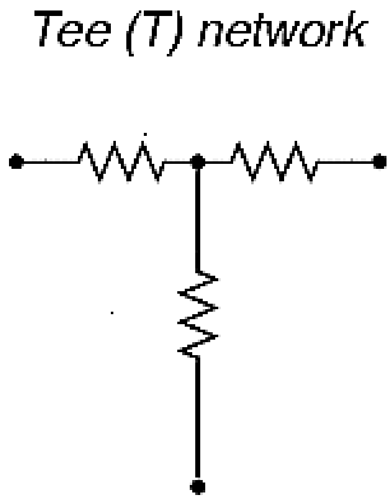
$$I_L = 3.82\left(\frac{4.82}{4.82+10}\right)$$

$$I_L = 1.24A$$

Star to delta and delta to start conversion

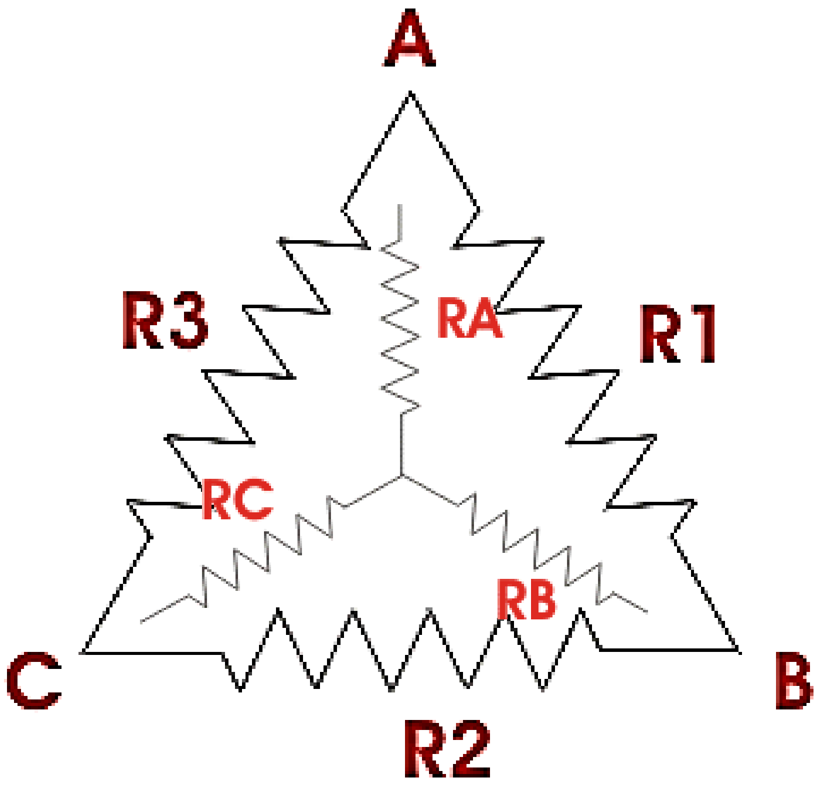






Delta to Star Conversion Formula

Delta to Start Conversion



$$R_A = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

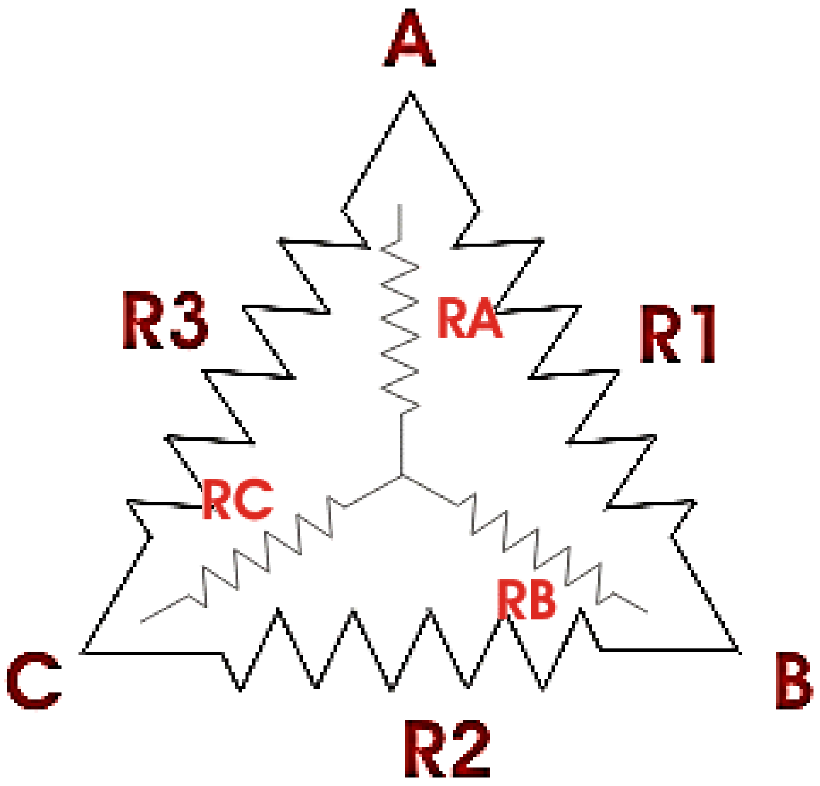
$$R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

NOTE: Denominator is common

Star to delta Conversion Formula

Star to Delta Conversion



$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

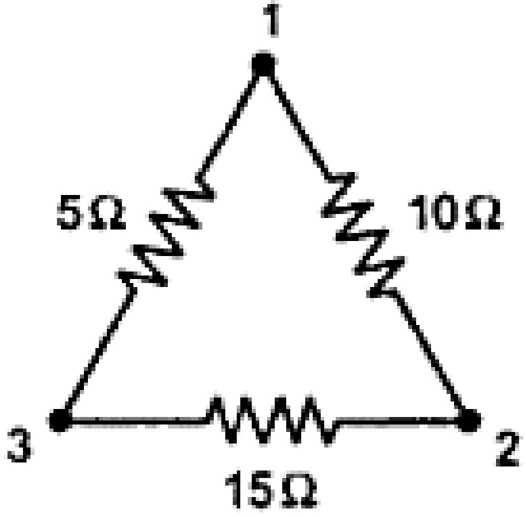
$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

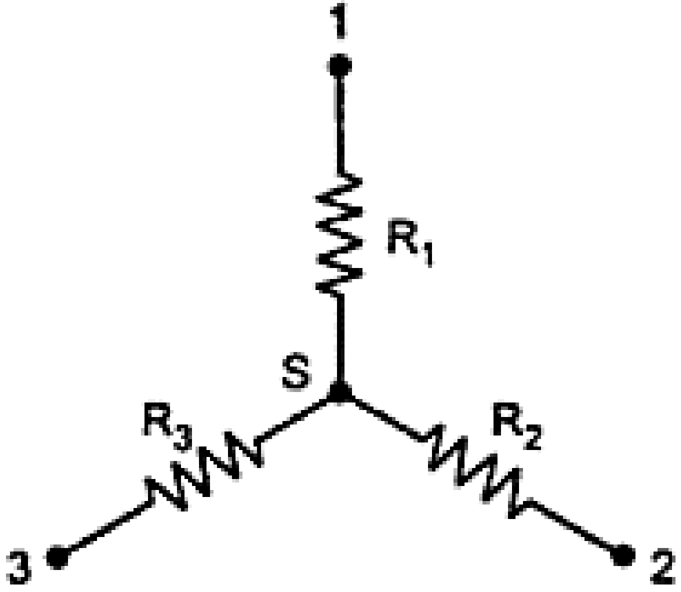
NOTE: Numerator is common

NOTE: Star to delta and delta to start conversion applicable to Capacitors and Inductors also, but should be in reactance format.

Examples

7. Convert the given network into equivalent star network





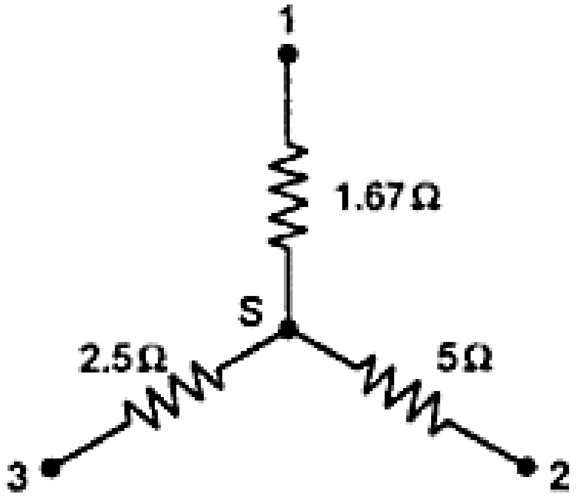
$$R_1 = \frac{5x10}{5+10+15} \Rightarrow 1.667\ \Omega$$

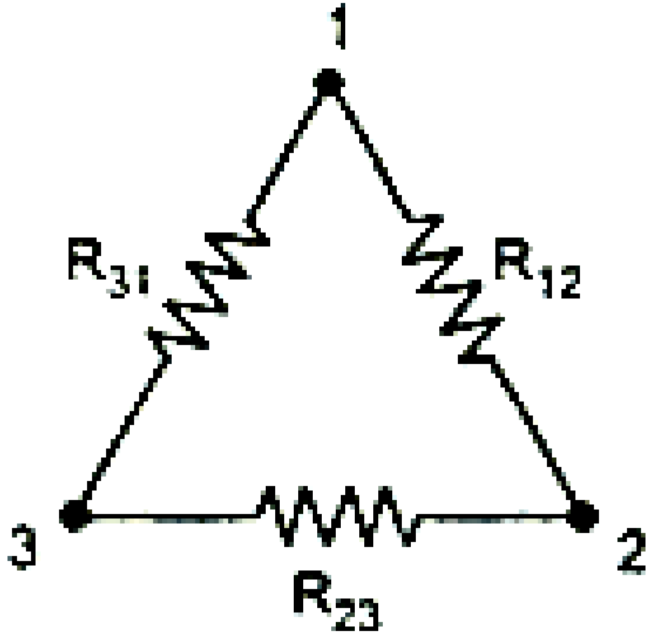
$$R_2 = \frac{10x15}{5+10+15} \Rightarrow 5\Omega$$

$$R_3 = \frac{5x15}{5+10+15} \Rightarrow 2.5\ \Omega$$

Examples

8. Convert the given network into equivalent Delta network





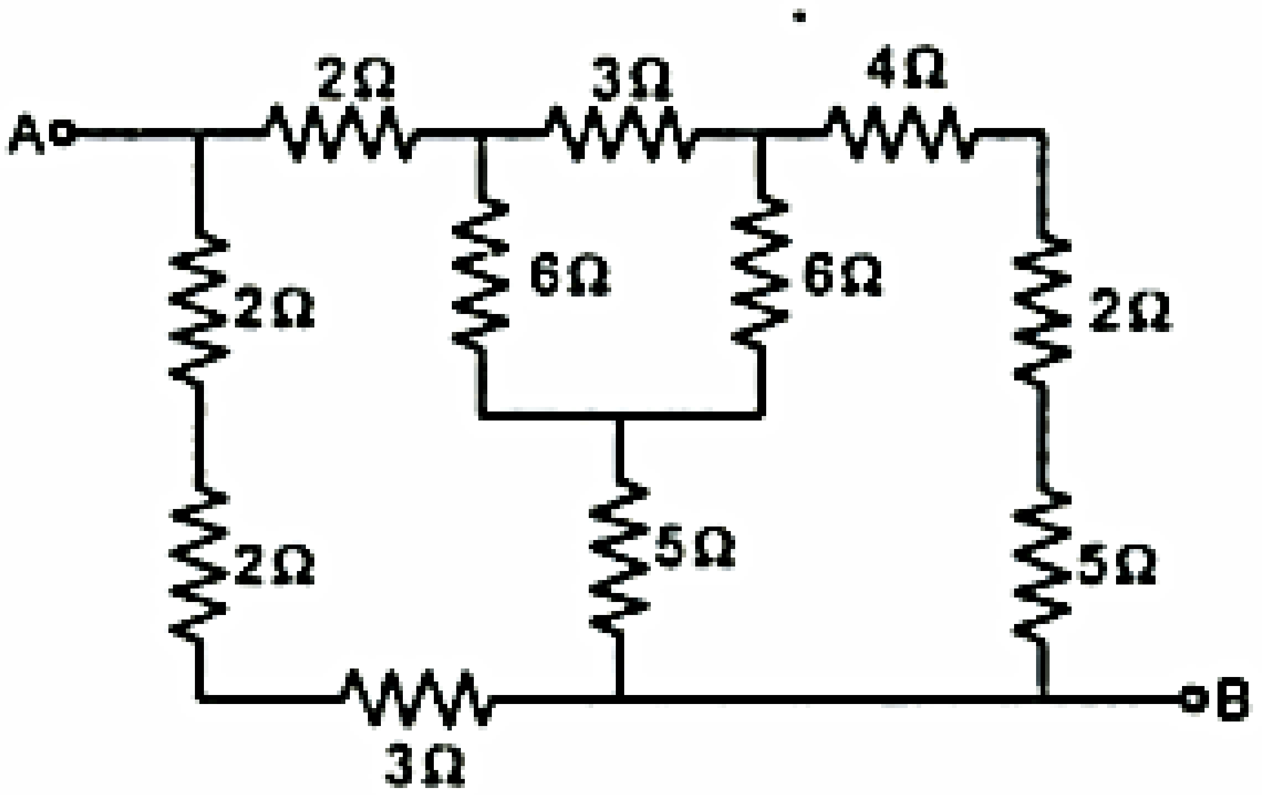
$$R_{31} = \frac{1.67x2.5 + 2.5x5 + 5x1.67}{5} \Rightarrow \mathbf{5\ \Omega}$$

$$R_{12} = \frac{1.67x2.5 + 2.5x5 + 5x1.67}{2.5} \Rightarrow \mathbf{10\ \Omega}$$

$$R_{23} = \frac{1.67x2.5 + 2.5x5 + 5x1.67}{1.67} \Rightarrow \mathbf{15\ \Omega}$$

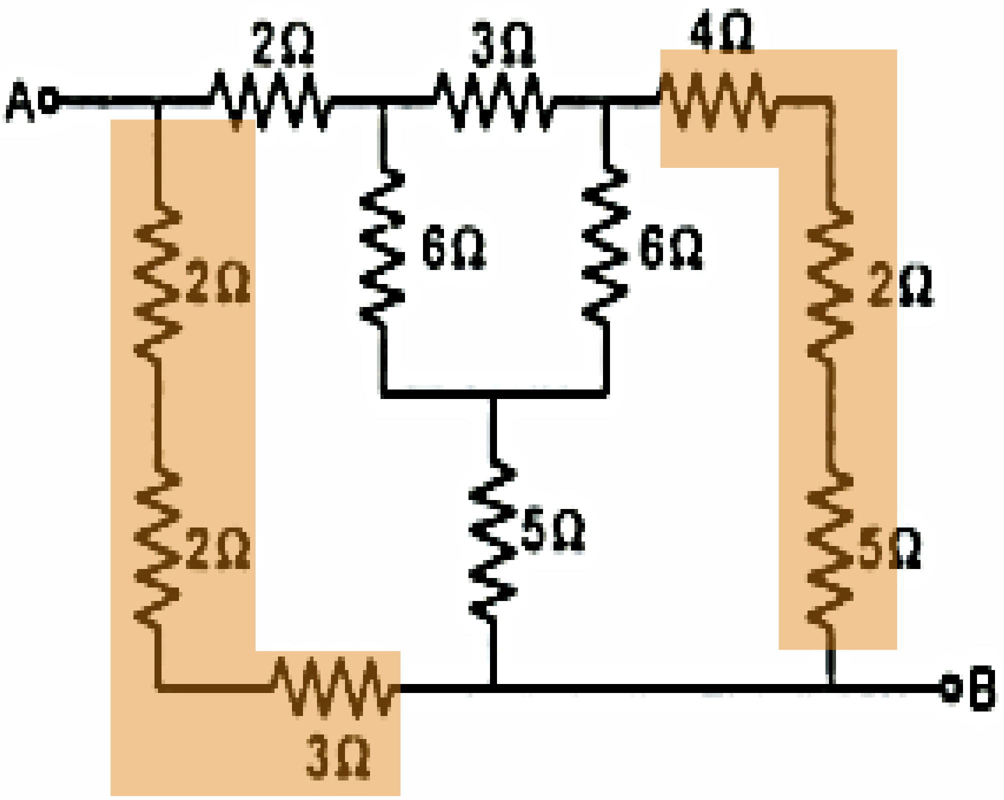
Examples

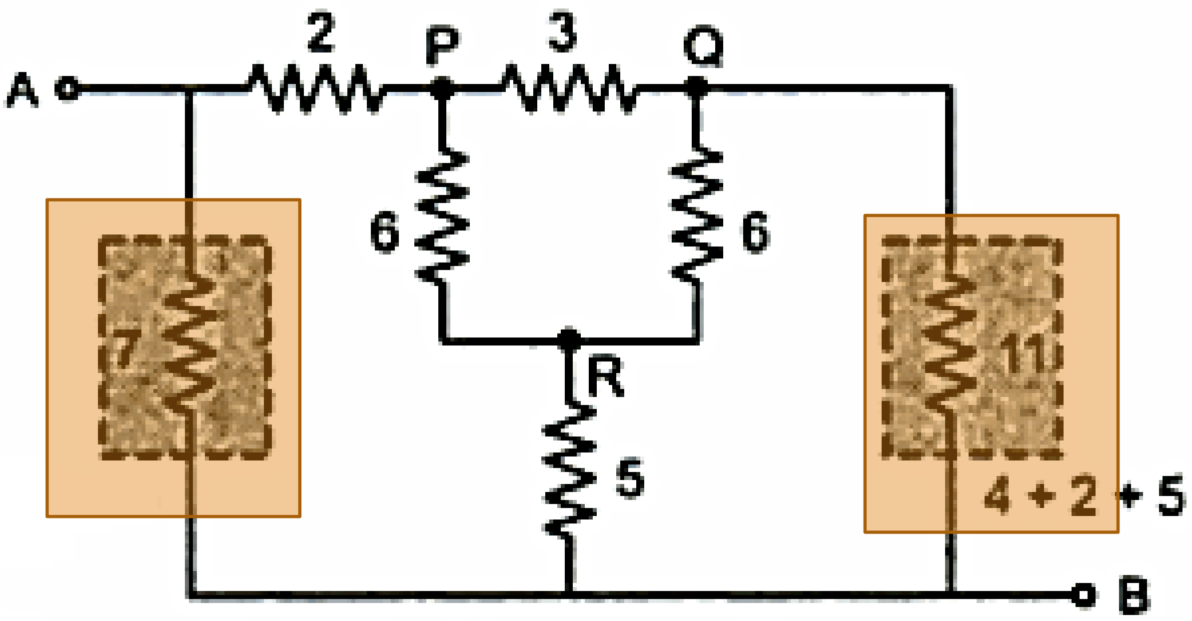
9. Find the equivalent resistance between the terminals A and B



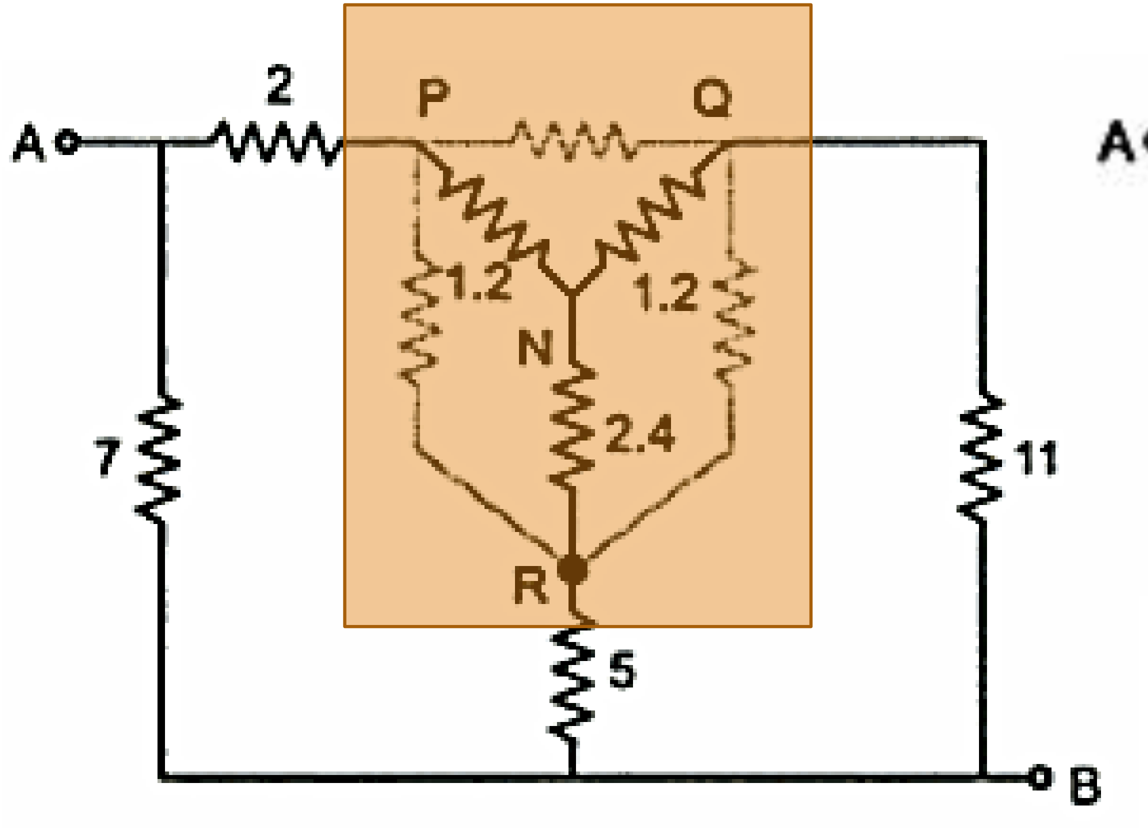
Examples

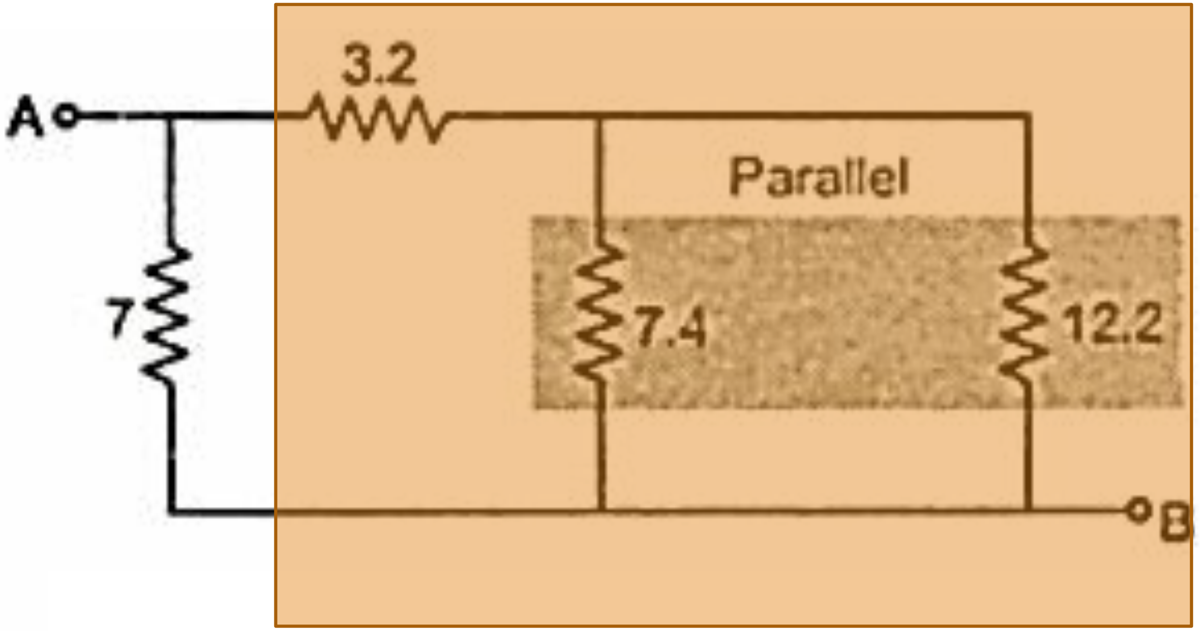
Combine the Series connected resistors



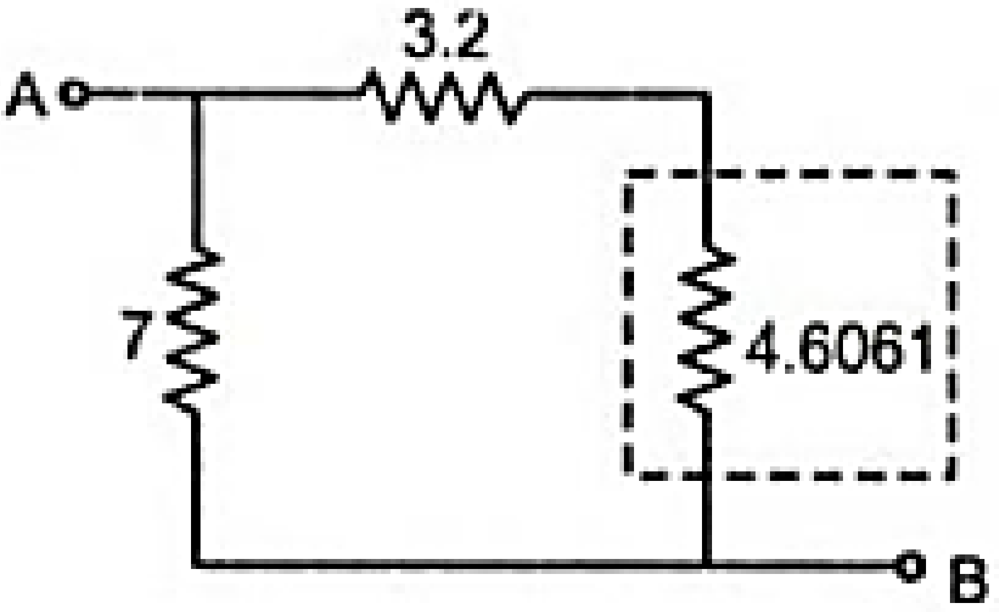


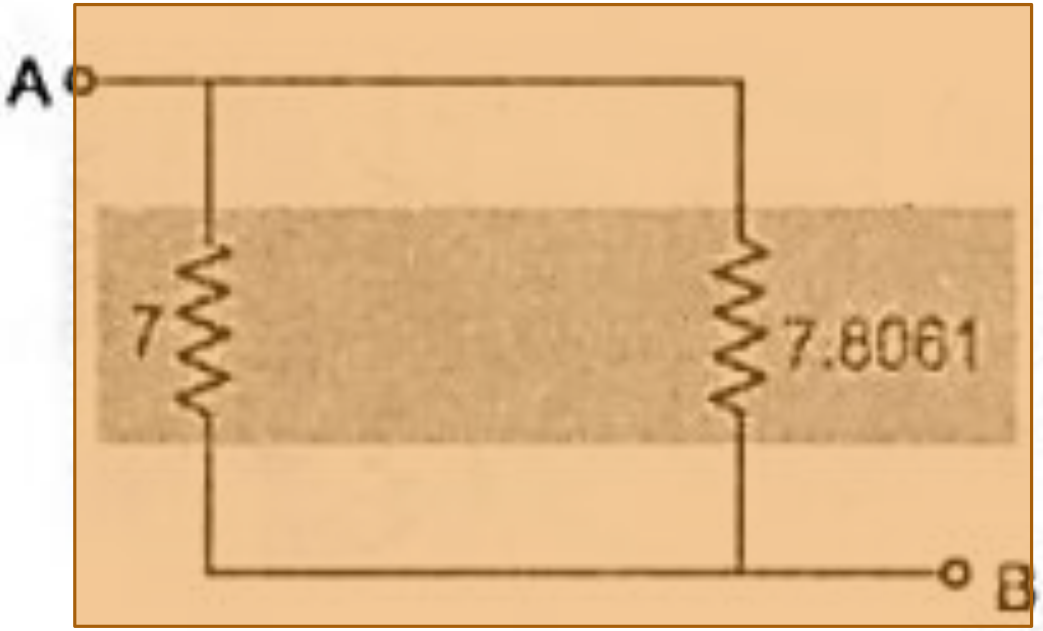
Examples





Examples

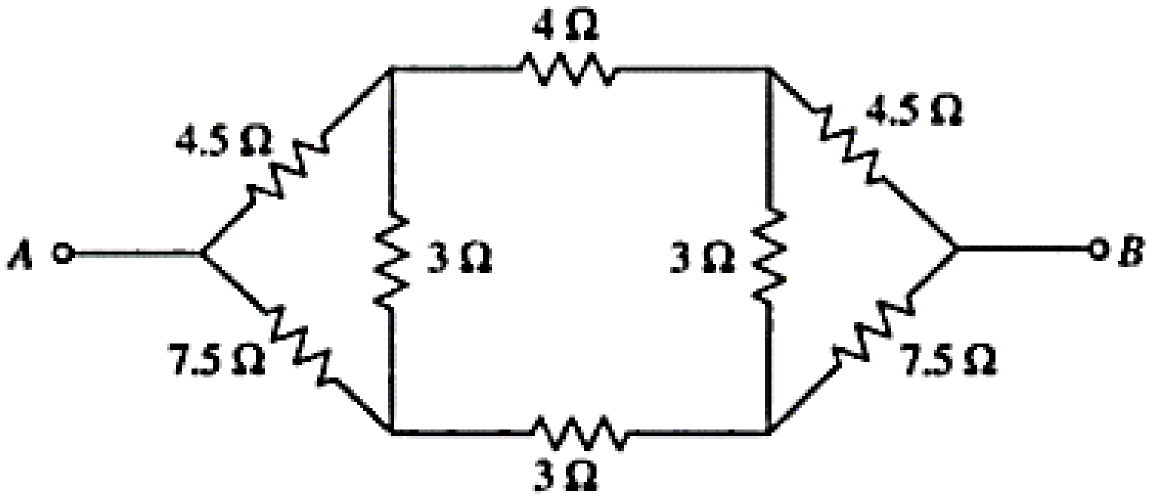


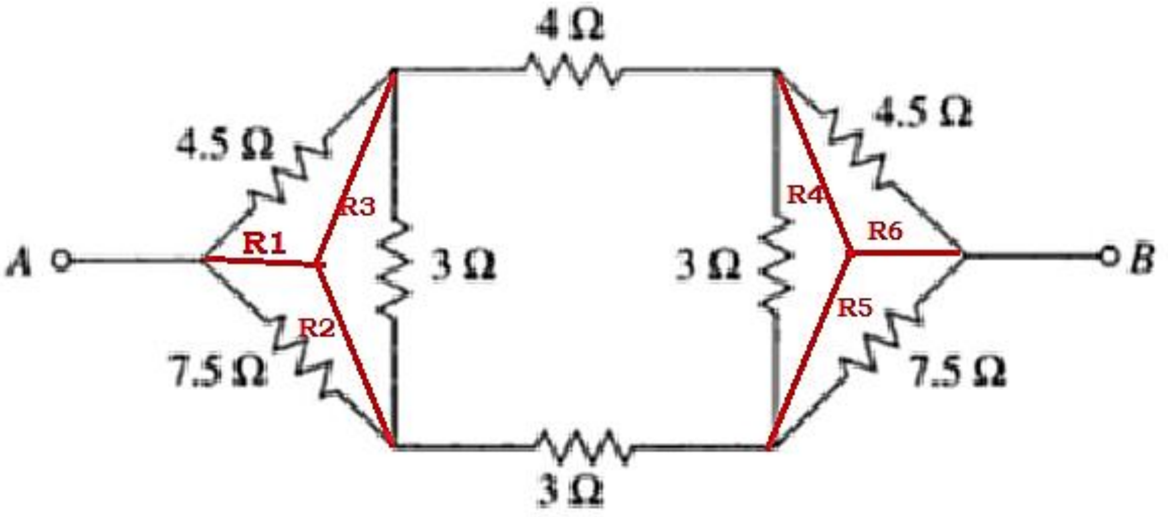


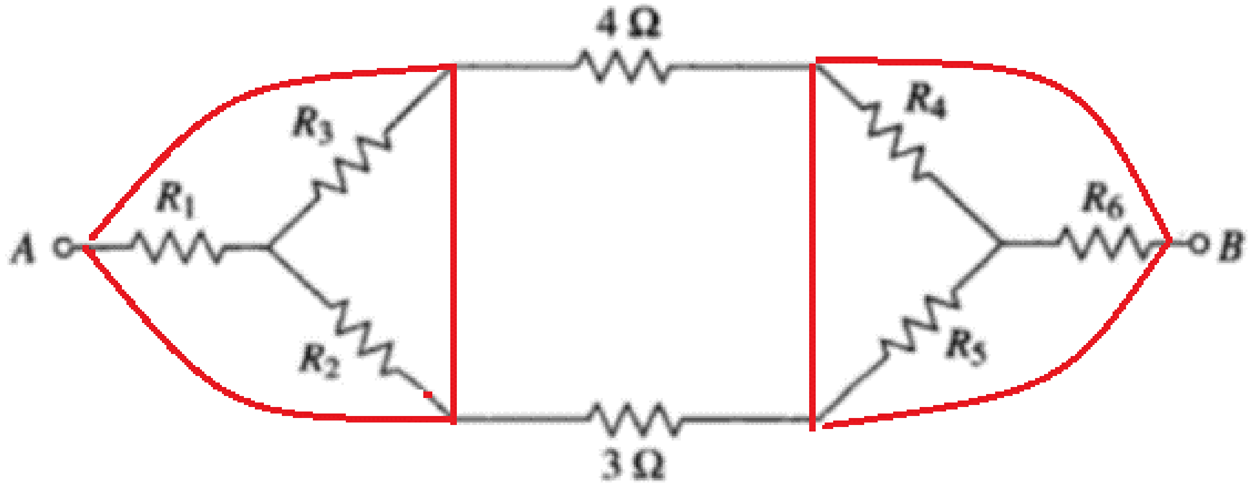
$$R_{AB} = 3.69\ \Omega$$

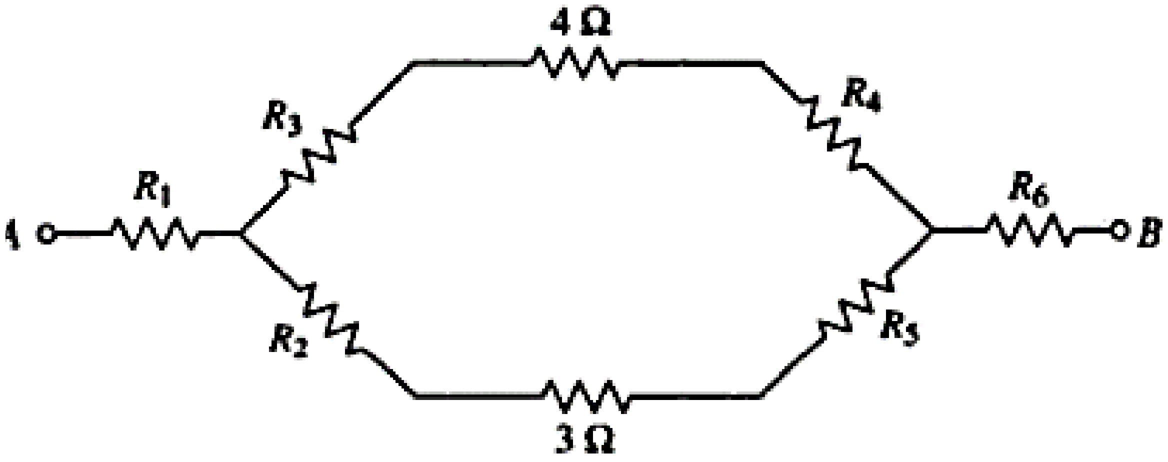
Examples

10. Find the equivalent resistance between the terminals A and B

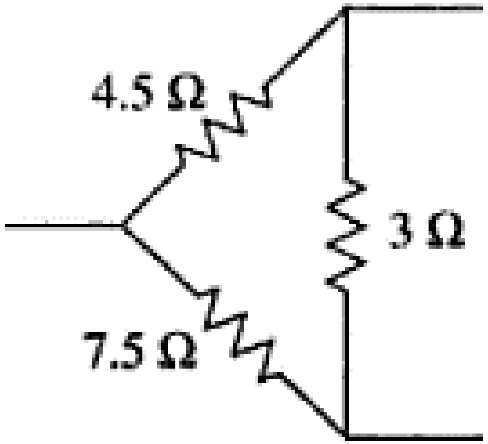


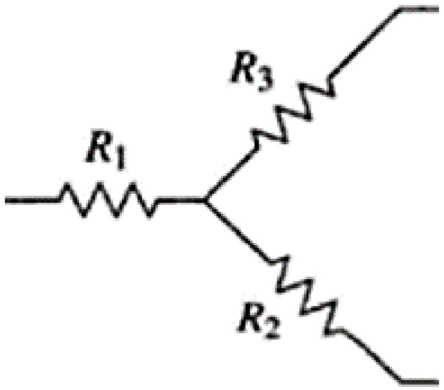


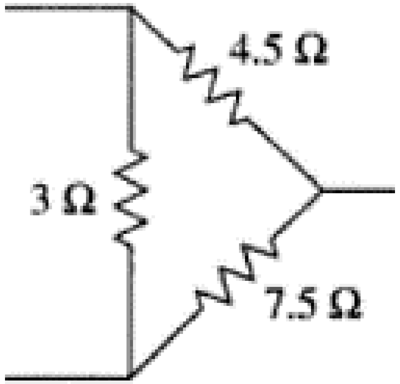


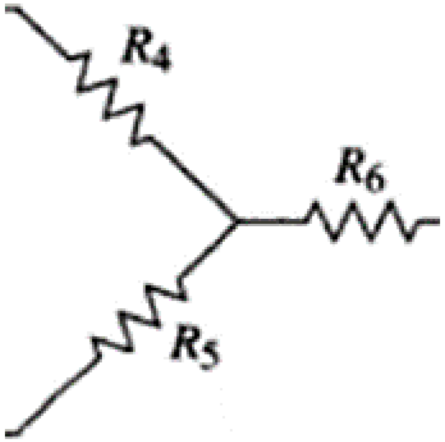


Examples







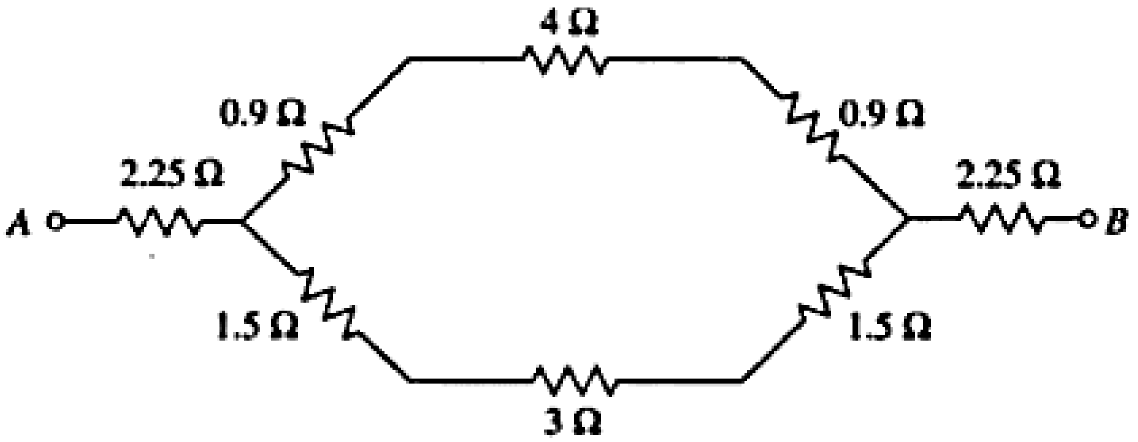


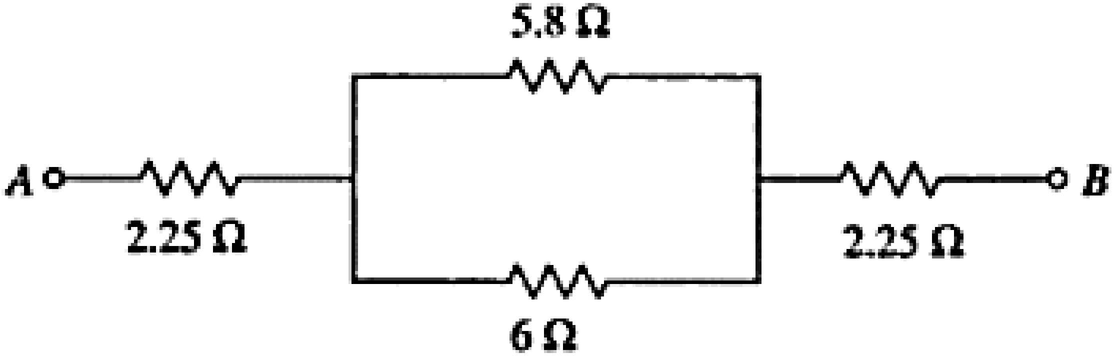
$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\ \Omega$$

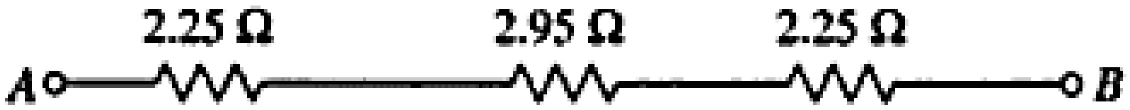
$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\ \Omega$$

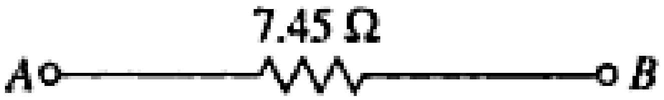
$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\ \Omega$$

Examples





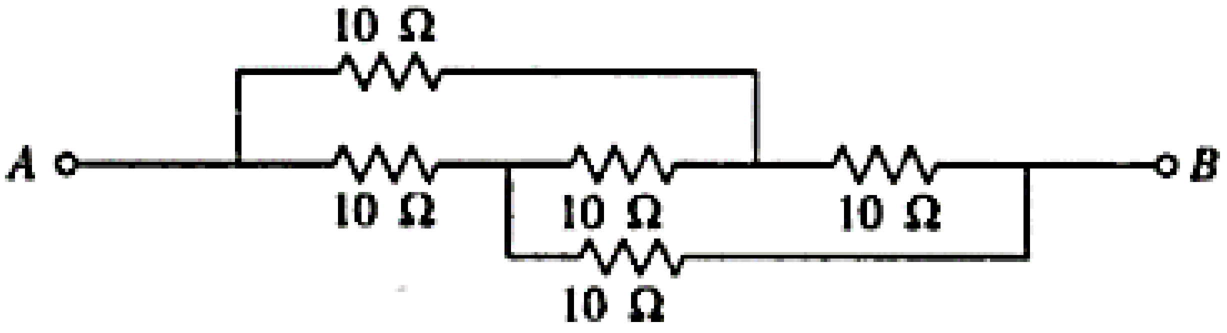


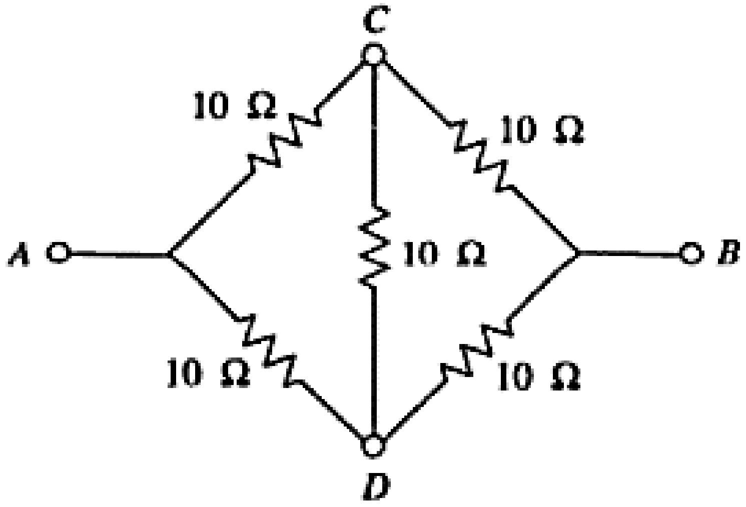


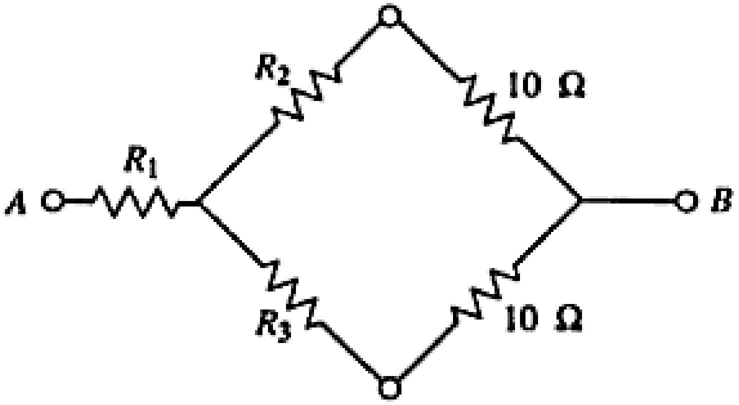
$$R_{AB} = 7.45\ \Omega$$

Examples

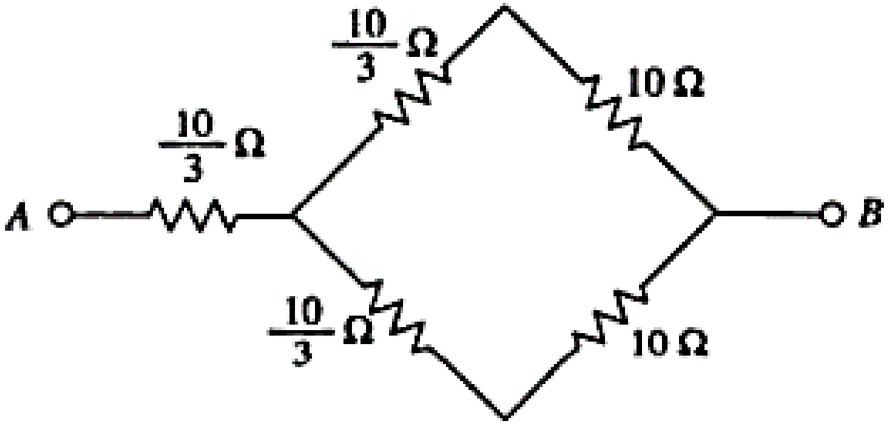
11. Find the equivalent resistance between the terminals A and B



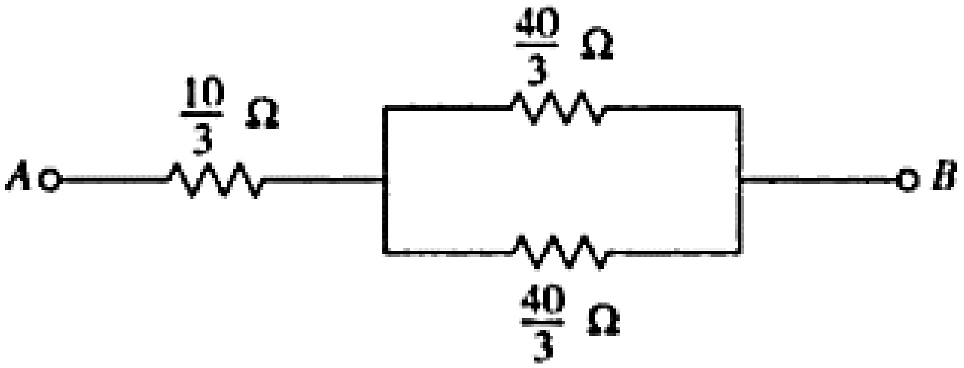


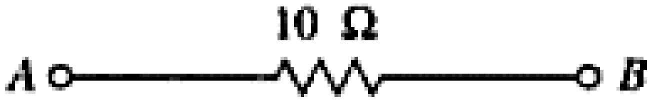


$$R_1 = R_2 = R_3 = \frac{10\times10}{10+10+10} = \frac{10}{3}\ \Omega$$



Examples





$$R_{AB} = 10\ \Omega$$

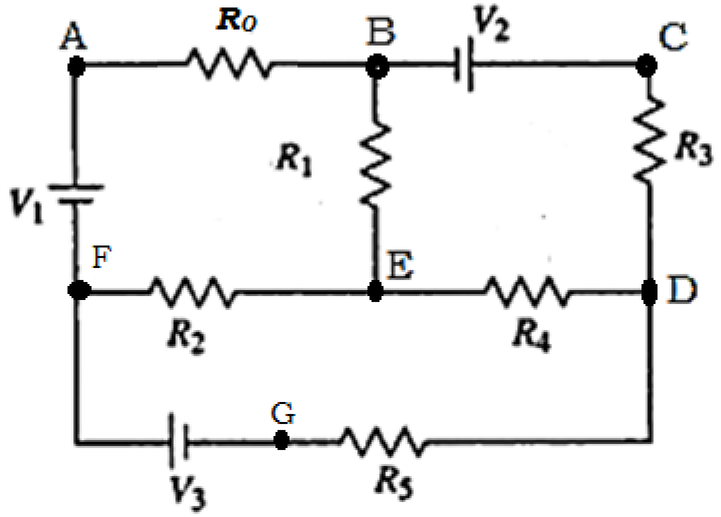
Network Analysis Techniques

Network Analysis: To study the behaviour (Finding the voltages, currents) of electrical circuits.

Types:

1. Mesh Analysis
2. Node Analysis

Terminologies and definitions



Loop: Any closed path
Example: A-B-E-F-A, B-C-D-E-B, E-D-G-F-E, A-B-C-D-E-F-A, A-B-C-D-G-F-A, A-B-E-D-G-F-A and B-C-D-E-F-G-D.

Mesh: Closed path without closed loops inside it.
Example: A-B-E-F-A, B-C-D-E-B and F-E-D-G-F

Node: Point or junction where two or more elements are connected together.
Example: A, B, C, D, E, F and G.

Fundamental Node: Point or Junction where current is dividing.
Example: B, E, D and F.

Note:

1. **All meshes are loops and vice-versa is not true.**
2. **All fundamental nodes are nodes and vice-versa is not true**

Network Analysis Techniques

KVL: Kirchhoff's Voltage Law
Statement:

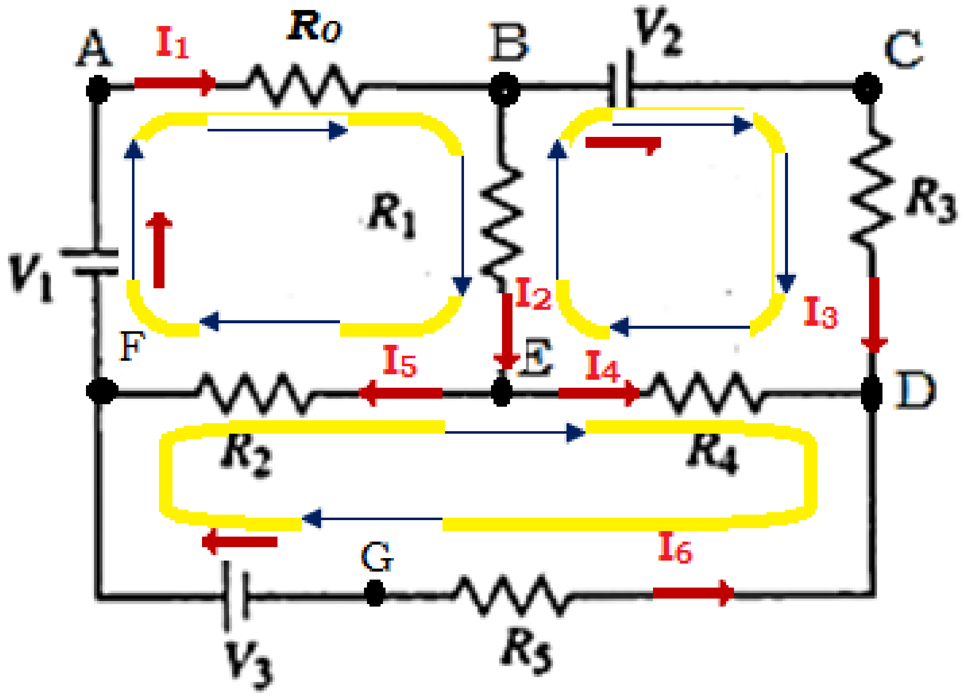
Algebraic sum of the voltages in any Loop is equal to zero.

$$i.e., \sum V_{loop} = 0$$

OR

Algebraic sum of the voltages applied is equal to the algebraic sum of the voltage developed across the elements in a loop.

$$i.e., \sum V_{Applied} = \sum V_{drop}$$



Example:
KVL to loop ABEFA

$$-V_1 + R_0 I_1 + R_1 I_2 + R_2 I_5 = 0;$$

OR

$$V_1 = R_0 I_1 + R_1 I_2 + R_2 I_5$$

Network Analysis Techniques

KCL: Kirchhoff's Current Law
Statement:

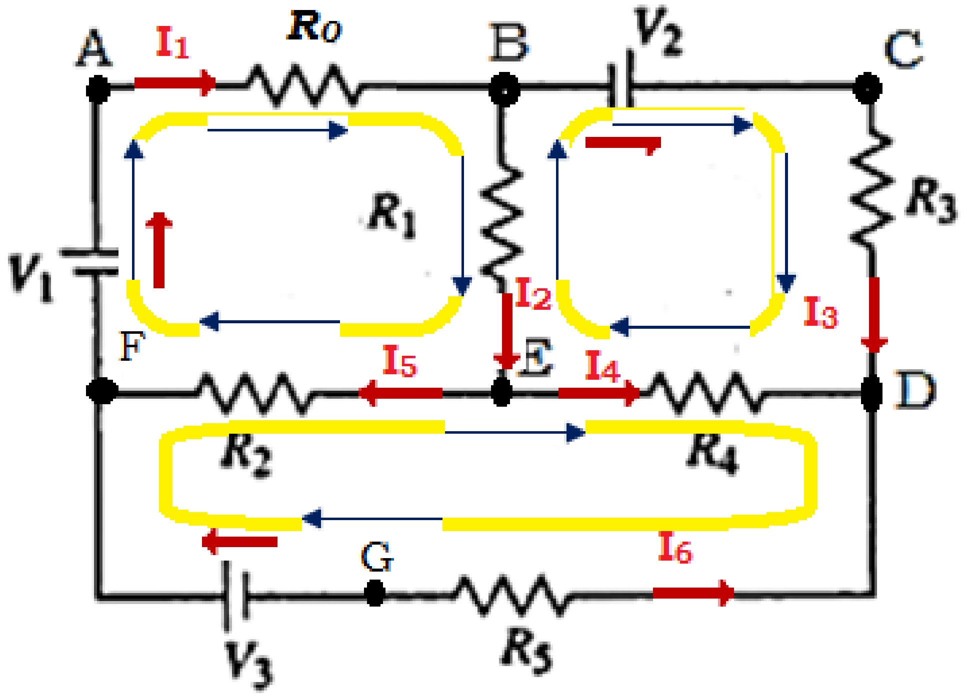
Algebraic sum of the branch Currents meeting at a node is equal to zero.

$$i.e., \sum I_{node} = 0;$$

OR

Algebraic sum of the Current entering the node is equal to the algebraic sum of the currents leaving the node.

$$i.e., \sum I_{Entering} = \sum I_{leaving}$$



Example:
At Node B
Apply KCL

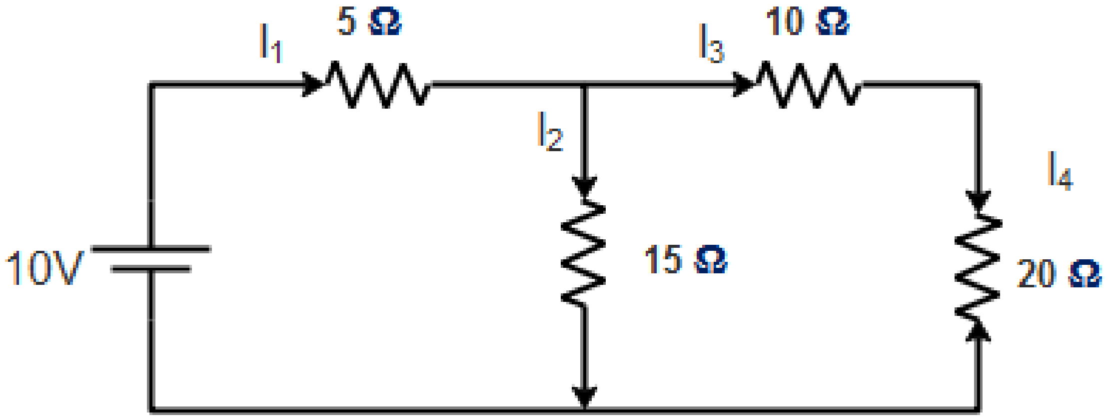
$$I_1 - I_2 - I_3 = 0;$$

OR

$$I_1 = I_2 + I_3\ ;$$

Examples:

1. Find the branch currents and voltages for the electrical circuit shown in figure.



$$-10+5I_1+15I_2=0 --- (1)$$
$$10I_3+20I_4-15I_2=0 --- (2)$$

$$I_3=I_4 --- (3)$$
$$I_1=I_2+I_3 --- (4)$$

Substitute 4 in 1

$$5(I_2+I_3)+15I_2=10$$
$$20I_2+5I_3=10 -- (5)$$

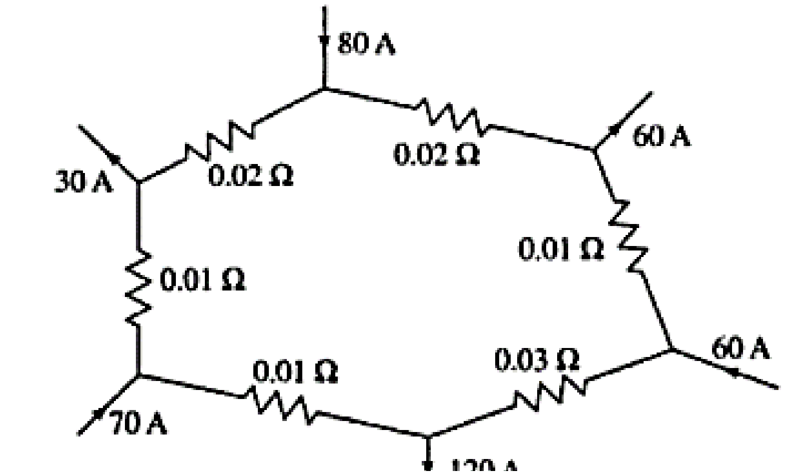
Substitute 3 in 2

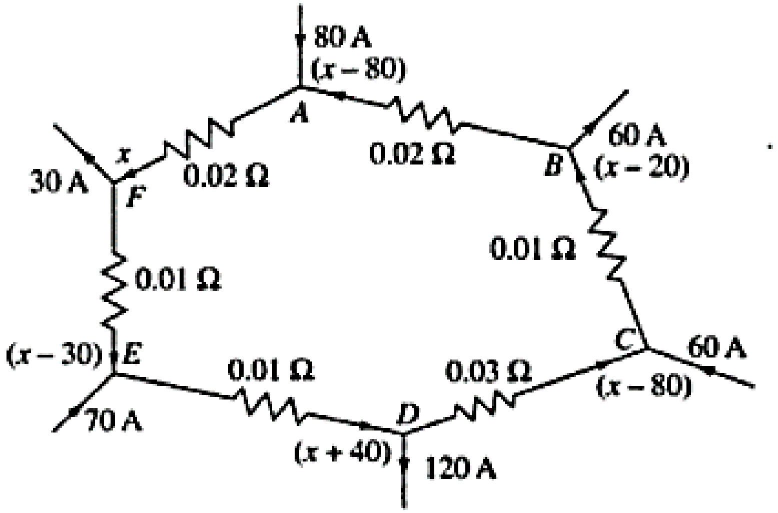
$$10I_3+20I_3-15I_2$$
$$-15I_2+30I_3=0 -- (6)$$

$$\boldsymbol{I_2} = \mathbf{0.4\ A}$$
$$\boldsymbol{I_3 = 0.2\ A}$$
$$\boldsymbol{I_4} = \mathbf{0.2\ A}$$
$$\boldsymbol{I_1} = \mathbf{0.6\ A}$$

Examples:

2. Find the branch currents for the electrical circuit shown in figure.

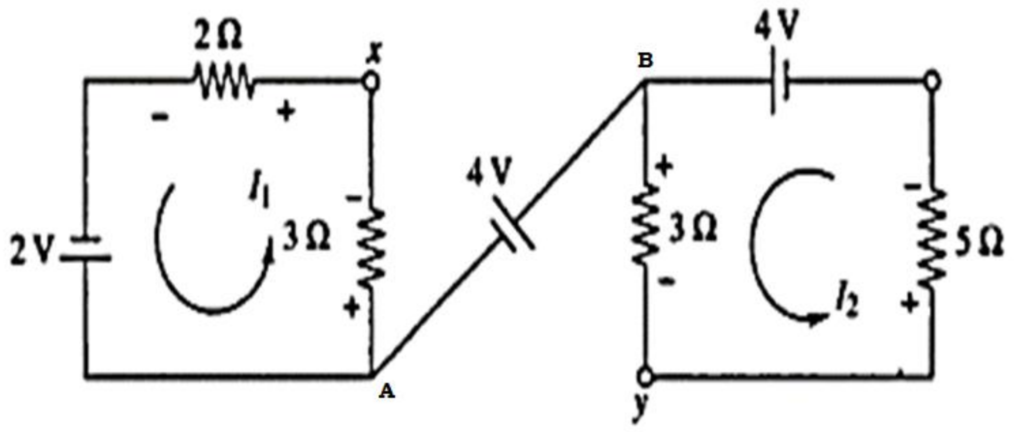




$$0.02(x-80)+0.02x+0.01(x-30)+0.01(x+40)$$
$$+0.03(x-80)+0.01(x-20)=0 \;---(1)$$
$$0.1x=4.1$$
$$x=41A$$

Examples:

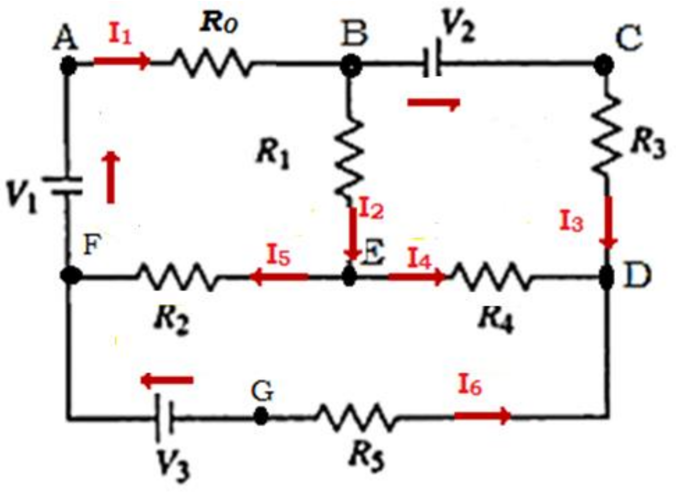
3. Find the voltage "V_{xy}" for the electrical circuit shown in figure.



$$-2 + 3I_1 + 2I_1 = 0 --- (1)$$
$$\boldsymbol{I_1 = 0.4A}$$
$$-4 + 3I_2 + 5I_2 = 0 -- (2)$$
$$\boldsymbol{I_2 = 0.5A}$$
$$\boldsymbol{V_{xy} = V_{xA} + V_{AB} + V_{By}}$$
$$\boldsymbol{V_{xy} = 3(-I_1) + (-4) + 3(I_2)}$$
$$\boldsymbol{V_{xy} = -3.7\ Volts}$$

Mesh Analysis

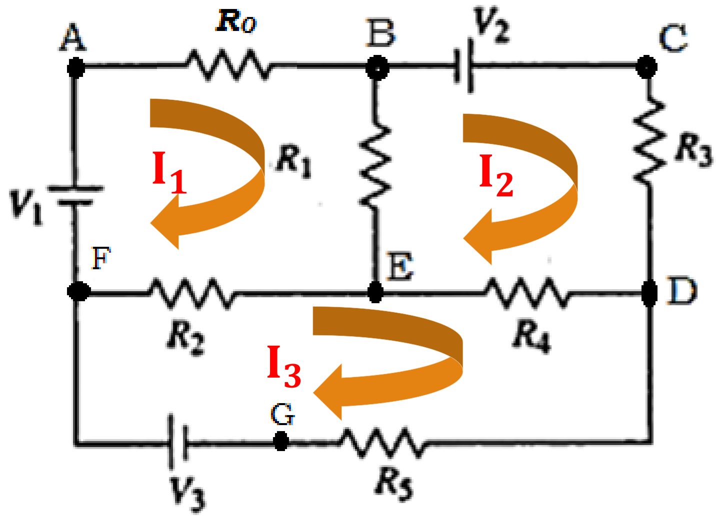
Branch Current Method



- Number of unknowns is equal to the number of branches.
- Difficult for complex circuits.

Mesh Current Method

- Number of unknowns is equal to the number of Meshes(Fundamental Loops)
- I_1, I_2 and I_3 are Mesh Currents



Mesh Analysis

Procedure to apply Mesh Analysis:

Step-1: As far as possible try to shift current sources into voltage sources without affecting the load elements.

Step-2: Identify the number of meshes (Fundamental Loops)

Step-3: Name the loops as Loop-1, Loop-2 and so on.

Step-4: Assign Mesh currents as I1, I2 etc.(or x, y etc.) to all the meshes and choose all the mesh currents direction are either clockwise or anticlockwise.

Step-5: Apply KVL to each mesh.

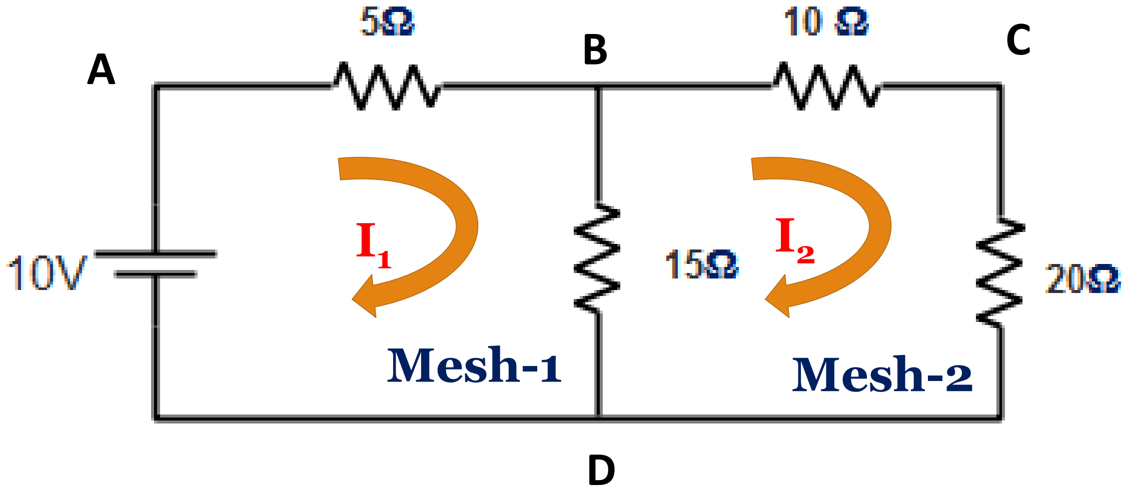
NOTE: Number of KVL equations is equal to number of Meshes (Mesh currents / Unknowns).

Step-6: Solve KVL equations using variable elimination method or by applying Cramer's Rule to find Mesh current.

Step-7: Find branch currents and /or branch voltages and/or powers from the mesh currents using Ohm's Law.

Examples:

1. Find the branch currents and voltages for the electrical circuit shown in figure.



Apply KVL to Mesh – 1

$$-10 + 5I_1 + 15(I_1 - I_2) = 0$$
$$20I_1 - 15I_2 = 10 --- (1)$$

Apply KVL to Mesh – 2

$$15(I_2 - I_1) + 10I_2 + 20I_2 = 0$$
$$-15I_1 + 45I_2 = 0 --- (2)$$

Solve equations (1) and (2)

We get

$$I_1 = 0.66A$$
$$I_2 = 0.22A$$

Branch Current

$$I_{DA} = I_{5V} = I_1 = 0.66A$$
$$I_{AB} = I_{5\Omega} = I_1 = 0.66A$$
$$I_{BC} = I_{10\Omega} = I_2 = 0.22A$$
$$I_{CD} = I_{20\Omega} = I_2 = 0.22A$$
$$I_{BD} = I_{15\Omega} = I_1 - I_2 = 0.44\ A$$

Branch Voltages

$$V_{DA} = V_{10V} = -10V$$
$$V_{AD} = V_{10V} = +10V$$
$$V_{AB} = V_{5\Omega} = 5xI_1 = 3.33V$$
$$V_{BC} = V_{10\Omega} = 10xI_2 = 2.22V$$
$$V_{CD} = V_{20\Omega} = 20xI_2 = 4.44V$$
$$V_{BD} = V_{15\Omega} = 15(I_1 - I_2) = 6.66V$$

Power

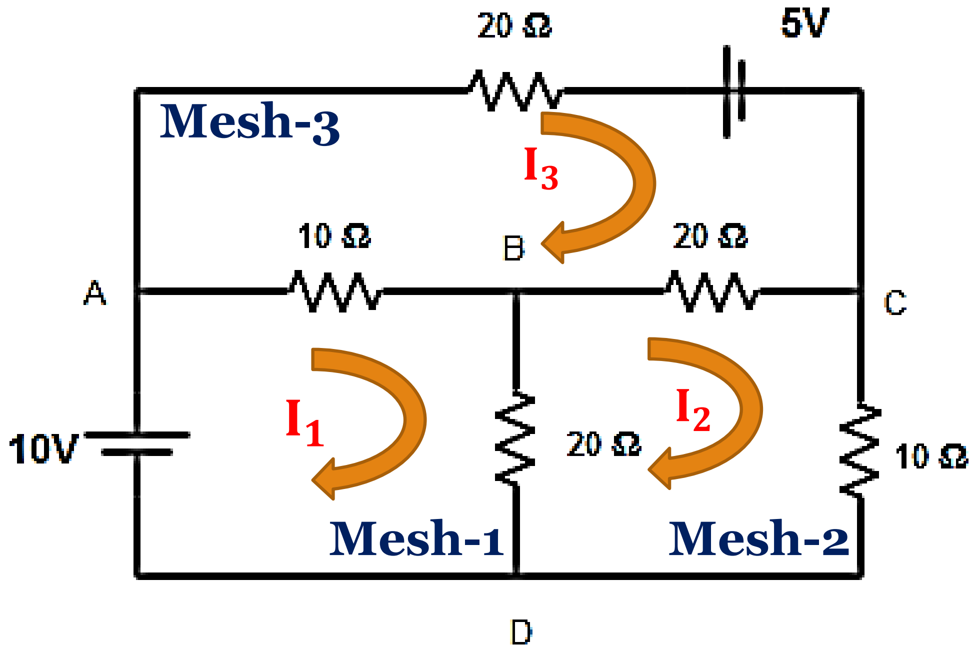
$$P_{10V} = 10xI_1 = 6.66W$$
$$P_{5\Omega} = 3.33x0.66 = 2.19W$$
$$P_{15\Omega} = 6.66x0.44 = 2.90W$$
$$P_{10\Omega} = 6.66x0.22 = 0.48W$$
$$P_{20\Omega} = 4.44x0.22 = 0.97W$$

Law of conservation of energy

$$P_{5V} = P_{5\Omega} + P_{15\Omega} + P_{10\Omega} + P_{20\Omega}$$

Examples:

2. Find the Mesh currents for the electrical circuit shown in figure.



Apply KVL to mesh – 1

$$-10 + 10(I_1 - I_3) + 20\,(I_1 - I_2) = 0$$

$$30I_1 - 20I_2 - 10I_3 = 10 --- (1)$$

Apply KVL to mesh – 2

$$20(I_2 - I_1) + 20(I_2 - I_3) + 10I_2 = 0$$

$$-20I_1 + 50I_2 - 20I_3 = 0 --- (2)$$

Apply KVL to mesh – 3

$$10(I_3 - I_1) + 20I_3 + 5 + 20(I_3 - I_2) = 0$$

$$-10I_1 - 20I_2 + 50I_3 = -5 --- (3)$$

Solve equations (1). (2) and (3) we get.

$$\mathbf{I_1 = 0.55A}$$

$$\mathbf{I_2 = 0.26A}$$

$$\mathbf{I_3 = 0.11A}$$

Cramer's Rule

$$\mathbf{a_1 I_1 + b_1 I_2 + c_1 I_3 = d_1}$$
$$\mathbf{a_2 I_1 + b_2 I_2 + c_2 I_3 = d_2}$$
$$\mathbf{a_3 I_1 + b_3 I_2 + c_3 I_3 = d_3}$$

$$\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

- *where, a, b and c are co − efficients of unknowns*
- d_1, d_2 *and* d_3 *are constant terms*

$$\Delta_1 = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix} \quad \Delta_2 = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \quad \Delta_3 = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

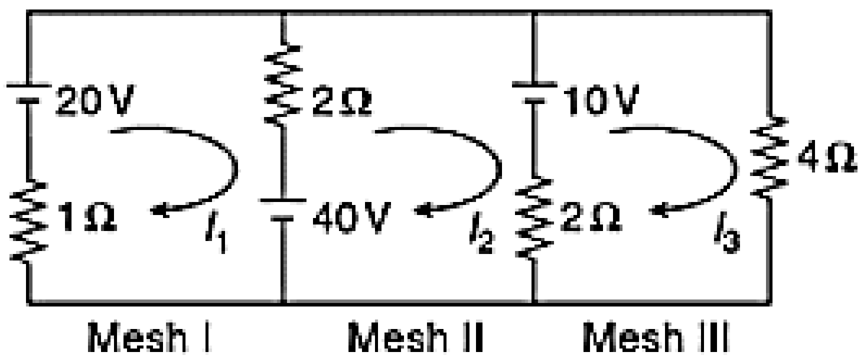
$$I_1 = \Delta_1 / \Delta$$

$$I_2 = \Delta_2 / \Delta$$

$$I_3 = \Delta_3 / \Delta$$

Examples:

3. Find the mesh current for the electrical circuit shown in figure suing mesh analysis.



Apply KVL to Mesh – I

$$I_1 - 20 + 2(I_1 - I_2) + 40 = 0$$

$$3I_1 - 2I_2 = -20 \text{ ---(1)}$$

Apply KVL to Mesh – II

$$-40 + 2(I_2 - I_1) + 10 + 2(I_2 - I_3) = 0$$

$$-2I_1 + 4I_2 - 2I_3 = 30 \text{ ---(2)}$$

Apply KVL to Mesh – III

$$2(I_3 - I_2) - 10 + 4I_3 = 0$$

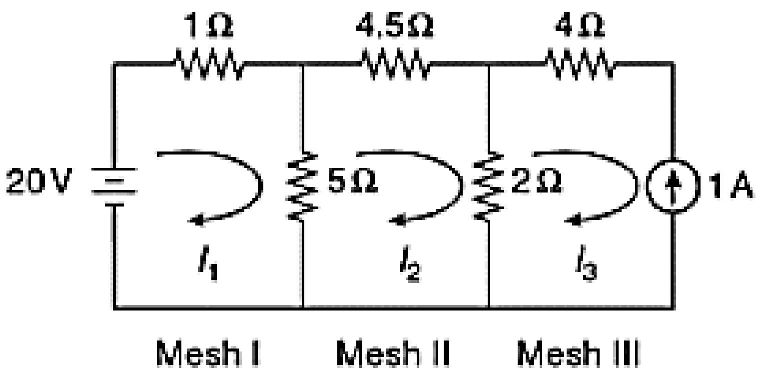
$$-2I_2 + 4I_3 = 10 \text{ ---(3)}$$

Solve equations (1), (2) and (3)

$$\mathbf{I_1 = 0A, I_2 = 10A \text{ and } I_3 = 5A}$$

Examples:

4. Find the mesh currents I1, I2 and I3 for the electrical circuit shown in figure suing mesh analysis.



Apply KVL to Mesh – I

$$-20 + I_1 + 5(I_1 - I_2) = 0$$

$$6I_1 - 5I_2 = 20 ---(1)$$

Apply KVL to Mesh – II

$$5(I_2 - I_1) + 4.5I_2 + 2(I_2 - I_3) = 0$$

$$-5I_1 + 11.5I_2 - 2I_3 = 0 ---(2)$$

Mesh – III

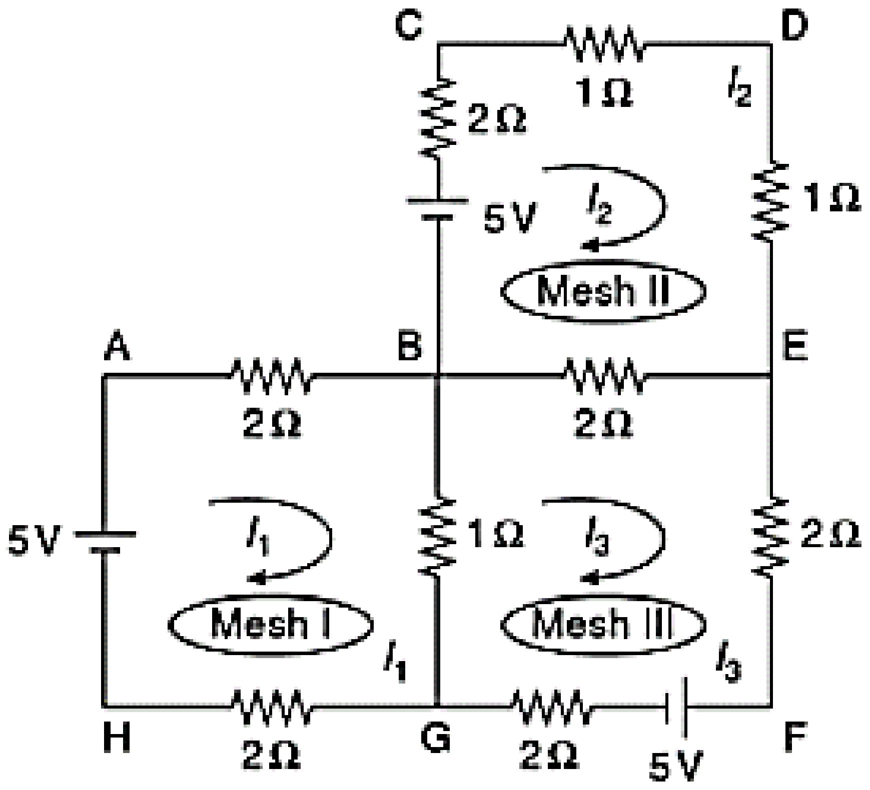
$$I_3 = -1A ---(3)$$

Solve equations (1), (2) and (3)

$$\mathbf{I_1 = 5A, I_2 = 2A \text{ and } I_3 = -1A}$$

Examples:

5. Find the mesh currents for the electrical circuit shown in figure suing mesh analysis.



Apply KVL to Mesh – I

$-5 + 2I_1 + 1(I_1 - I_3) + 2I_1 = 0$

$5I_1 - I_2 = 5 --- (1)$

Apply KVL to Mesh – II

$-5 + 2I_2 + I_2 + I_2 + 2(I_2 - I_3)=0$

$6I_2 - 2I_3 = 5 --- (2)$

Apply KVL toMesh – III

$1(I_3 - I_1) + 2(I_3 - I_2) + 2I_3 + 5 + 2I_3 = 0$

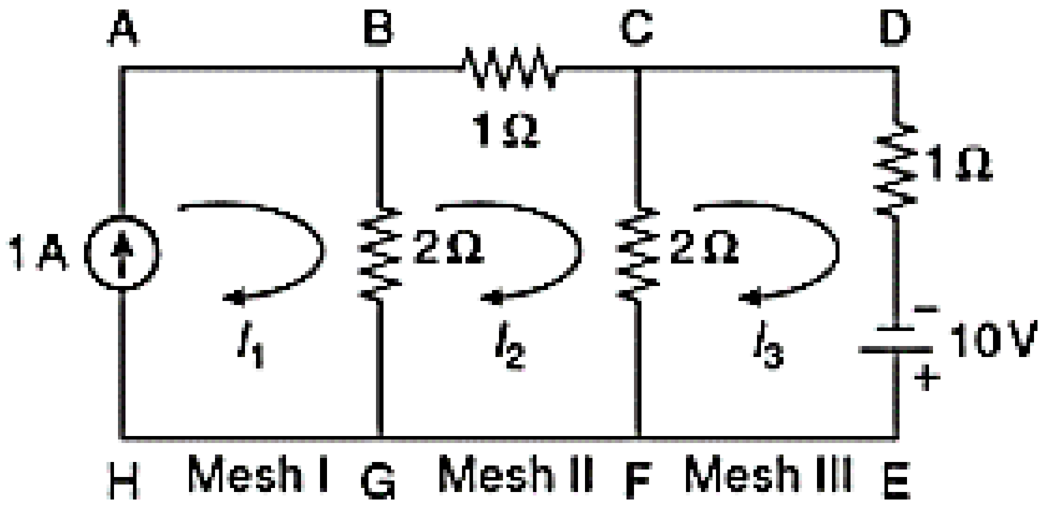
$-I_1 - 2I_2 + 7I_3 = -5 --- (3)$

Solve equations (1), (2)and (3)

$\mathbf{I_1 = 1.14A, I_2 = 0.71A \text{ and } I_3 = -0.34A}$

Examples:

6. Find the mesh currents I1, I2 and I3 for the electrical circuit shown in figure suing mesh analysis.



Mesh – I

$I_1 = 1A ---(1)$

Apply KVL to Mesh – II

$2(I_2 - I_1) + I_2 + 2(I_2 - I_3) = 0$

$-2I_1 + 5I_2 - 2I_3 = 0 ---(2)$

Apply KVL toMesh – III

$2(I_3 - I_2) + I_3 - 10 = 0$

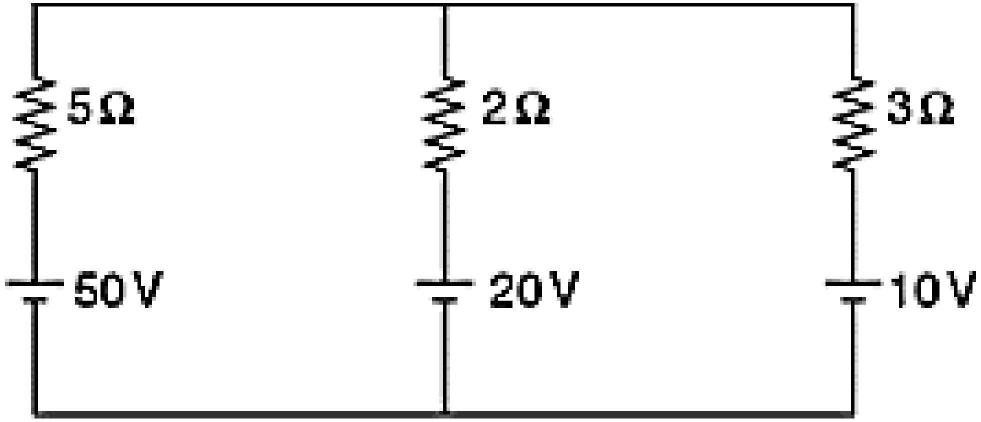
$-2I_2 + 3I_3 = 10 ---(3)$

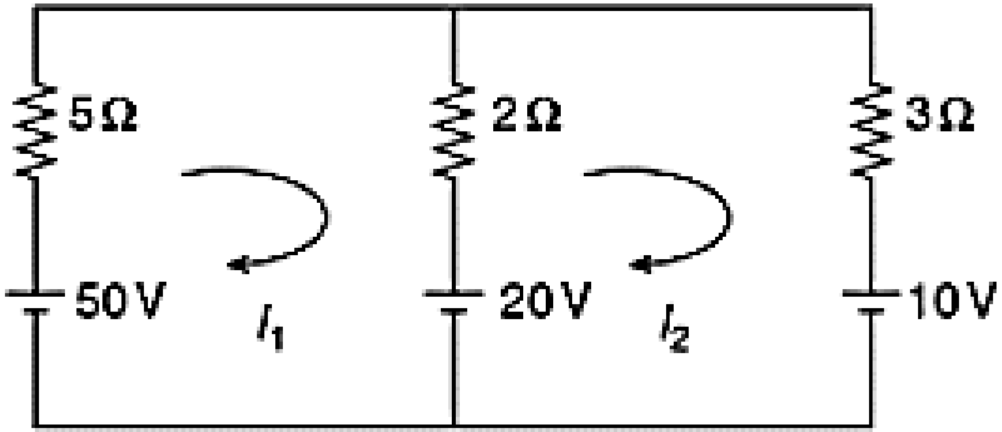
Solve equations (1), (2)and (3)

$\mathbf{I_1 = 1\ A, I_2 = 2.36\ A\ and\ I_3 = 4.9\ A}$

Examples:

7. Find the mesh currents for the electrical circuit shown in figure using mesh analysis.





Apply KVL to Mesh – I

$$5I_1 + 2(I_1 - I_2) + 20 - 50 = 0$$
$$7I_1 - 2I_2 = 30 \;---(1)$$

Apply KVL to Mesh – II

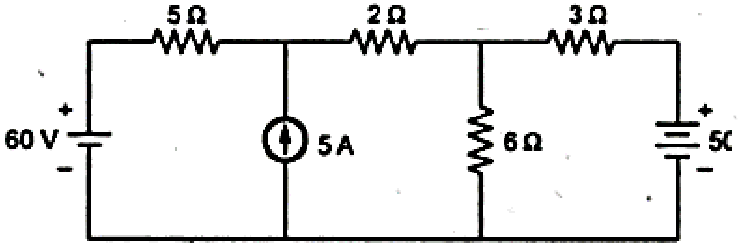
$$2(I_2 - I_1) + 3I_2 + 10 - 20 = 0$$
$$-2I_1 + 5I_2 = 10 \;---(2)$$

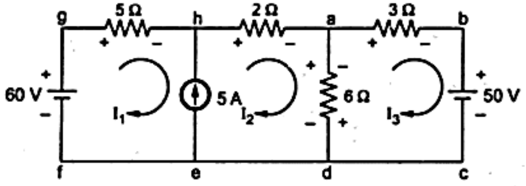
Solve equations (1) and(2)

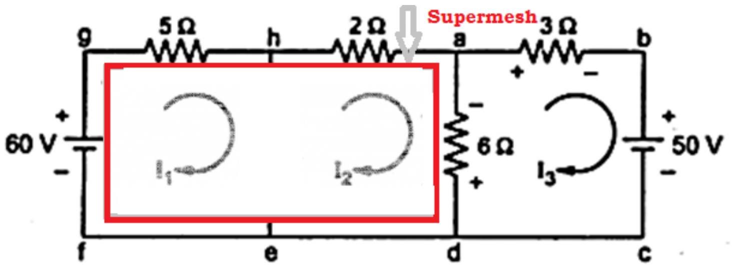
$\mathbf{I_1 = 5.48A, I_2 = 4.19\ A}$

Examples:

8. Find the mesh currents for the electrical circuit shown in figure suing mesh analysis.







- 5A Current source is common to meshes 1 and 2 Consider current source separately
- i.e., $I_2 - I_1 = 5 ---- (1)$
- combine the meshes 1 and 2 without considering common current source to form a single mesh, Called Supermesh.

Apply KVL to supermesh

$$5I_1 + 2I_2 + 6(I_2 - I_3) - 60 = 0$$

$$5I_1 + 8I_2 - 6I_3 = 60 --- (2)$$

Apply KVL toMesh – III

$$6(I_3 - I_2) + 3I_3 + 50 = 0$$

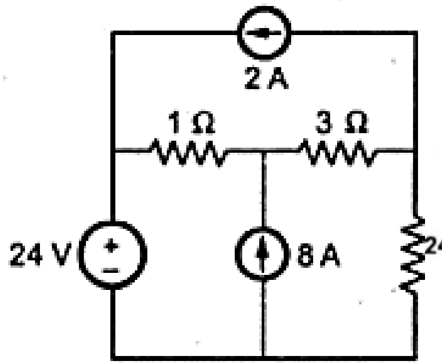
$$-6I_2 + 9I_3 = -50 --- (3)$$

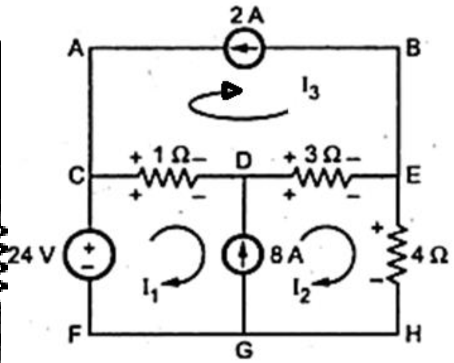
Solve equations (1), (2)and (3)

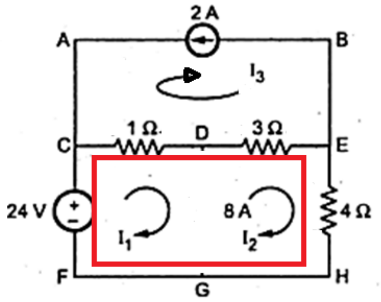
$I_1 = 0.74$ A, $I_2 = 5.74$ A and $I_3 = -1.72$ A

Examples:

9. Find the mesh currents for the electrical circuit shown in figure suing mesh analysis.







- 8 A Current source is common to meshes 1 and 2 Consider current source separately
- i.e., $I_2 - I_1 = 8 ----(1)$
- combine the meshes 1 and 2 without considering common current source to form a single source, Called Supermesh.

Apply KVL to supermesh

$$(I_1 - I_3) + 3(I_2 - I_3) + 4I_2 - 24 = 0$$

$$I_1 + 7I_2 - 4I_3 = 24 ---(2)$$

Apply KVL toMesh – III

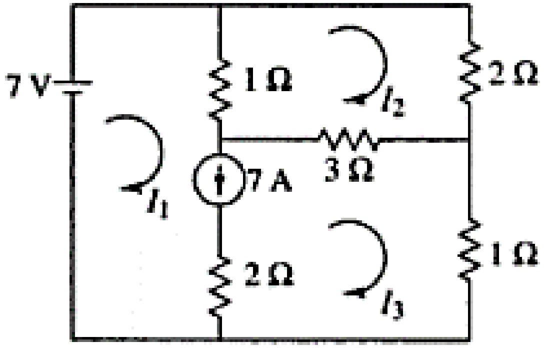
$$I_3 = -2\ A ---(3)$$

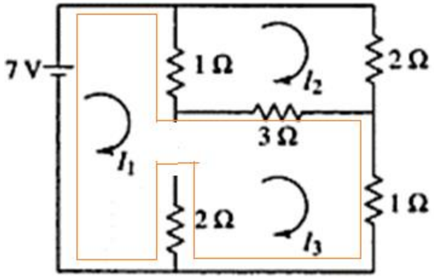
Solve equations (1), (2)and (3)

$\mathbf{I_1 = -5\ A, I_2 = 3\ A\ and\ I_3 = -2\ A}$

Examples:

10. Find the mesh currents for the electrical circuit shown in figure using mesh analysis.





- 7 A Current source is common to meshes 1 and 3
  Consider current source separately
- i.e., $I_3 - I_1 = 7 ----(1)$
- combine the meshes 1 and 3 without considering common current source to form a single source, Called Supermesh.

Apply KVL to supermesh

$$1(I_1 - I_2) + 3(I_3 - I_2) + I_3 + 2(I_3 - I_1) + 2(I_1 - I_3) - 7 = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 ---(2)$$

Apply KVL toMesh – III

$$(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0 ---(3)$$

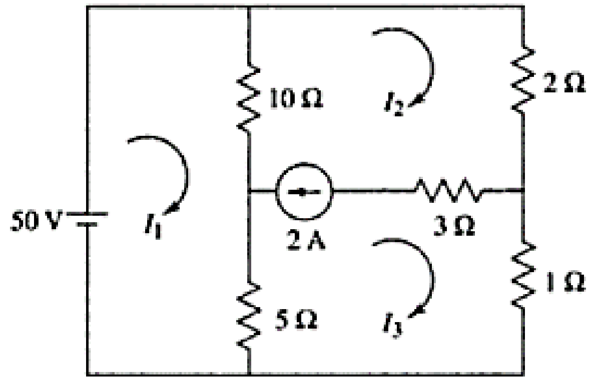
$$-I_1 + 6I_2 - 3I_3 = 0 ---(2)$$

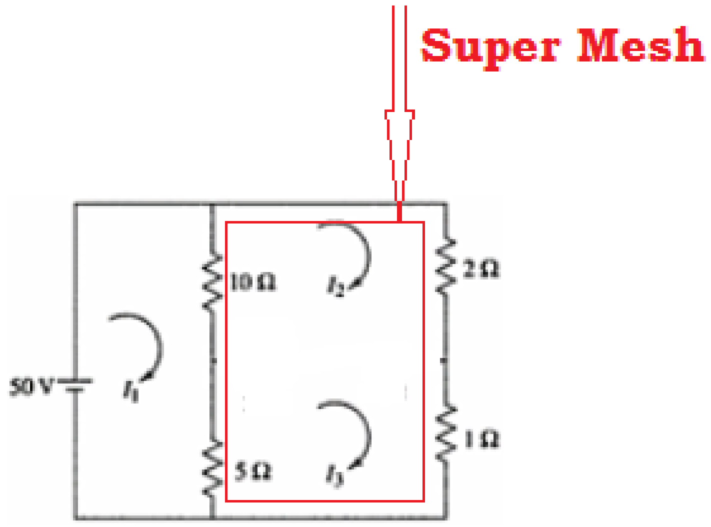
Solve equations (1), (2)and (3)

$\mathbf{I_1 = 9\ A, I_2 = 2.5\ A\ and\ I_3 = 2\ A}$

Examples:

11. Find the mesh currents for the electrical circuit shown in figure suing mesh analysis.





$$I_2 - I_3 = 2 ---(1)$$

KVL to supermesh

$$2I_2 + 1_3 + 5(I_3 - I_1) + 10(I_2 - I_1) = 0$$

$$-15I_1 + 12I_2 + 6I_3 = 0 ---(2)$$

KVL to mesh 1

$$10(I_1 - I_2) + 5(I_1 - I_3) - 50 = 0$$

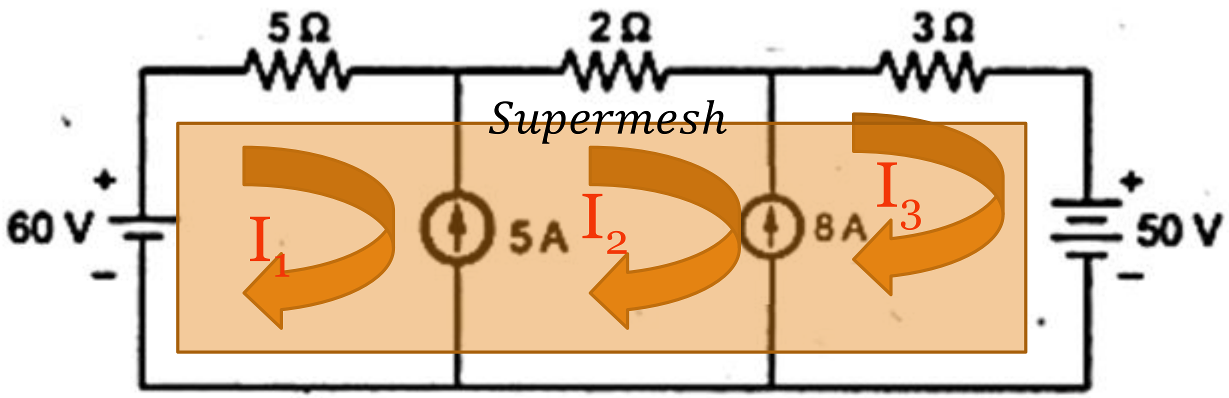
$$15I_1 - 10I_2 - 5I_3 = 50 ---(3)$$

Solve equations (1), (2) and (3)

I_1=20A, I_2=17.33A and I_3=15.33A

Examples:

12. Find the mesh currents for the electrical circuit shown in figure suing mesh analysis.



$$I_2 - I_1 = 5 --- (1)$$
$$I_3 - I_2 = 8 --- (2)$$

KVL to Supermesh

$$5I_1 + 2I_2 + 3I_3 + 50 - 60 = 0$$
$$5I_1 + 2I_2 + 3I_3 = 10 --- (3)$$

Solve the equations (1), (2) and (3)

I_1=-3.9A

I_2=1.1A

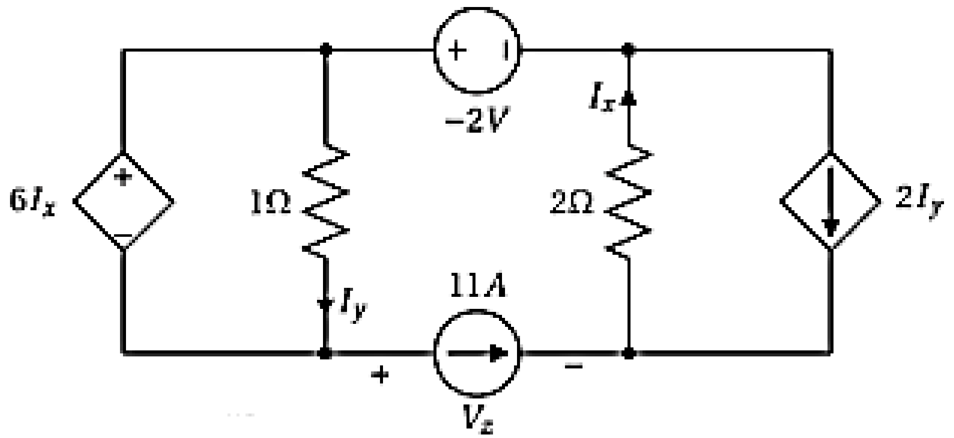
I_3=9.1A

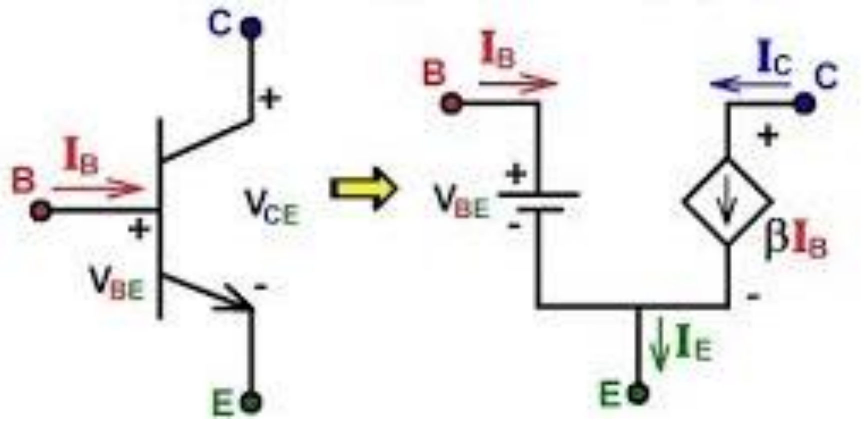
Dependent/Controlled Sources

Dependent Sources

Current and voltage of source depends on some other current and voltage.

***Example*:**

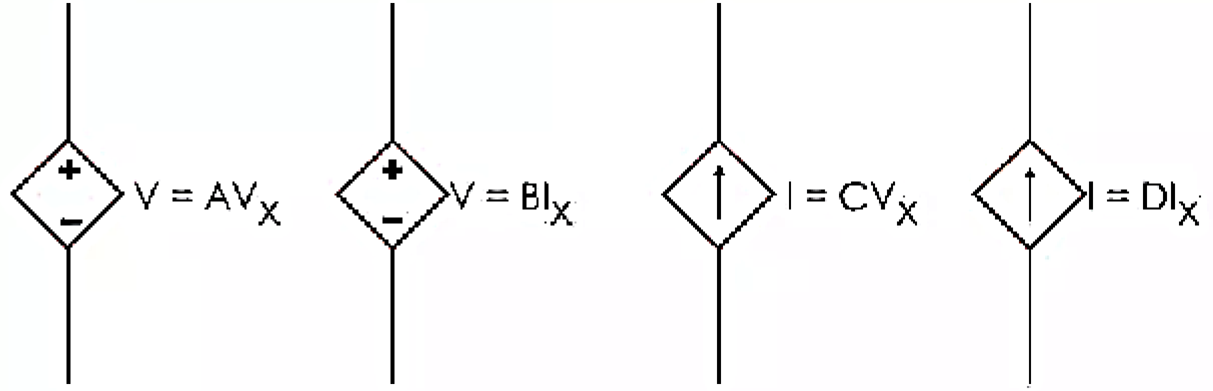




Applications: Analysis of amplifiers

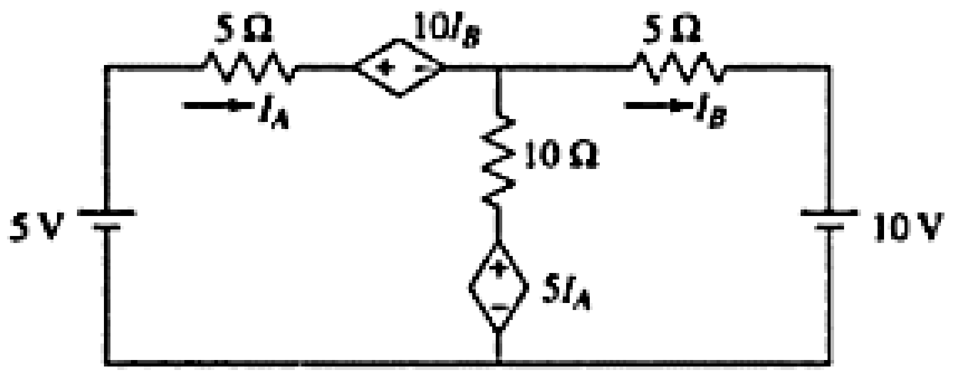
Types:

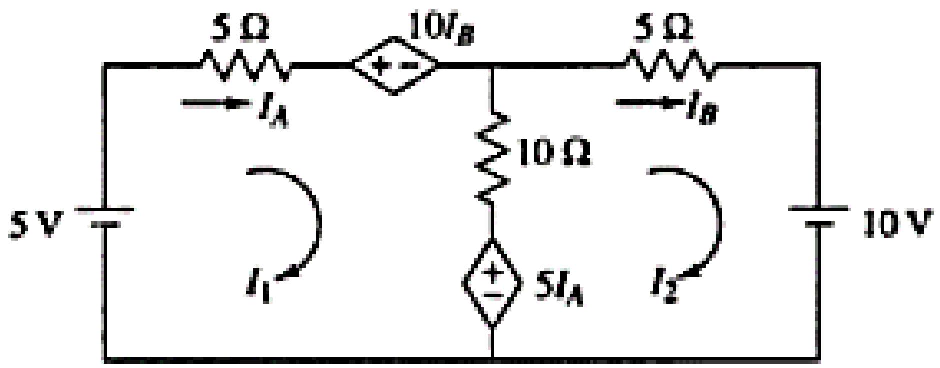
1. Voltage Dependent Voltage source
2. Current Dependent Voltage source
3. Voltage Dependent Current Source
4. Current dependent Current source



Examples:

1. Find the mesh currents for the electrical circuit shown in figure.





Obtain the control variables in terms of mesh currents.

$$I_A = I_1; I_B = I_2$$

KVL to Mesh-1

$$-5 + 5I_1 + 10I_B + 10(I_1 - I_2) + 5I_A = 0$$
$$-5 + 5I_1 + 10I_2 + 10(I_1 - I_2) + 5I_1 = 0$$
$$20I_1 = 5$$

Therefore $I_1 = 0.25A$

KVL to Mesh-2

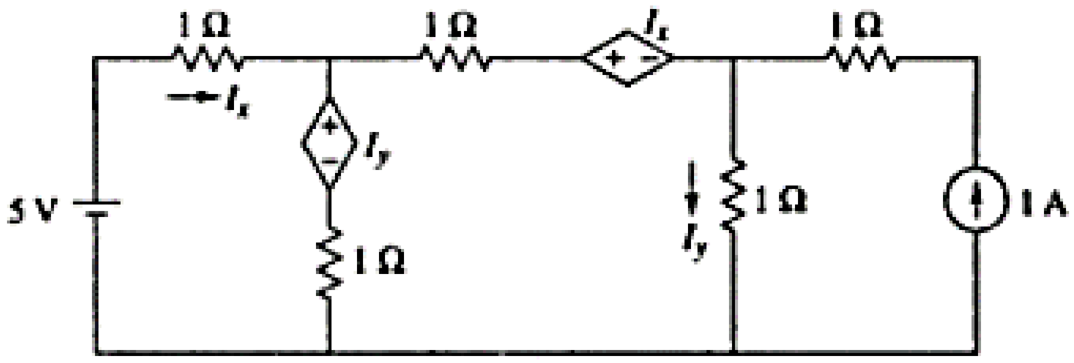
$$-5I_A + 10(I_2 - I_1) + 5I_2 + 10 = 0$$
$$-5I_1 + 10(I_2 - I_1) + 5I_2 + 10 = 0$$
$$-15I_1 + 15I_2 = -10$$

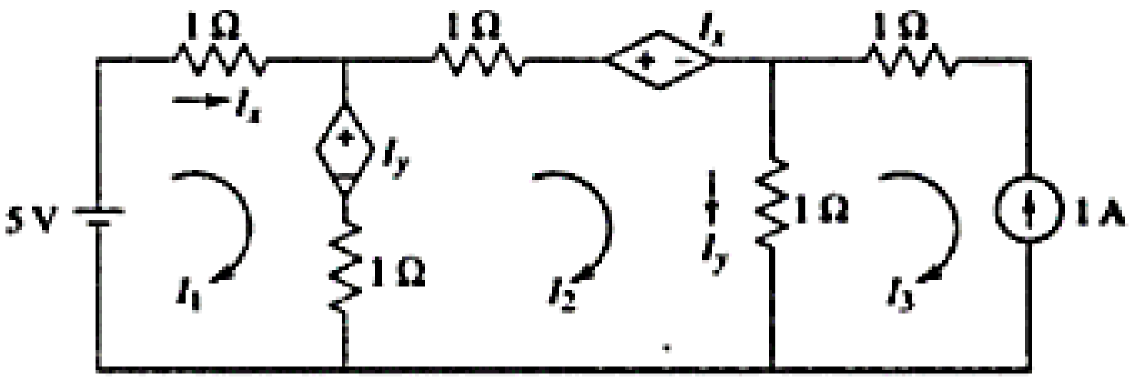
W.K.T., $I_1 = 0.25A$

Therefore $I_2 = -0.416A$

Examples:

2. Find the mesh currents for the electrical circuit shown in figure.





Obtain the control variables in terms of mesh currents.

$$I_x = I_1; I_y = I_2 - I_3$$

KVL to mesh-1

$$-5 + 1I_1 + I_y + 1(I_1 - I_2) = 0$$

$$-5 + I_1 + I_2 - I_3 + I_1 - I_2 = 0$$

$$2I_1 - I_3 = 5 --- (1)$$

KVL to mesh-2

$$1(I_2 - I_1) - I_y + 1I_2 + I_x + 1(I_2 - I_3) = 0$$

$$I_2 - I_1 - I_2 + I_3 + I_2 + I_1 + I_2 - I_3 = 0$$

$$2I_2 = 0$$

$$I_2 = 0A.$$

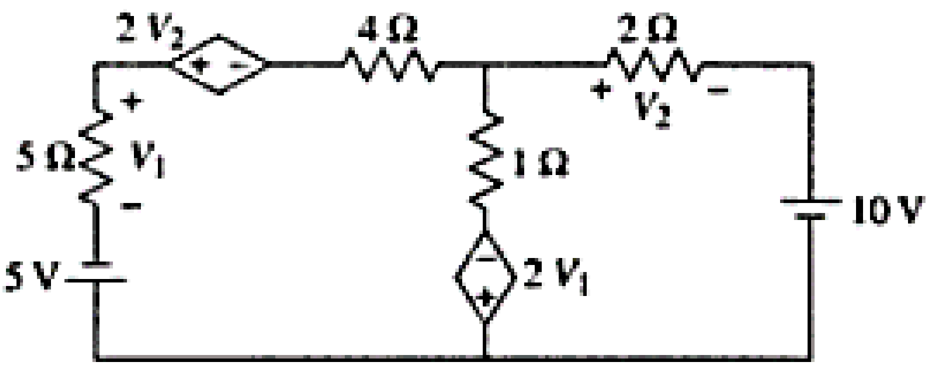
Mesh-3

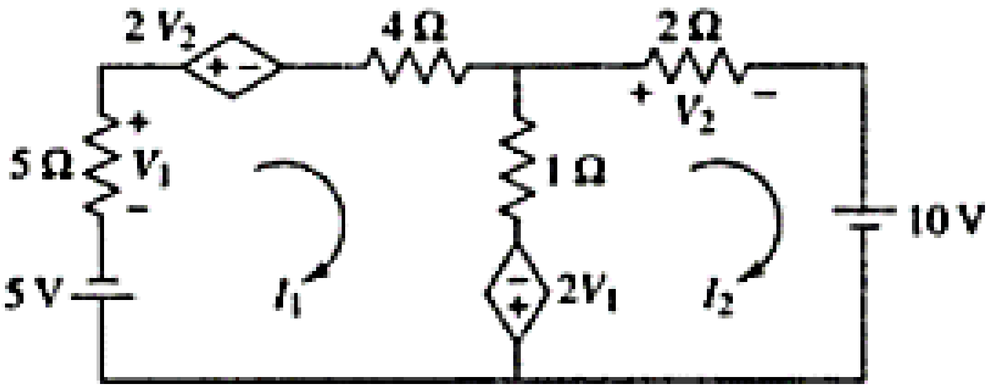
$$I_3 = -1A$$

$$I_1 = 2A \ , I_2 = 0A \text{ and } I_3 = -1A$$

Examples:

3. Find the mesh currents for the electrical circuit shown in figure.





Obtain the control variables in terms of mesh currents.

$$V_1 = -5I_1; V_2 = 2I_2;$$

Apply KVL to mesh-1

$$+5 + 5I_1 + 2V_2 + 4I_1 + 1(I_1 - I_2) - 2V_1 = 0$$
$$5 + 5I_1 + 4I_2 + 4I_1 + I_1 - I_2 + 10I_1 = 0$$
$$20I_1 + 3I_2 = -5 --- (1)$$

Apply KVL to mesh-2

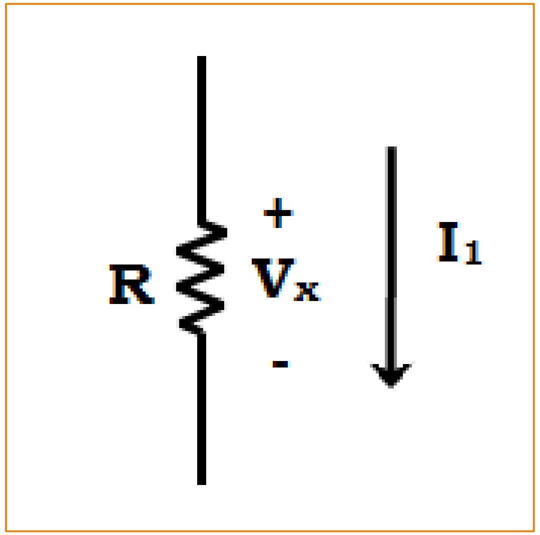
$$2V_1 + 1(I_2 - I_1) + 2I_2 + 10 = 0$$
$$-10I_1 + I_2 - I_1 + 2I_2 + 10 = 0$$
$$-11I_1 + 3I_2 = -10 --- (2)$$

$$I_1 = 0.161\ A \ \ and\ I_2 = -2.7\ A$$

Examples:

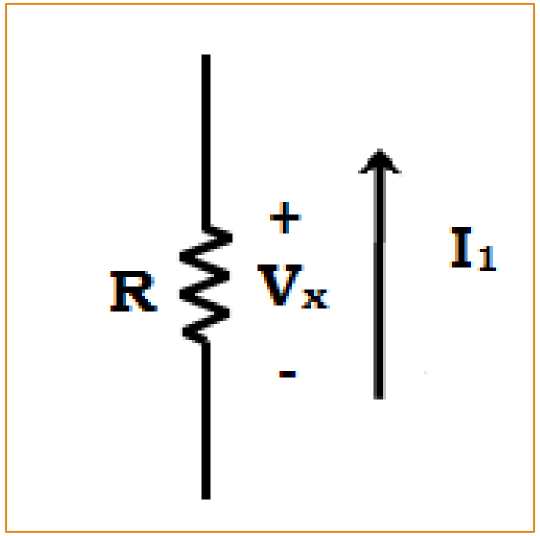
Obtaining the relationship between control variables and mesh currents

Case(i)



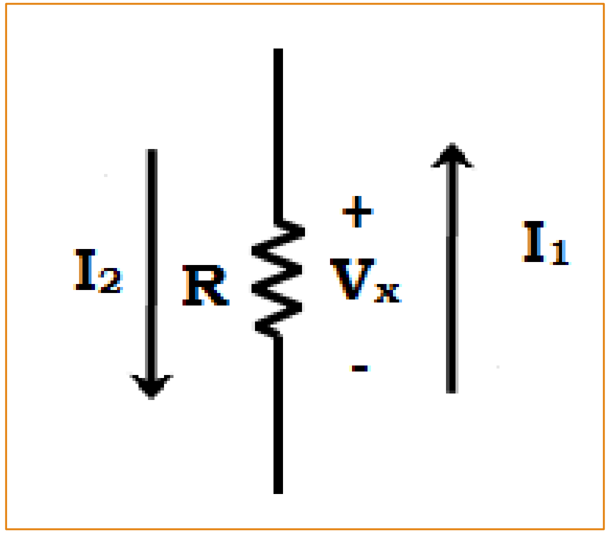
$V_x = RI_1$

Case(ii)



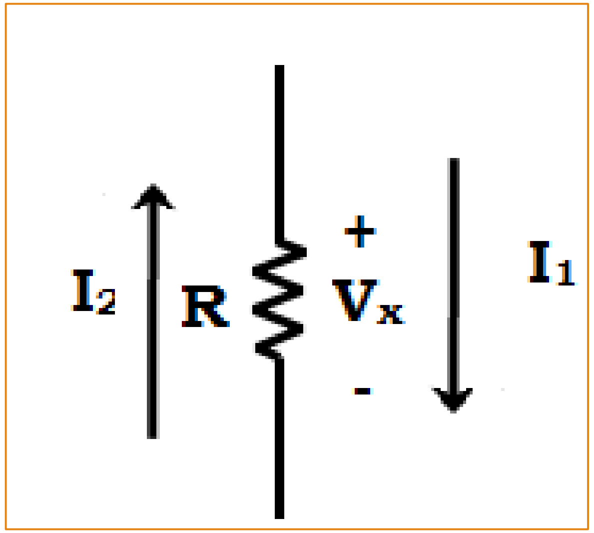
$V_x = R(-I_1)$

Case(iii)



$V_x = R(I_2 - I_1)$

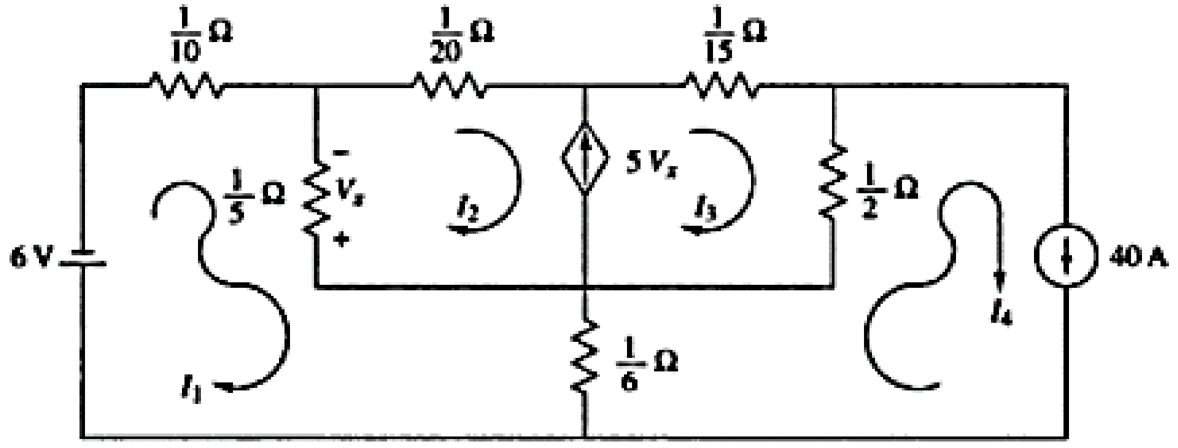
Case(iv)



$V_x = R(I_1 - I_2)$

Examples:

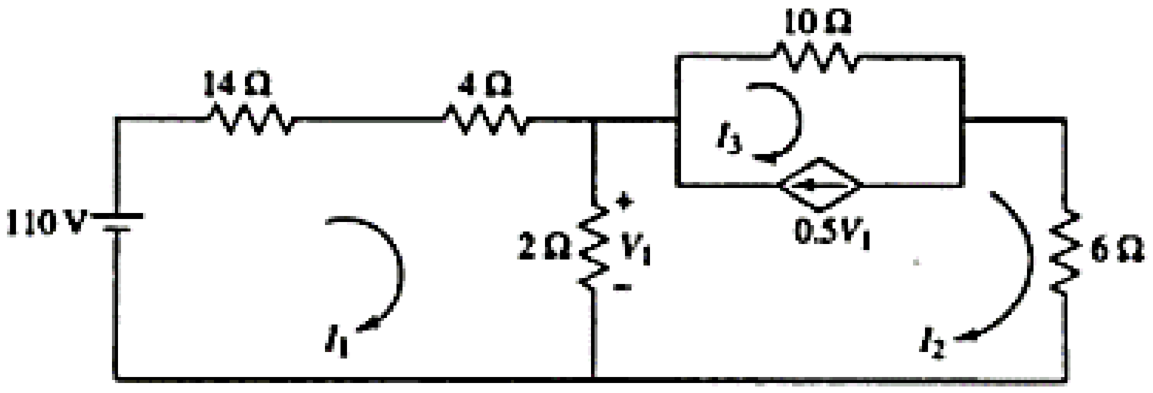
4. Find the mesh currents I1, I2, I3 and I4 for the electrical circuit shown in figure.



Obtain the control variables in terms of mesh currents.

Examples:

5. Find the mesh currents I1 and I2 for the electrical circuit shown in figure.

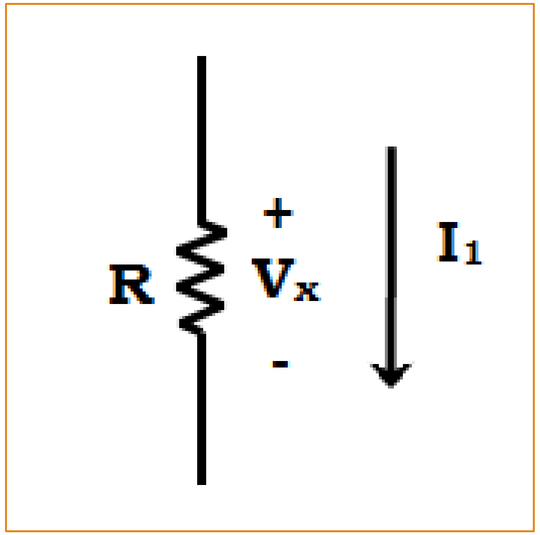


Obtain the control variables in terms of mesh currents.

Examples:

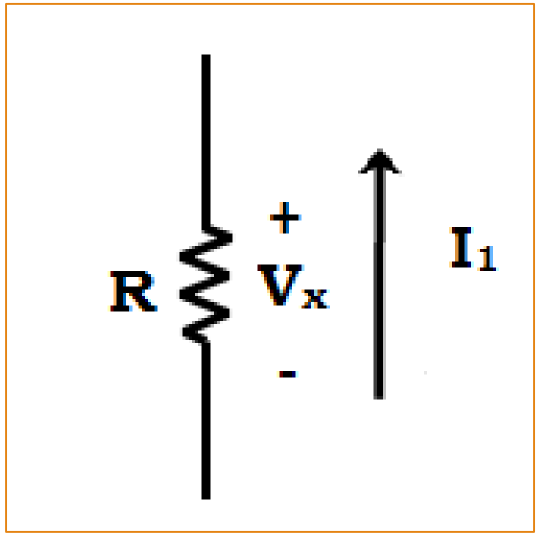
Obtaining the relationship between control variables and mesh currents

$Case(i)$



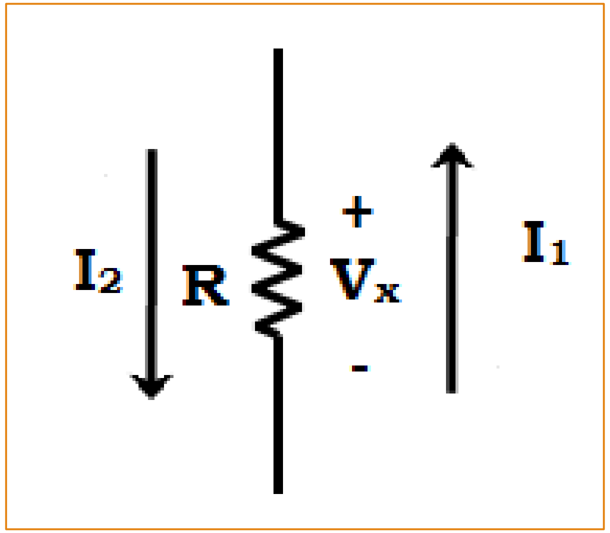
$V_x = RI_1$

$Case(ii)$



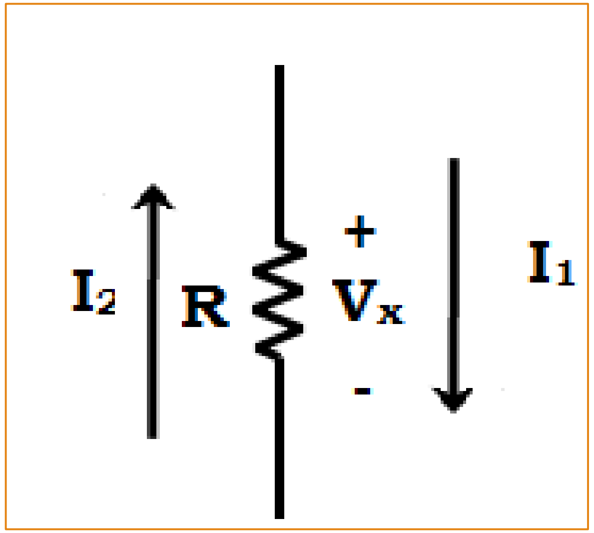
$V_x = R(-I_1)$

$Case(iii)$



$V_x = R(I_2 - I_1)$

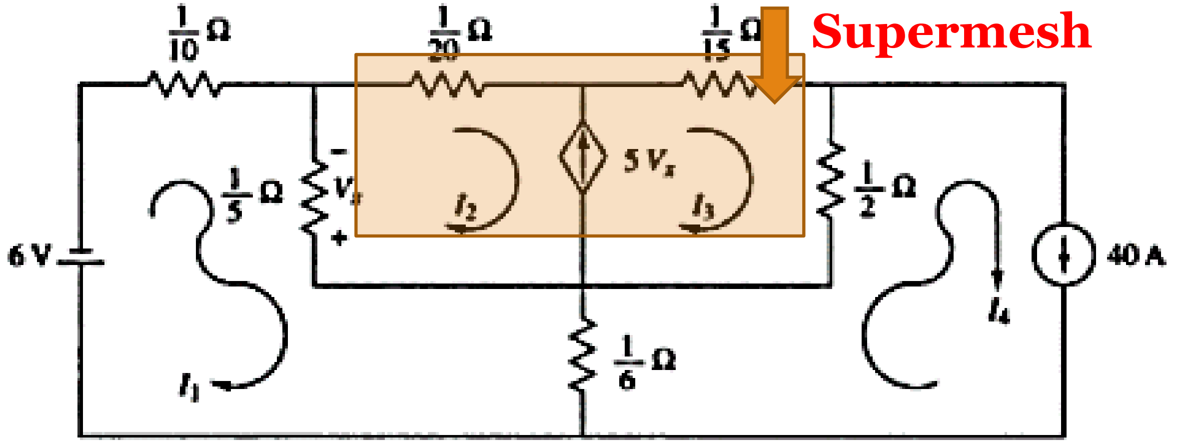
$Case(iv)$



$V_x = R(I_1 - I_2)$

Examples:

4. Find the mesh currents I1, I2, I3 and I4 for the electrical circuit shown in figure.



Obtain the control variables in terms of mesh currents.

$$V_x = \frac{1}{5}(I_2 - I_1) ---(1)$$

KVL at mesh-1

$$+6 + \frac{1}{10}I_1 + \frac{1}{5}(I_1 - I_2) + \frac{1}{6}(I_1 - I_4) = 0$$

$$0.46I_1 + 0.2I_2 - 0.166I_4 = -6 ---(2)$$

5Vx current source is common to mesh-2 and 3.

$$I_3 - I_2 = 5V_x$$

$$I_3 - I_2 = 5.\frac{1}{5}(I_2 - I_1)$$

$$I_1 - 2I_2 + I_3 = 0 ---(3)$$

KVL to supermesh

$$\frac{1}{20}I_2 + \frac{1}{15}I_3 + \frac{1}{2}(I_3 - I_4) + \frac{1}{5}(I_2 - I_1) = 0$$

$$-0.2I_1 + 0.25I_2 + 0.56I_3 - 0.4I_4 = 0 ----(4)$$

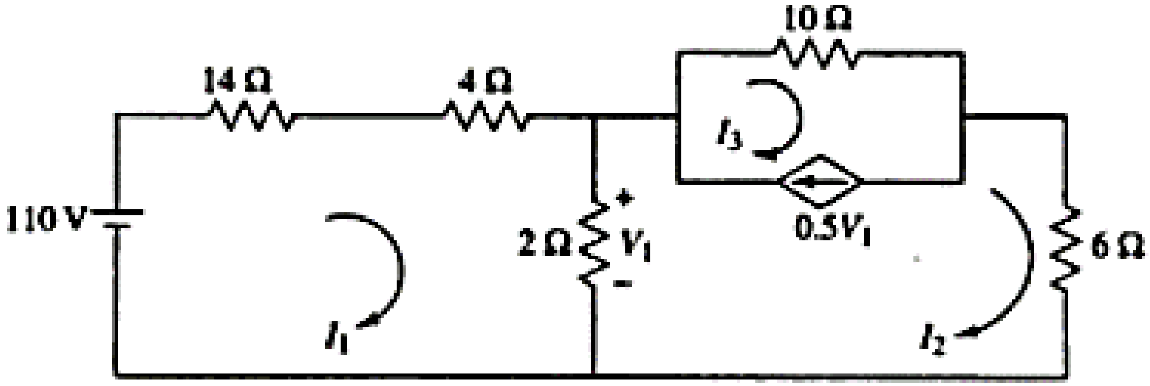
At mesh 4

$$I_4 = 40A$$

$$I_1 = 10A, I_2 = 20A, I_3 = 30A \text{ and } I_4 = 40A$$

Examples:

5. Find the mesh currents I1 and I2 for the electrical circuit shown in figure.



Obtain the control variables in terms of mesh currents.

$$V_1 = 2(I_1 - I_2) ---(1)$$

KVL to mesh 1

$$-110 + 14I_1 + 4I_1 + 2(I_1 - I_2) = 0$$
$$20I_1 - 2I_2 = 110 ---(2)$$

0.5V_1 current source is common to mesh-2 and 3,

$$0.5V_1 = I_3 - I_2$$
$$0.5\big(2\,(I_1 - I_2)\big) = I_3 - I_2$$
$$I_1 = I_3 --(3)$$

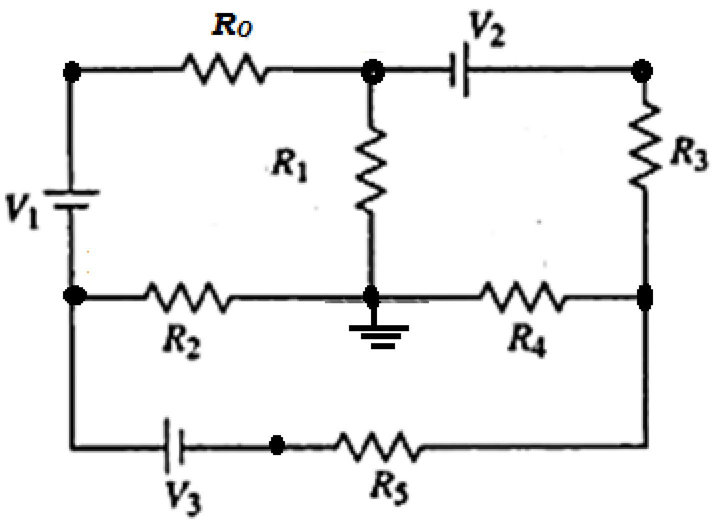
KVL to supermesh

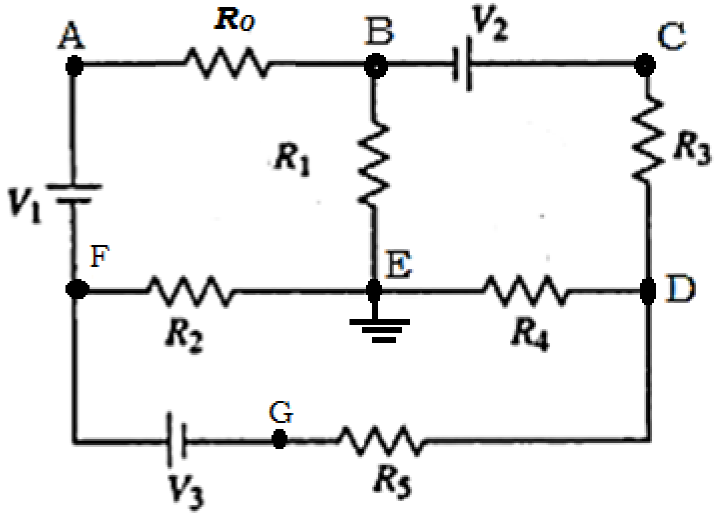
$$10I_3 + 6I_2 + 2(I_2 - I_1) = 0$$
$$-2I_1 + 8I_2 + 10I_3 = 0 --(4)$$

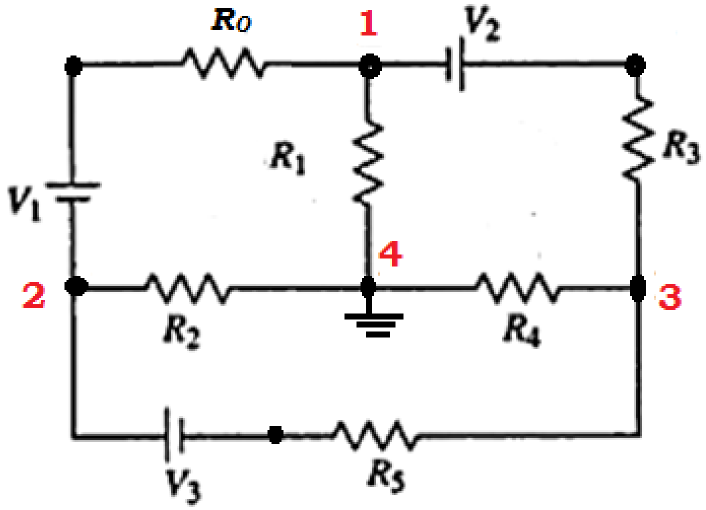
Solve equations (2, 3 and 4)
We get.

$$I_1 = 5A, I_2 = -5\ A\ and\ I_3 = 5A$$

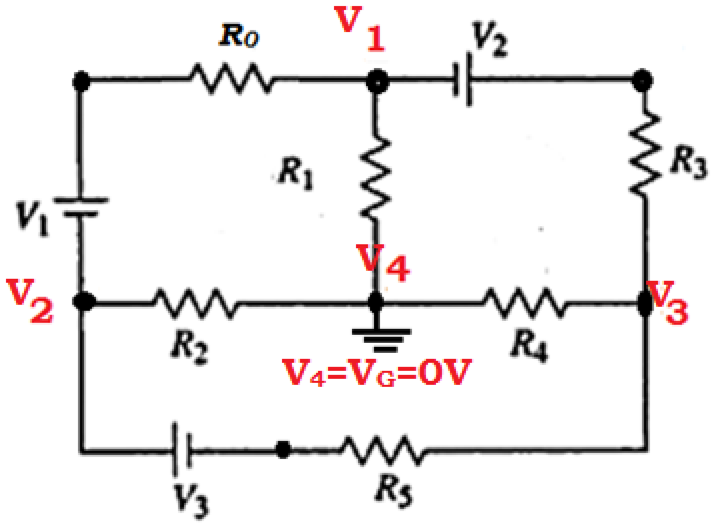
Node Analysis







- **Node:** A junction or a point where two or more elements are connected.
- Example:
- **Fundamental Node** : A node where Current division takes place
- Number of unknowns is equal to the number of nodes-1.
- V_1, V_2, V_3 ... are node voltages



Node Analysis

Procedure to apply Node Analysis:

Step-1: As for as possible try to convert voltage sources into current sources, without affecting the load elements.

Step-2: Identify the number of fundamental nodes.

Step-3: Name the nodes and assign node voltages as V1, V2, V3…

NOTE: Ground potential is always zero.

Step-4: Assign branch currents to each branch as I1, I2, I3 etc., and choose the directions randomly.

Step-5: Apply KCL at each node

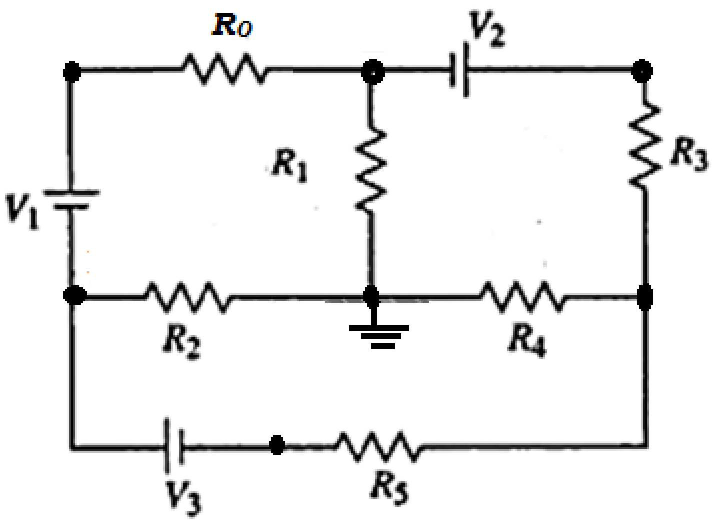
NOTE: Number of KCL equations is equal to the number of nodes-1/number of unknowns.

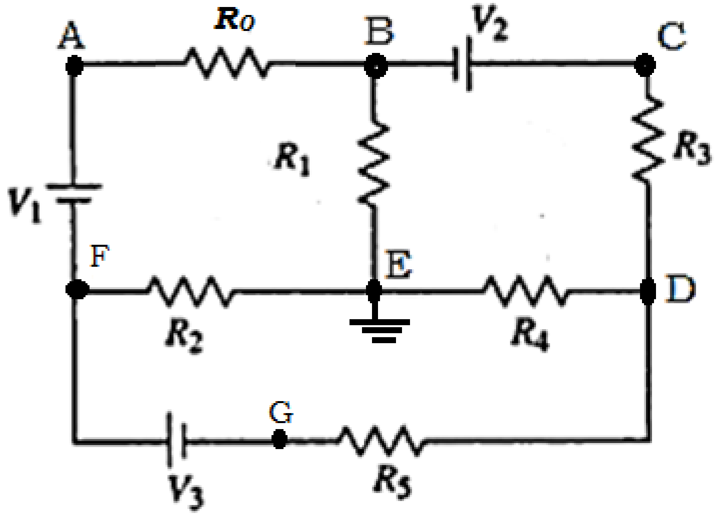
Step-6: Replace branch currents in terms of node voltages.

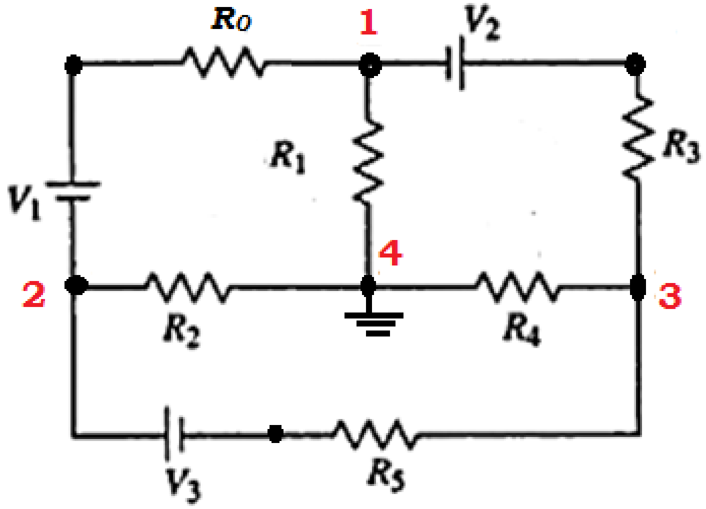
Step-7: Solve KCL equations using any mathematical technique to find node voltages.

Step-8: Find the branch currents/branch voltages/power from node voltages using ohm's law.

Node Analysis







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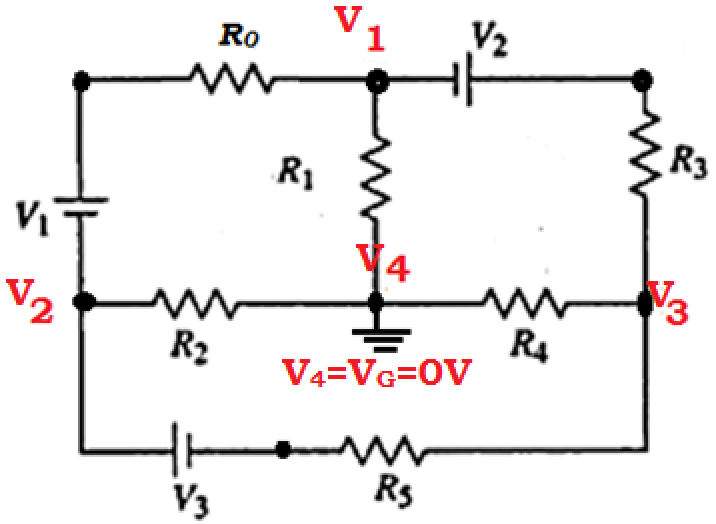

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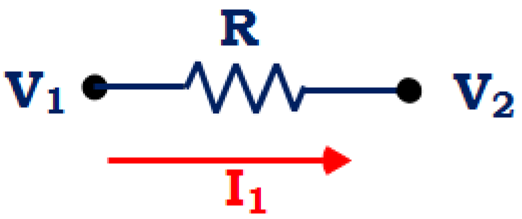
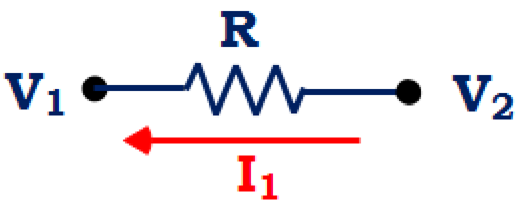
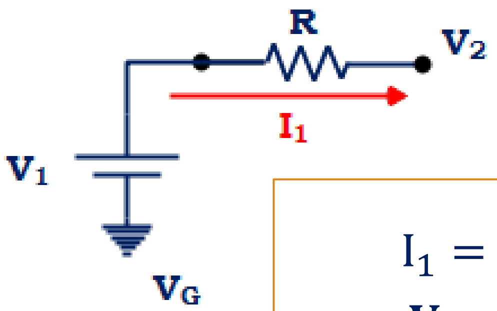
Node Analysis-Tips



$$I_1 = \frac{V_1 - V_2}{R}$$

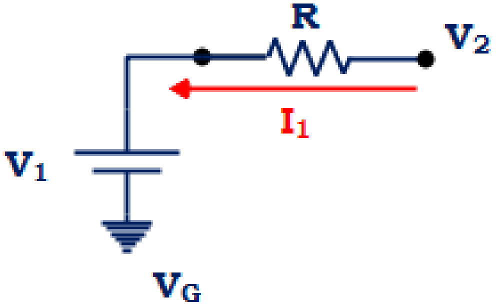


$$I_1 = \frac{V_2 - V_1}{R}$$



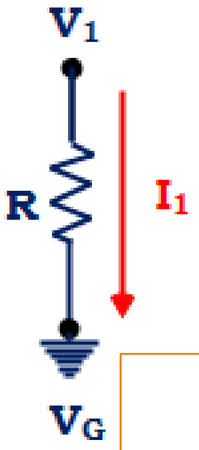
$$I_1 = \frac{V_G - (-V_1) - V_2}{R}$$

$$I_1 = \frac{V_1 - V_2}{R} \text{ (Since } V_G = 0V\text{)}$$



$$I_1 = \frac{V_2 - V_1 - V_G}{R}$$

$$I_1 = \frac{V_2 - V_1}{R} \text{ (Since } V_G = 0V\text{)}$$



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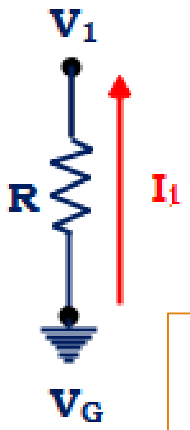
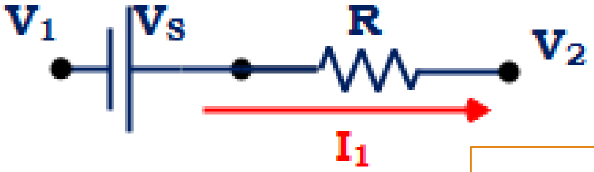


$$I_1 = \frac{V_G - V_1}{R}$$

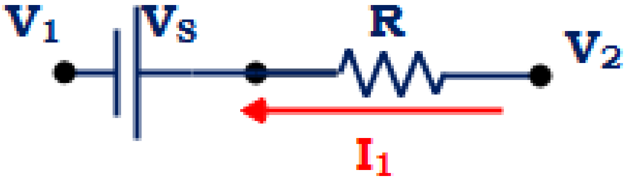
$$I_1 = \frac{-V_1}{R} \text{ (Since } V_G = 0V\text{)}$$

Node Analysis-Tips

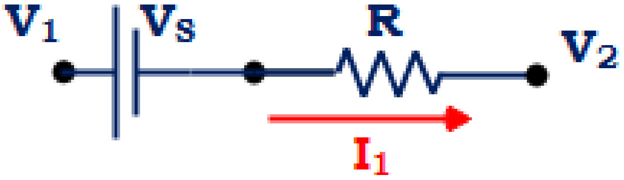


$$I_1 = \frac{V_1 - (-V_s) - V_2}{R}$$

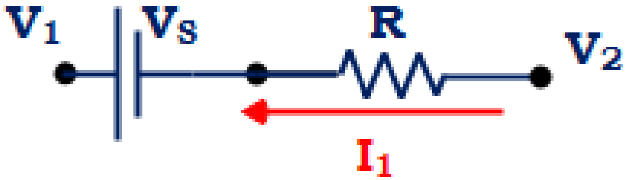
$$\mathbf{I_1 = \frac{V_1 + V_s - V_2}{R}}$$



$$\mathbf{I_1 = \frac{V_2 - V_s - V_1}{R}}$$



$$\mathbf{I_1 = \frac{V_1 - V_s - V_2}{R}}$$

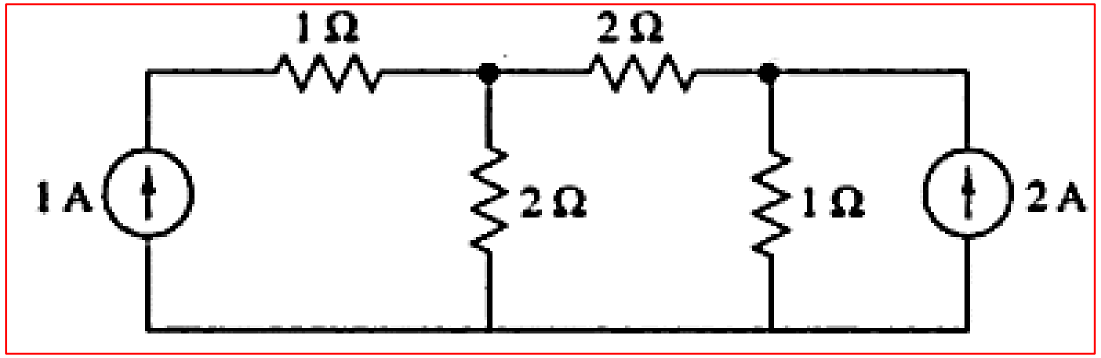


$$I_1 = \frac{V_2 - (-V_s) - V_1}{R}$$

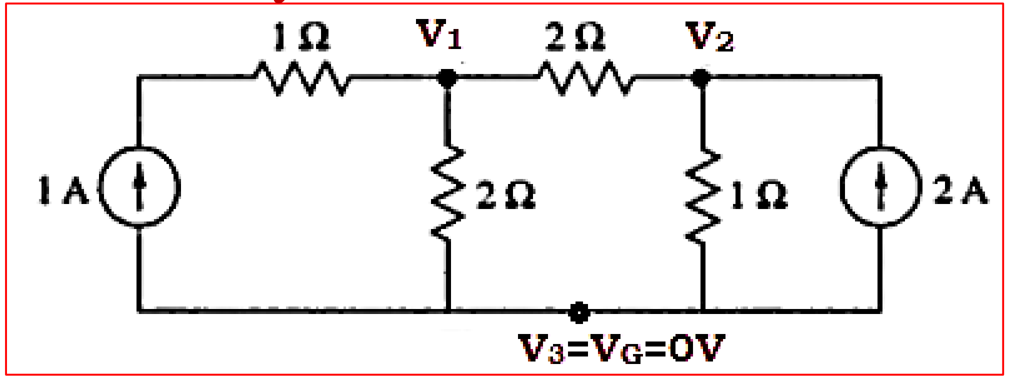
$$\mathbf{I_1 = \frac{V_2 + V_s - V_1}{R}}$$

Node Analysis-Examples

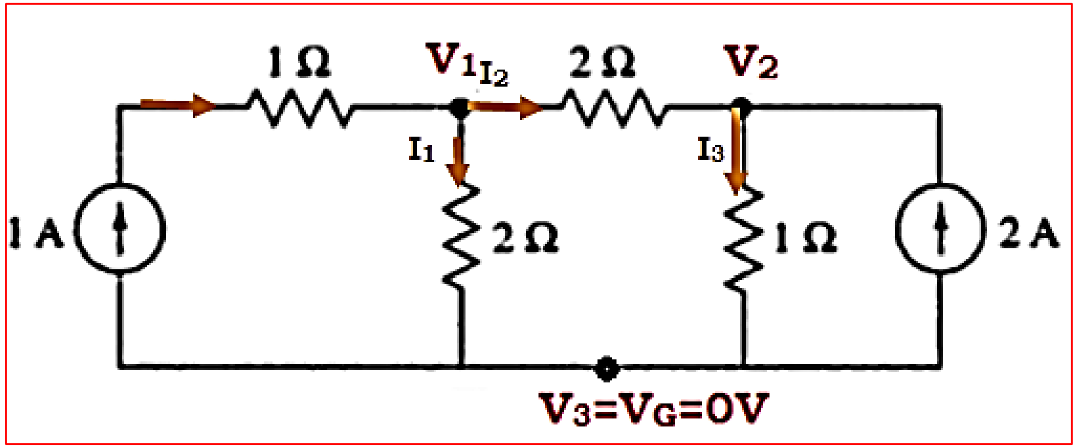
1. Find the branch currents and branch voltages for the electrical circuit shown in figure.



Identify the fundamental nodes



Assign branch currents



KCL at node 1

$$1 = I_1 + I_2$$

Express branch currents in terms of node voltages

$$1 = \frac{V_1 - V_G}{2} + \frac{V_1 - V_2}{2}$$

$$1 = V_1 - 0.5V_2 ---(1)$$

KCL at node 2

$$I_2 + 2 = I_3$$

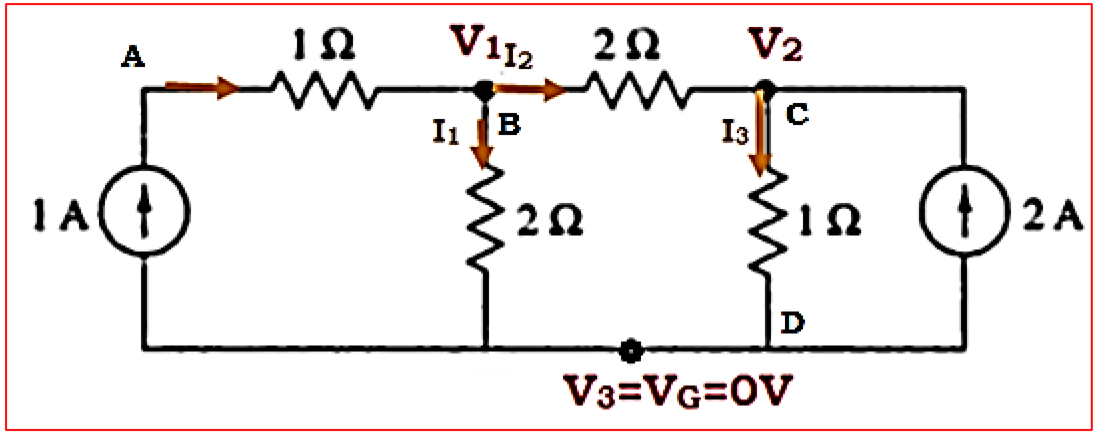
$$\frac{V_1 - V_2}{2} + 2 = \frac{V_2 - V_G}{1}$$

$$0.5V_1 - 1.5V_2 = -2 ---(2)$$

Answer:
V_1=2V
V_2=2V

Node Analysis-Examples

Branch currents



$$I_{AB} = 1A$$

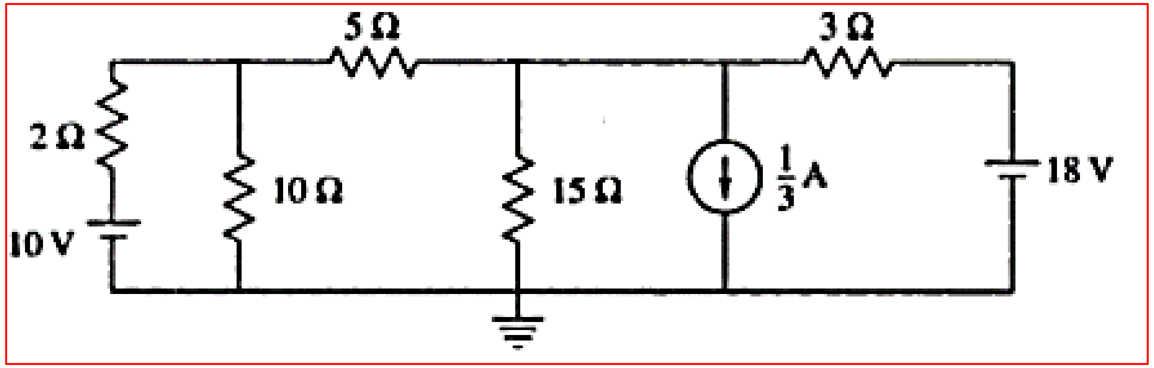
$$I_{BD} = I_1 = \frac{V_1}{2} = 1A$$

$$I_{BC} = I_2 = \frac{V_1 - V_2}{2} = 0A$$

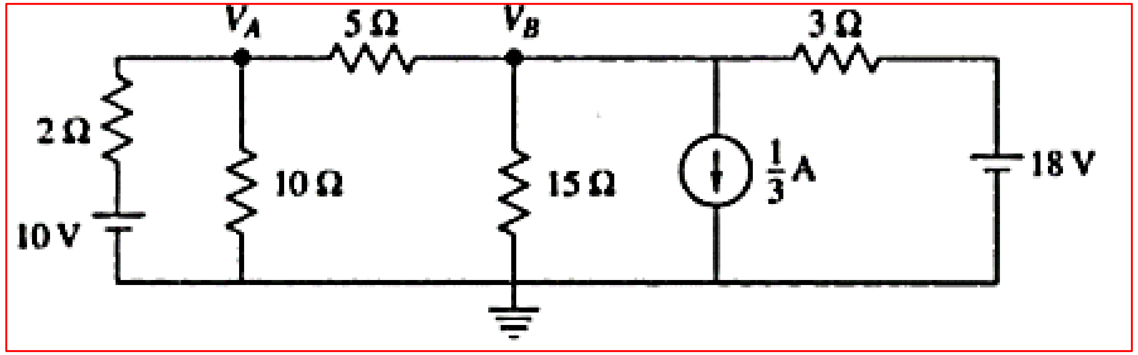
$$I_{CD} = I_3 = \frac{V_2}{1} = 2A.$$

Node Analysis-Examples

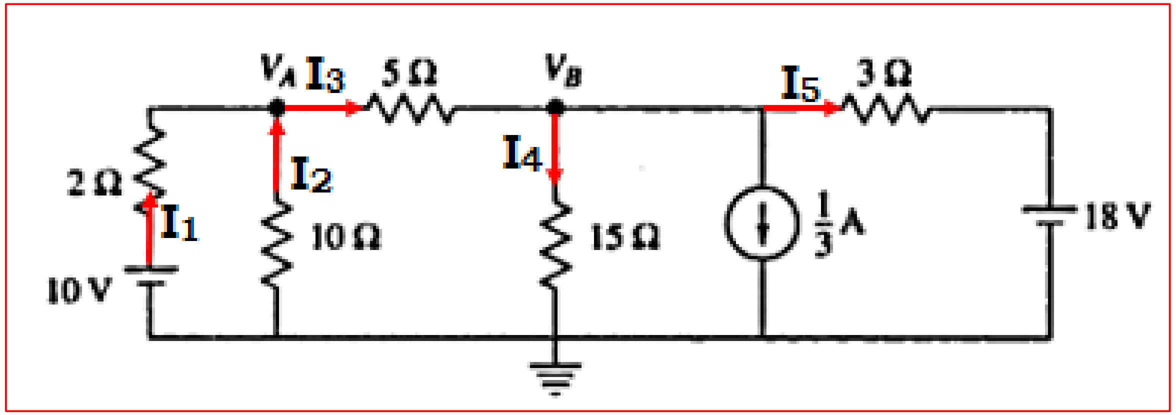
2. Find the Node voltages for the electrical circuit shown in figure.



Identify the fundamental nodes



Assign branch currents



Answer: $V_A = 9.39V$ **and** $V_B = 12.58V$

KCL at node A

$$I_1 + I_2 = I_3$$

Express branch currents in terms of node voltages

$$\frac{10 - V_A}{2} + \left(-\frac{V_A}{10}\right) = \frac{V_A - V_B}{5} \;---(1)$$

$$8V_A - 2V_B = 50 \;---(1)$$

KCL at node B

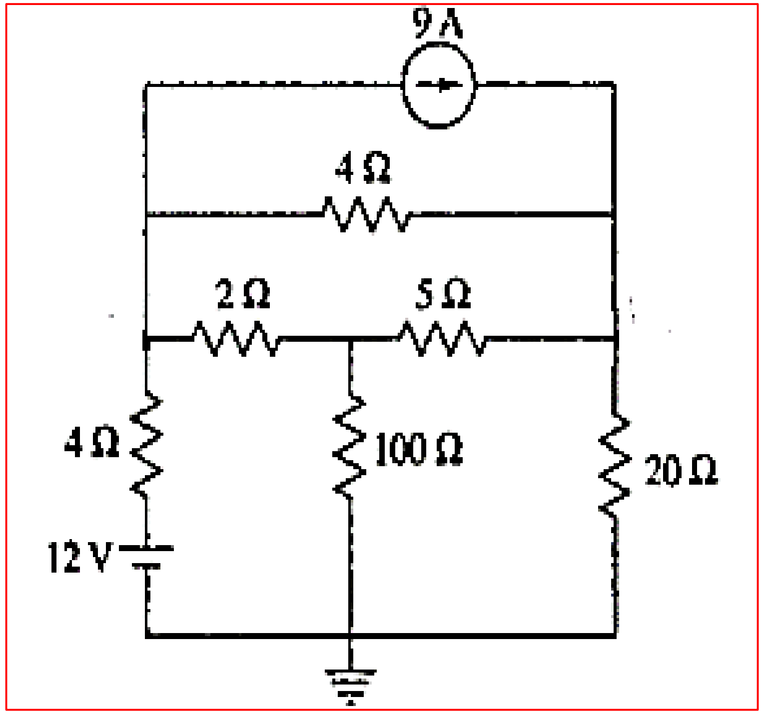
$$I_3 = I_4 + I_5 + \frac{1}{3}$$

$$\frac{V_A - V_B}{5} = \frac{V_B}{15} + \frac{V_B - 18}{3} \;---(2)$$

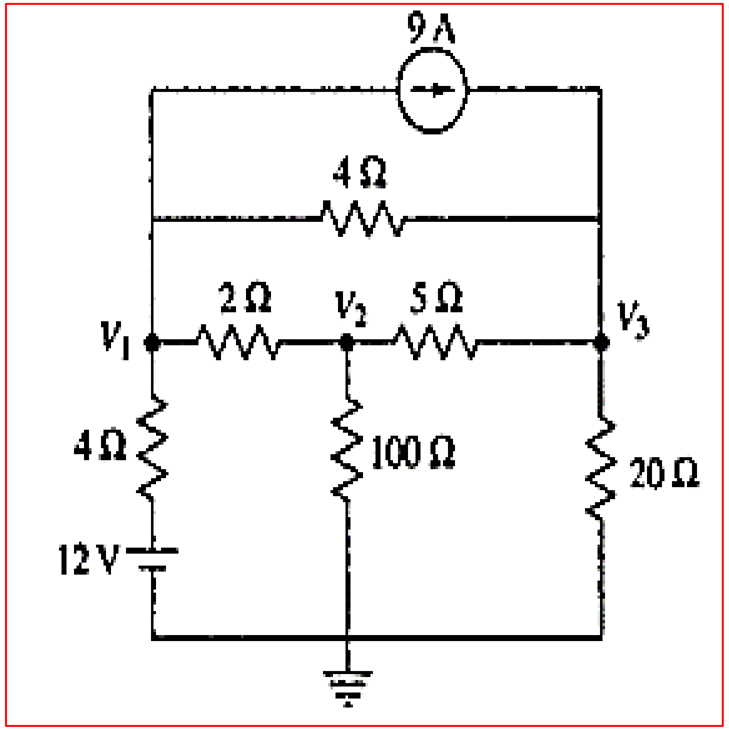
$$3V_A - 9V_B = -30 \;---(2)$$

Node Analysis-Examples

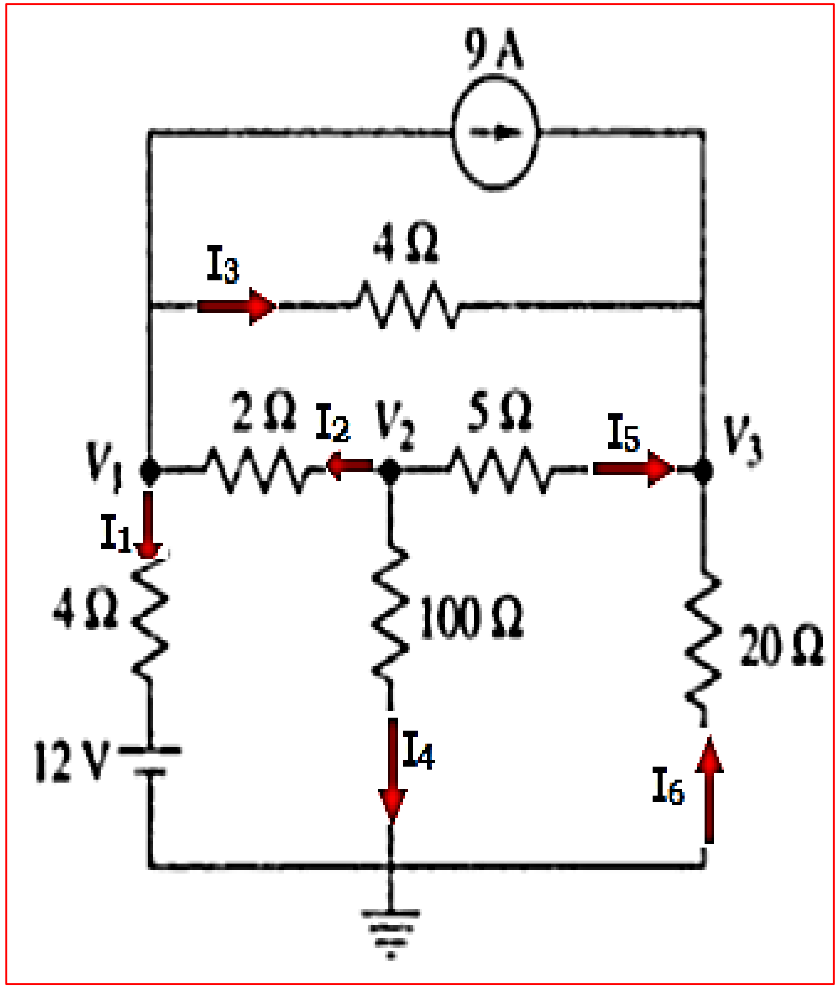
3. Find the Node voltages for the electrical circuit shown in figure.



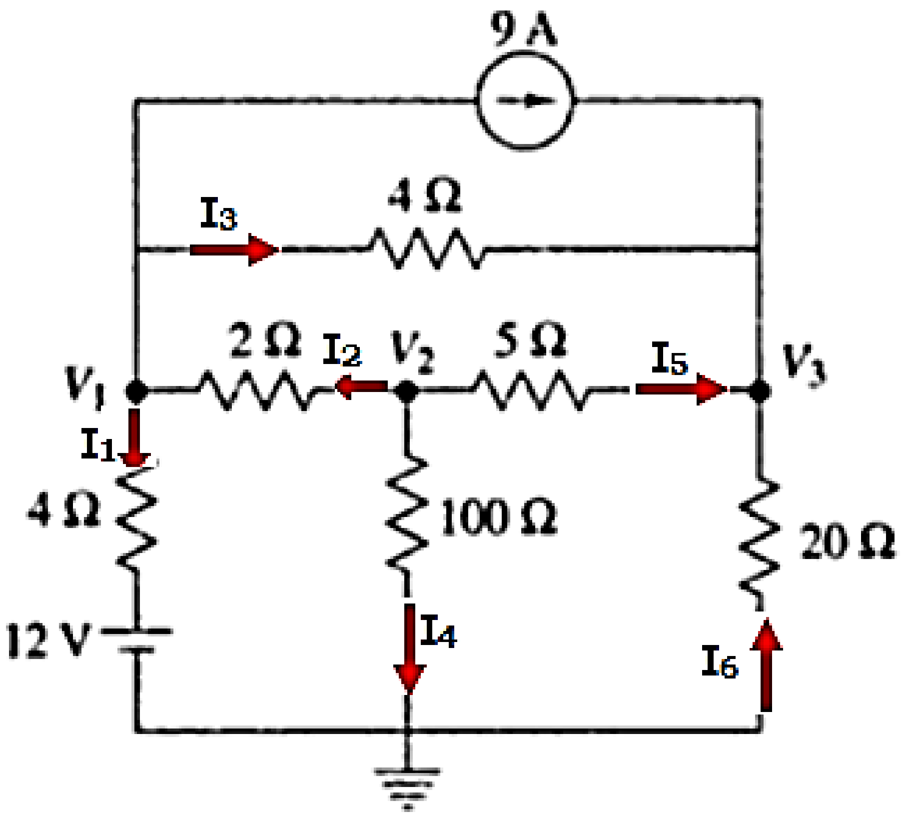
Identify the fundamental nodes



Assign branch currents



Node Analysis-Examples



KCL at node 1, $I_2 = I_1 + I_3 + 9$

$$\frac{V_2 - V_1}{2} = \frac{V_1 - 12}{4} + \frac{V_1 - V_3}{4} + 9$$

$$-V_1 + 0.5V_2 + 0.25V_3 = 6 ---(1)$$

KCL at node 2, $I_2 + I_4 + I_5 = 0$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{100} + \frac{V_2 - V_3}{5} = 0$$

$$-0.5V_1 + 0.71V_2 - 0{,}2V_3 = 0 ---(2)$$

KCL at node 3, $9 + I_3 + I_5 + I_6 = 0$

$$9 + \frac{V_1 - V_3}{4} + \frac{V_2 - V_3}{5} + \frac{-V_3}{20} = 0$$

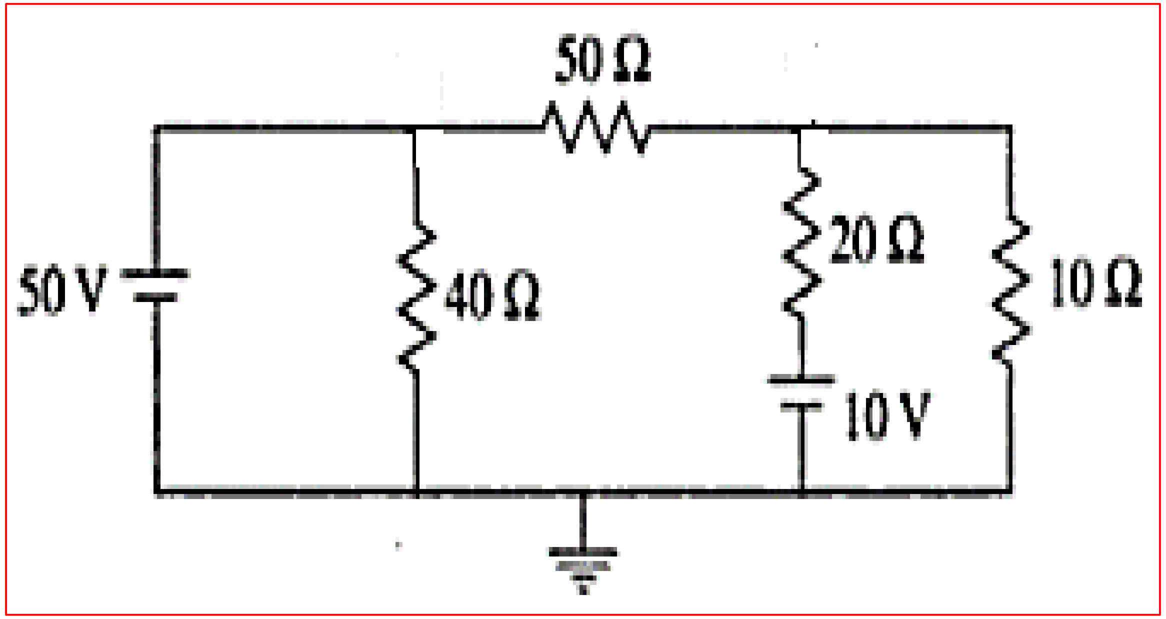
$$0.25V_1 + 0.2V_2 - 0.5V_3 = -9 ---(3)$$

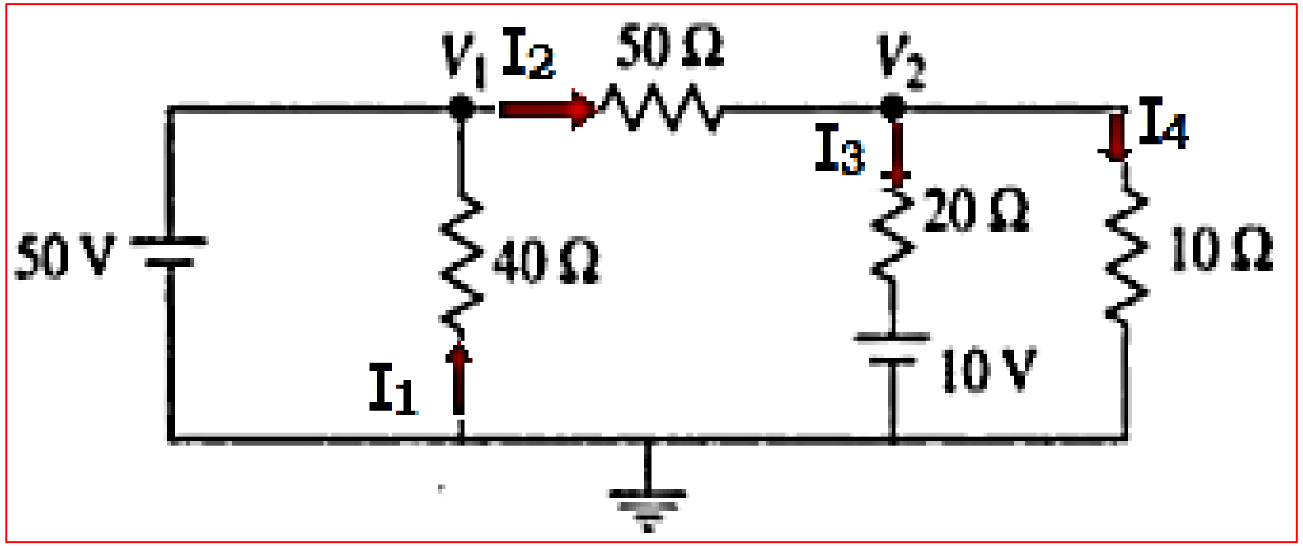
Answer: $\mathbf{V_1 = 6.35V}$ **,** $\mathbf{V_2 = 11.76V}$ **and** $\mathbf{V_3 = 25.88V}$

Node Analysis-Examples

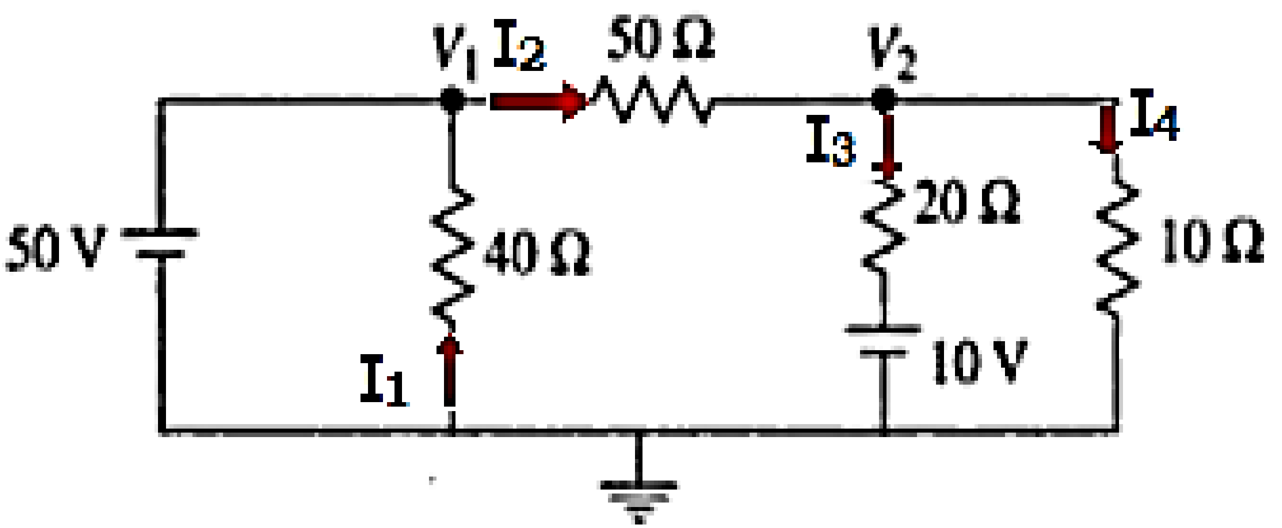
4. Find the V1 and V2 using Node voltage analysis for the electrical circuit shown in figure.

Identify the fundamental nodes and assign branch currents





Node Analysis-Examples



$$at\ node - 1$$

50V source is connected between non reference node -1 to ground node.

$$V_1 = 50V --- (1)$$

Apply KCL at node-2

$$I_2 = I_3 + I_4$$

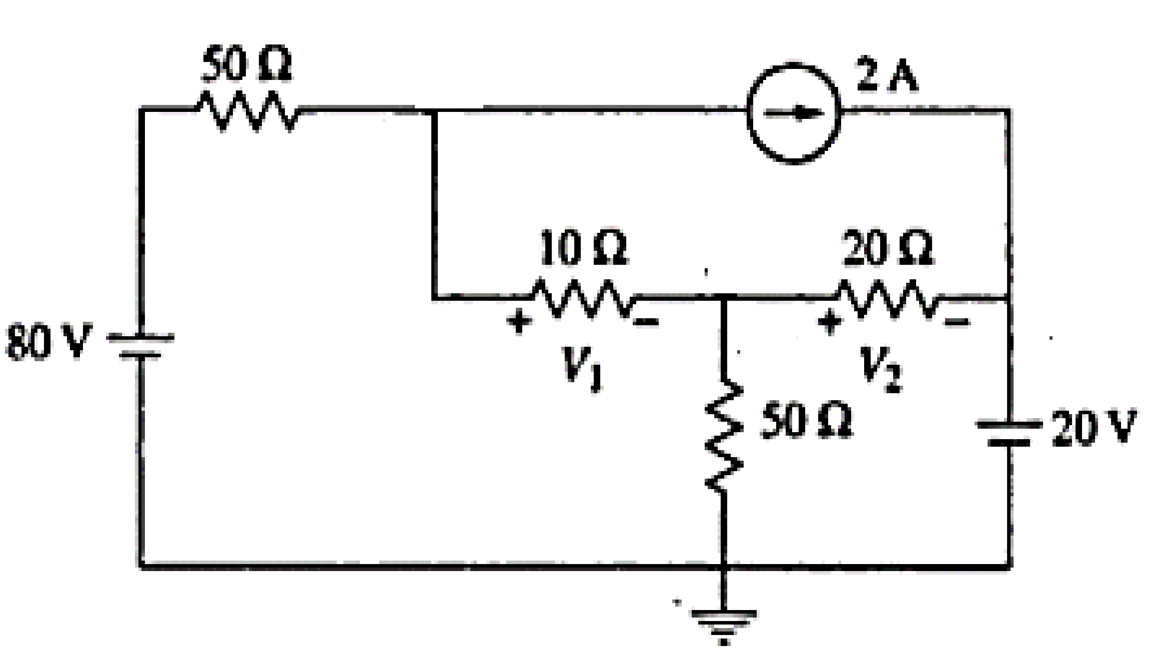
$$\frac{V_1 - V_2}{50} = \frac{V_2 - 10}{20} + \frac{V_2}{10}$$

$$0.02V_1 - 0.17V_2 = -0.5 --- (2)$$

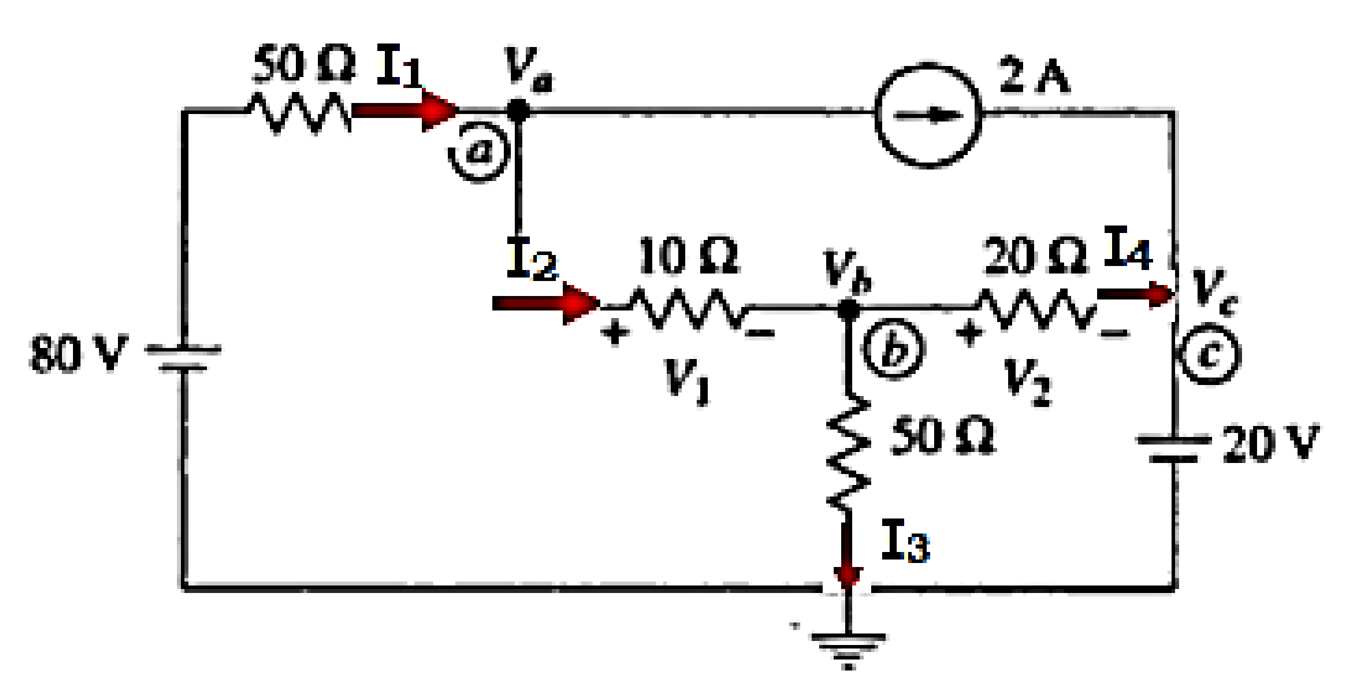
Answer: $V_1 = 50V$ and $V_2 = 8.82V$

Node Analysis-Examples

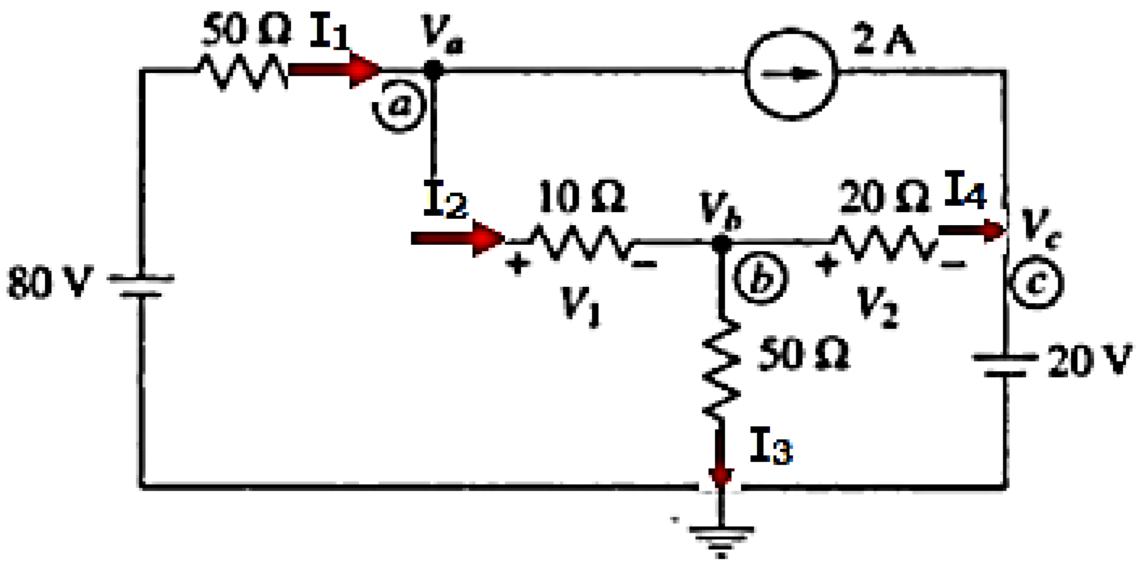
5. Find the V1 and V2 using Node voltage analysis for the electrical circuit shown in figure.



Identify the fundamental nodes and assign branch currents



Node Analysis-Examples



$$V_c = 20V \;---(1)$$

KCL at node a.

$$I_1 = I_2 + 2$$

$$\frac{80 - V_a}{50} = \frac{V_a - V_b}{10} + 2$$

$$-0.12V_a + 0.1V_b = 0.4 \;---(2)$$

KCL at node b

$$I_2 = I_3 + I_4$$

$$\frac{V_a - V_b}{10} = \frac{V_b}{50} + \frac{V_b - V_c}{20}$$

$$0.1V_a - o.17V_b + 0.05V_c = 0 \;---(3)$$

Answer: $\mathbf{V_a = 3.08V, V_b = 7.69V\,, V_c = 20V,}$

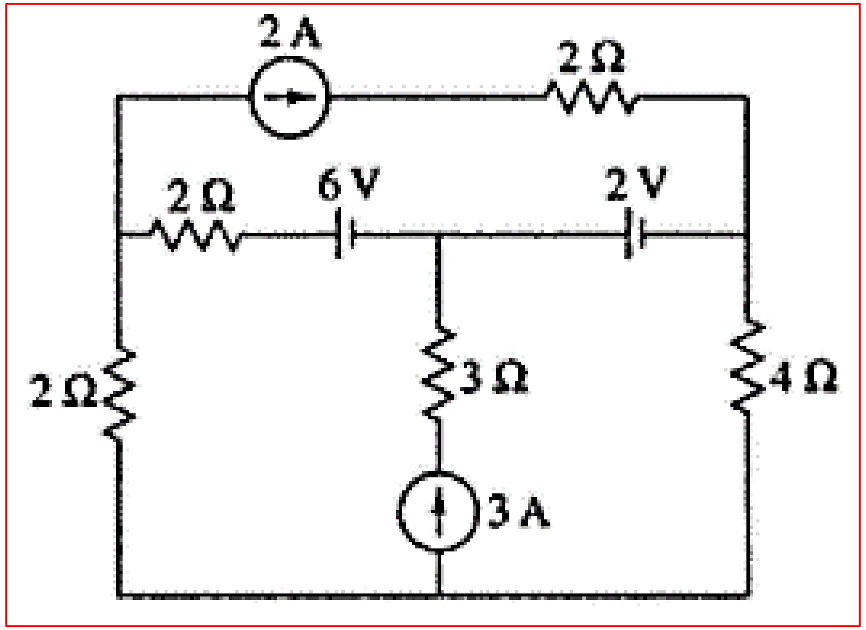
$$V_1 = V_a - V_b; V_2 = V_b - V_c$$

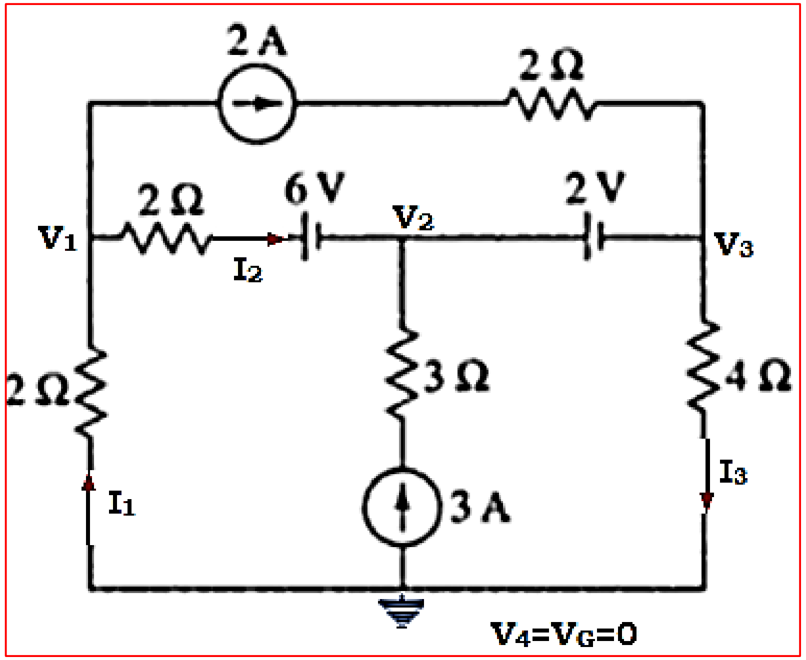
$$\mathbf{V_1 = -4.61V \text{ and } V_3 = -12.31V}$$

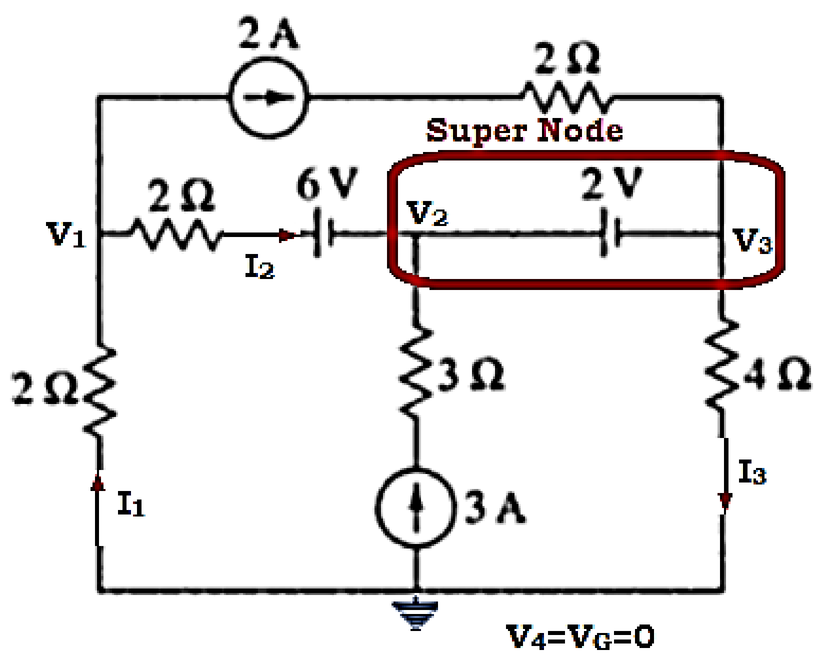
Node Analysis-Examples

6. Find Node voltages for the electrical circuit shown in figure.

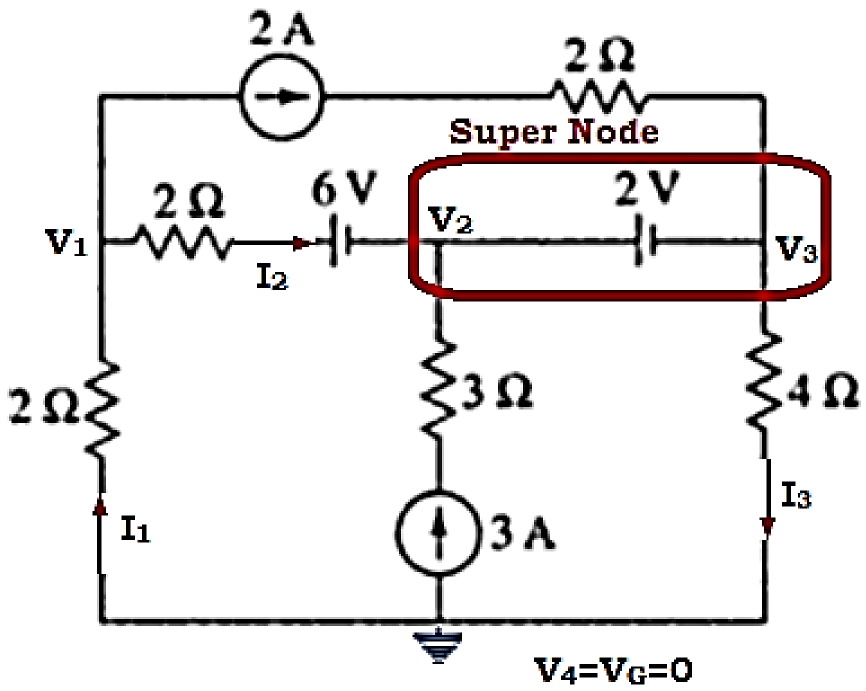
Identify the fundamental nodes and assign branch currents







Node Analysis-Examples



NOTE: if an ideal voltage source is connected between two non reference nodes, consider that common voltage source independently and write mathematical equation corresponding to the common voltage sources.

$$V_2 - V_3 = 2 --- (1)$$

For further analysis, no need to consider that voltage source. Combine nodes 2 and 3 to form a single node called as super node.

Apply KCL at super node.

$$I_2 + 3 + 2 = I_3$$

$$\frac{V_1 - 6 - V_2}{2} + 5 = \frac{V_3}{4} ---- (2); 0.5V_1 - 0.5V_2 - 0.25V_3 = -2$$

Apply KCL at node1

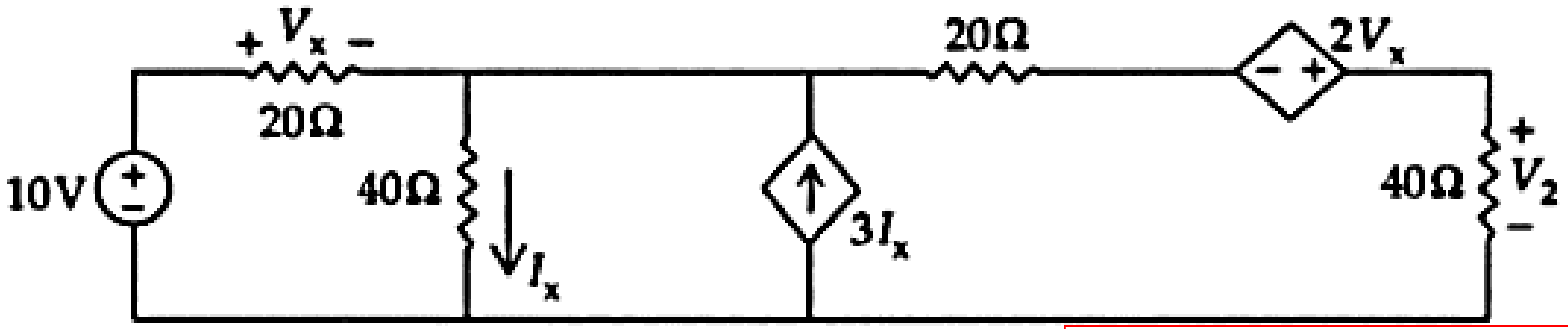
$$I_1 = I_2 + 2$$

$$\left(-\frac{V_1}{2}\right) = \frac{V_1 - 6 - V_2}{2} + 2 --- (3); -V_1 + 0.5V_2 = -1$$

Answer: $V_1 = 4V$; $V_2 = 6V$; $V_3 = 4V$.

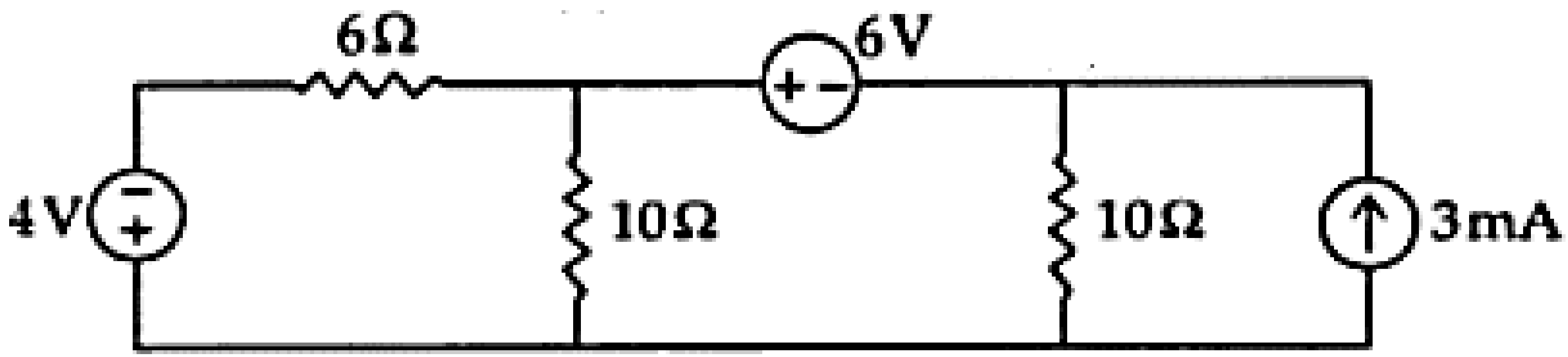
Node Analysis-Practice problems

1. Carryout nodal analysis and find V_2.



$V_2 = 0.06\ Volts$

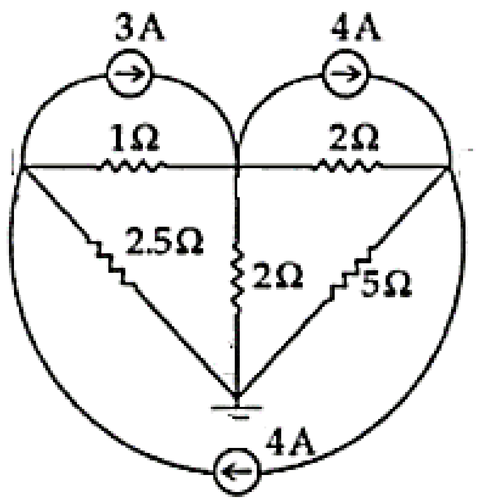
2. Carryout nodal analysis and find node voltages.



$V_1 = -0.17V\ and\ V_2 - 6.17V$

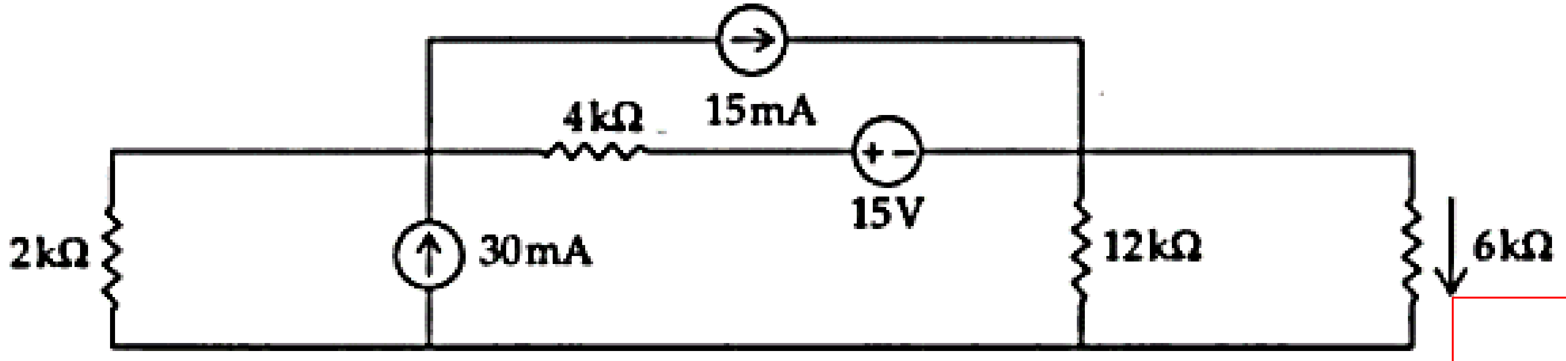
Node Analysis-Practice problems

3. Carryout nodal analysis and find voltage across 2 Ohms resistor (Connected Vertically).



$V_2(Vertically\ connected) = 0.31\ Volts$

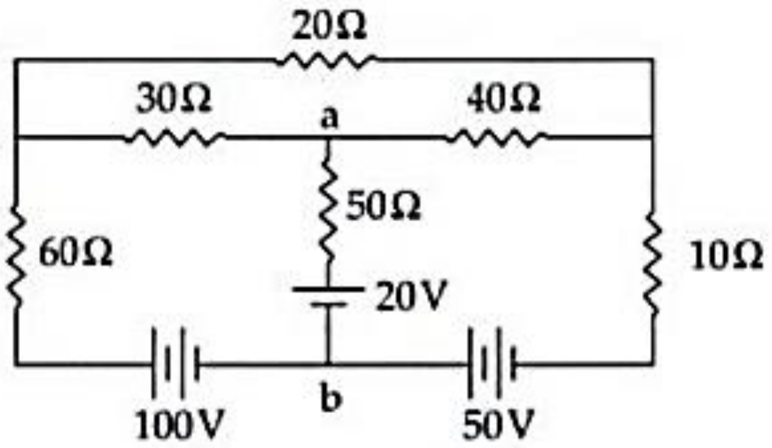
4. Find the power dissipated in 6KOhms resistor using node voltage analysis.



$P = 0.29\ Watts$

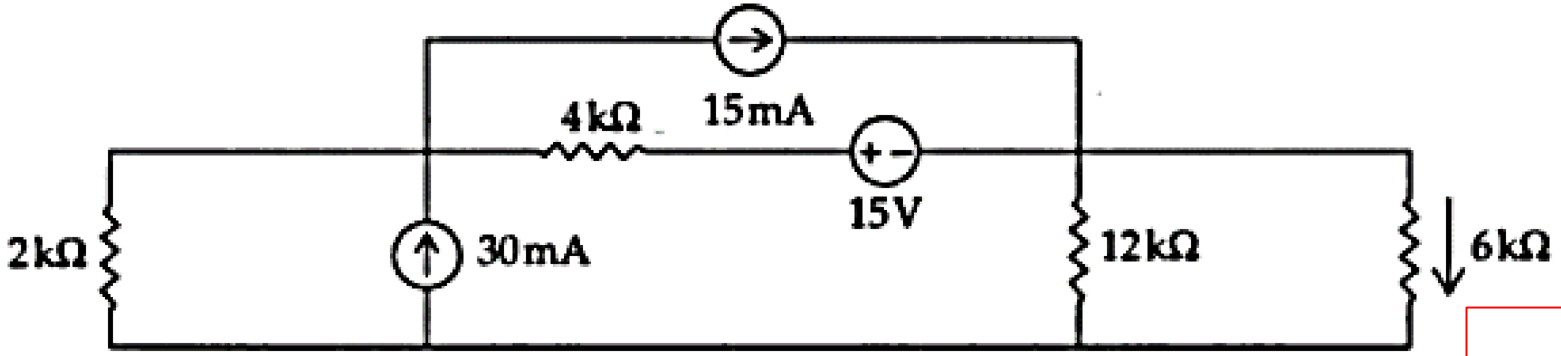
Node Analysis-Practice problems

5. Find the current through 50 Ohms resistor using mesh analysis.



$I_{50\ Ohms} = 0.24A\ (b\ to\ a)$

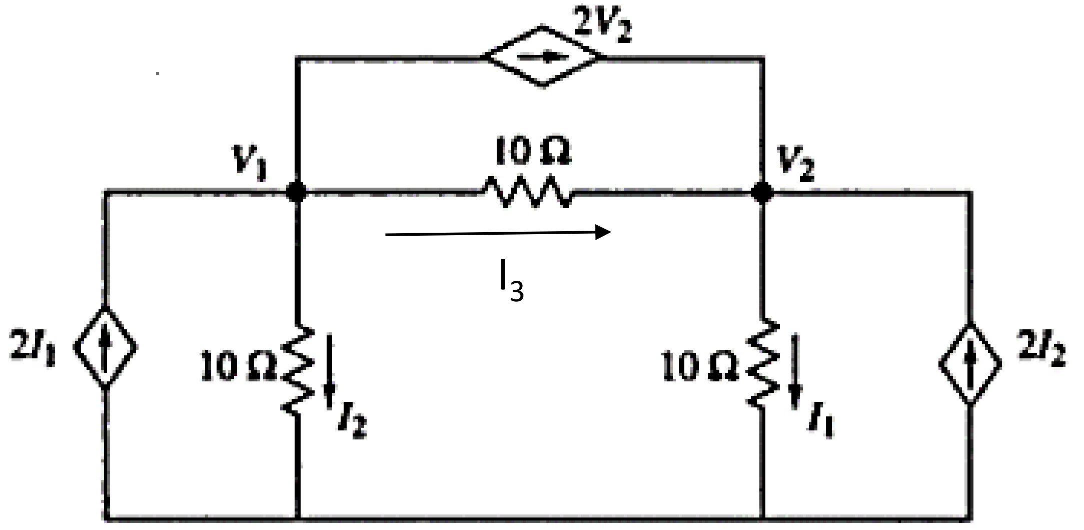
6. Find the power dissipated in 6KOhms resistor using Mesh analysis.



$P = 0.29\ Watts$

Node Analysis-Examples

7. Find V1 and V2 using Node voltage method for the electrical circuit shown in figure.



$$I_1, I_2 \; and \; V_2$$

Express control variables in terms of node voltages.

$$I_1 = \frac{V_2}{10}; I_2 = \frac{V_1}{10} \; and \; V_2$$

Apply KCL at node-1.

$$2I_1 = I_2 + I_3 + 2V_2$$

$$2\left(\frac{V_2}{10}\right) = \left(\frac{V_1}{10}\right) + \frac{V_1 - V_2}{10} + 2V_2$$

$$0.2V_2 = 0.1V_1 + 0.1V_1 - 0.1V_2 + 2V_2$$

$$\boldsymbol{V_2 = 0 \; Volts}$$

Apply KCL at node-2

$$I_3 + 2V_2 + 2I_2 = I_1$$

$$\frac{V_1 - V_2}{10} + 2V_2 + 2\left(\frac{V_1}{10}\right) = \left(\frac{V_2}{10}\right)$$

$$0.1V_1 - 0.1V_2 + 2V_2 + 0.2V_1 = 0.1V_2$$

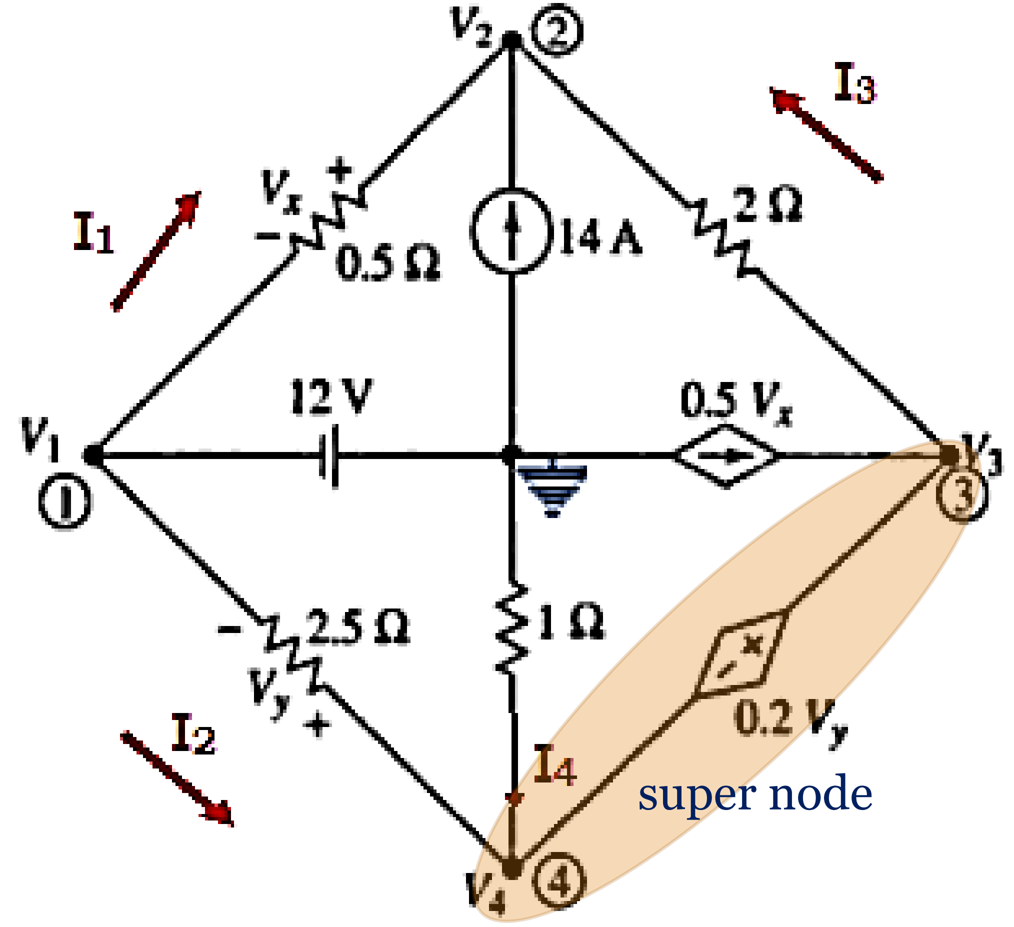
$$0.3V_1 + 1.8V_2 = 0 - -(1)$$

$$\boldsymbol{V_1 = 0 \; Volts}.$$

Answer: $V_1 = 0V$ and $V_2 = 0V$

Node Analysis-Examples

8. Find Node voltages for the electrical circuit shown in figure.



V_x *and* V_y *are control varoables.*

Express control variables in terms of node voltages.

$$V_x = V_2 - V_1 \;---(1)$$
$$V_y = V_4 - V_1 \;---(2)$$

Node-1

$$V_1 = -12\,Volts \;---(3)$$

Apply KCL at node 2

$$I_1 + I_3 + 14 = 0$$

$$\frac{V_1 - V_2}{0.5} + \frac{V_3 - V_2}{2} + 14 = 0 \;---(4);\; 2V_1 - 2V_2 + 0.5V_3 - 0.5V_2 = -14$$

$$\mathbf{2V_1 - 2.5V_2 + 0.5V_3 = -14}$$

$0.2V_y$ voltage source is between V_3 and V_4.

$$0.2V_y = V_3 - V_4 \;---(5);\; 0.2(V_4 - V_1) = V_3 - V_4 \;---(5)$$

$$\mathbf{-0.2V_1 - V_3 + 2V_4 = 0}$$

Apply KCL at super node.

$$I_2 + I_4 + 0.5V_x = I_3$$

$$\frac{V_1 - V_4}{2.5} + (-V_4) + 0.5(V_2 - V_1) = \frac{V_3 - V_2}{2} \;---(6)$$

$$\mathbf{-0.9V_1 + V_2 - 0.5V_3 - 1.4V_4 = 0}$$

Answer: $V_1 = -12, V_2 - 4V, V_3 = 0\ V$ and $V_4 = -2V$

AC Quantities

AC quantities are represented in two different formats.

Polar form

Format=$M\angle\emptyset$

Where, M is the magnitude and ∅ is the phase angle

Rectangular form

$x + jy$

Where, x is the real part and y is the imaginary part.

Conversion from rectangular to polar

Given: x+jy, **To find**: M and ∅

$$M = \sqrt{x^2 + y^2} \text{ and } \emptyset = \tan^{-1}(\frac{y}{x})$$

Conversion from polar to rectangular

Given : M and ∅, **To find**: x and y

$$x = M\cos\emptyset \text{ and } y = M\sin\emptyset$$

AC Quantities

Conversions are used to perform mathematical calculations.

For addition and subtraction-Rectangular form

Consider

$$A = x_1 + jy_1 \text{ and } B = x_2 + jy_2$$
$$A + B = (x_1 + x_2) + j(y_1 + y_2)$$

Similarly
$$A - B = (x_1 - x_2) + j(y_1 - y_2)$$

For multiplication and division-Polar form

Consider

$$A = M_1\angle \emptyset_1 \text{ and } B = M_2\angle \emptyset_2$$
$$A * B = M_1 * M_2\angle \emptyset_1 + \emptyset_2$$

Similarly
$$\frac{A}{B} = \frac{M_1}{M_2}\angle \emptyset_1 - \emptyset_2$$

Also

$$A * B = (x_1 + jy_1)(x_2 + jy_2)$$
$$= x_1x_2 + jx_1y_2 + jx_2y_1 + j^2y_1y_2$$
$$= (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1) \quad \text{since } j^2 = -1$$

NOTE:

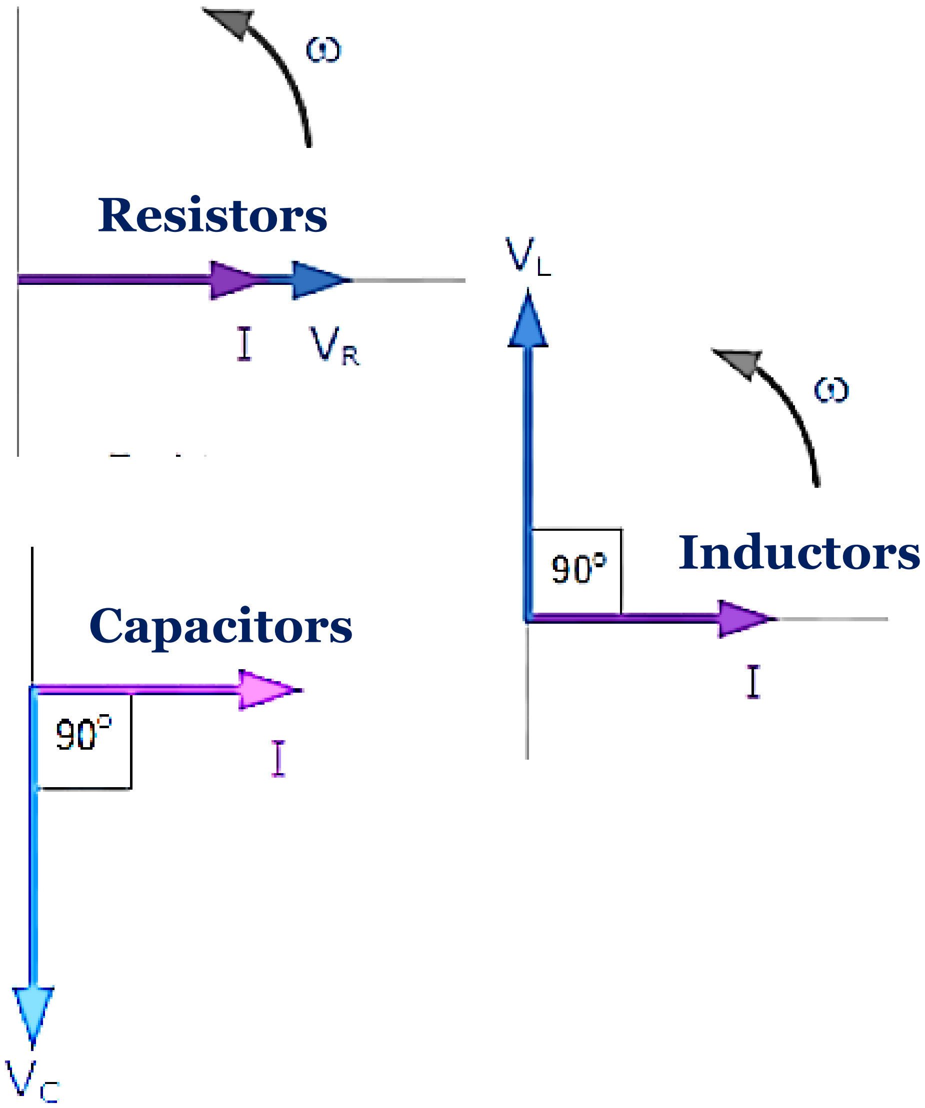
$$A = 10; A = 10\angle 0^0 \Rightarrow 10 + j0$$
$$A = 10\angle 90^0 \Rightarrow 0 + j10$$
$$A = 10\angle -90^0 \Rightarrow 0 - j10$$
$$A = 10 + j0 \Rightarrow 10 \text{ or } 10\angle 0^0$$
$$A = j10 \Rightarrow 10\angle 90^0$$
$$A = -j10 \Rightarrow 10\angle -90^0 \text{ or } -10\angle 90^0$$
$$A = \frac{1}{j} \Rightarrow -j$$
$$A = j * j \Rightarrow j^2 = -1.$$

AC Quantities

Resistors:
R Ohms (Same for both DC and AC)
Voltage in phase with the current

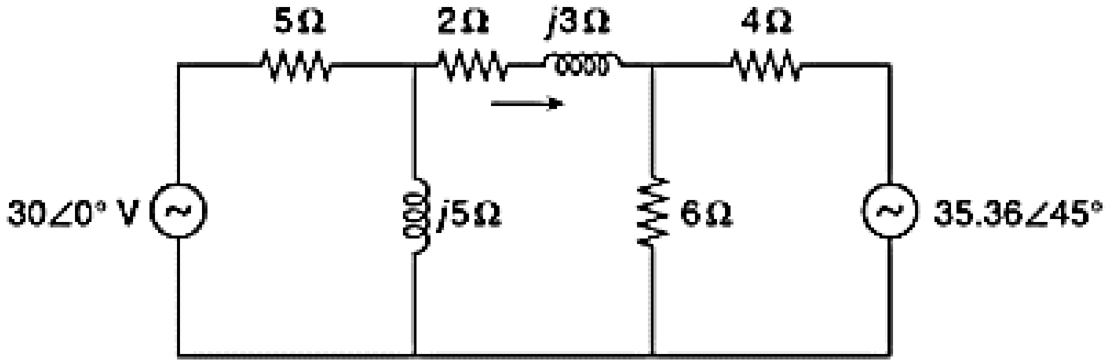
Capacitors:
C Farads (DC analysis)
-jX_C Ohms (Capacitive reactance AC analysis) (Negative sign-Voltage lags current by 90°)
Where, $X_C = \frac{1}{2\pi f C} \Rightarrow 1/wC, where, f\ is\ the\ frequency.$

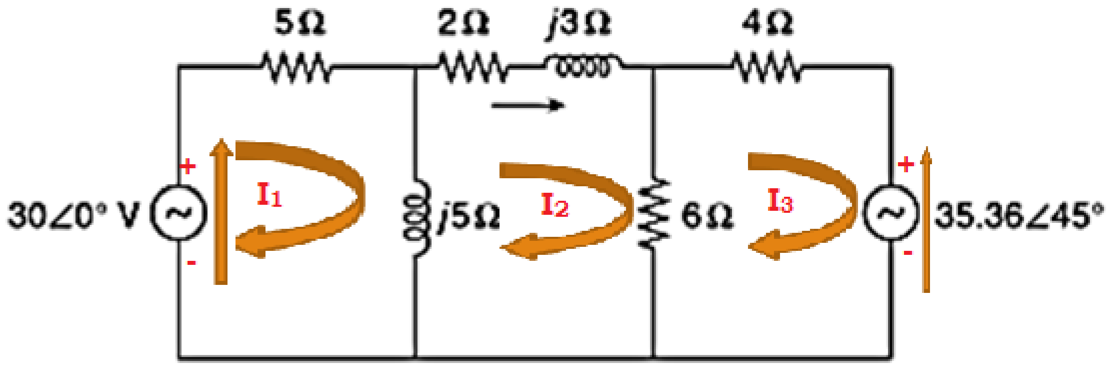
Inductors:
L Henry (DC analysis)
+jX_L Ohms (Inductive reactance AC analysis) (Positive sign-voltage leads current by 90°)
Where, $X_L = 2\pi f L \Rightarrow wL, where, f\ is\ the\ frequency.$



Mesh Analysis-Examples

1. Find the mesh currents using mesh analysis





KVL at mesh-1

$$5I_1 + j5(I_1 - I_2) - 30\angle 0^0 = 0$$

$$\mathbf{(5 + j5)I_1 - j5I_2 = 30\angle 0^0 --- (1)}$$

KVL at mesh-2

$$2I_2 + j3I_2 + 6(I_2 - I_3) + j5(I_2 - I_1) = 0$$

$$\mathbf{(-j5)I_1 + (8 + j8)I_2 - 6I_3 = 0 --- (2)}$$

KVL at mesh-3

$$4I_3 + 35.36\angle 45^0 + 6(I_3 - I_2) = 0$$

$$\mathbf{(-6)I_2 + (10)I_3 = -35.36\angle 45^0 --- (3)}$$

Solve equations (1), (2) and (3), we get

I_1, I_2 and I_3

Mesh Analysis-Examples

Cramer's rule:

$$\Delta = \begin{bmatrix} (5+j5) & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix}, X = \begin{bmatrix} 30+j0 \\ 0 \\ -25-j25 \end{bmatrix}$$

$$\Delta = (5+j5)[(8+j8)(10)-6(6)] - (-j5)[-j5(10)-0]$$
$$\Delta = (5+j5)[80+j80-36] + j5[-j50]$$
$$\Delta = (5+j5)(44+j80) + 250$$
$$\Delta = 220 + j400 + j220 - 400 + 250$$
$$\Delta = \mathbf{70 + j620 \text{ or } 623.9\angle 83.55^0}$$

$$\Delta_1 = \begin{bmatrix} 30+j0 & -j5 & 0 \\ 0 & 8+j8 & -6 \\ -25-j25 & -6 & 10 \end{bmatrix}$$

$$\Delta_1 = 30[(8+j8)10-36] + j5\big(-6(25+j25)\big)$$
$$\Delta_1 = 30[(80+j80)-36] + j5(-150-j150)$$
$$\Delta_1 = 30[44+j80] - j750 + 750$$

$$\Delta_1 = 1320 + j2400 - j750 + 750$$
$$\Delta_1 = \mathbf{2070 + j1650 \text{ or } 2647.14\angle 38.55^0}$$

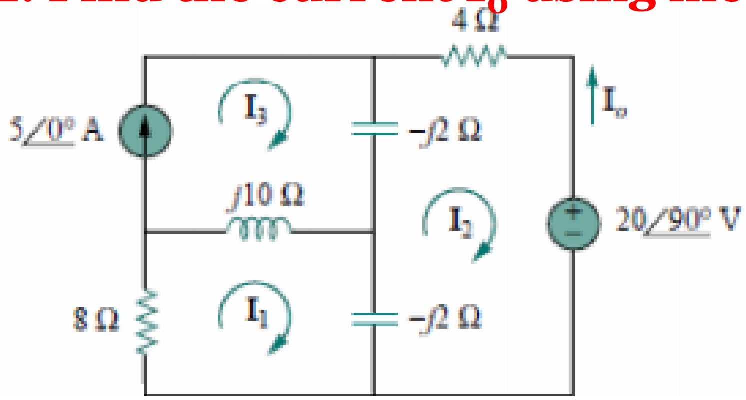
Similarly evaluate Δ_2 *and* Δ_3

Therefore $\boldsymbol{I_1} = \frac{\Delta_1}{\Delta} = \mathbf{4.24\angle 45.01\,^0 A}$

Similarly evaluate $I_2 = \frac{\Delta_2}{\Delta}$ and $I_3 = \frac{\Delta_3}{\Delta}$

Mesh Analysis-Examples

2. Find the current I_o using mesh analysis



Applying KVL to mesh 1, we obtain:

$$(8 + j10 - j2)I_1 - (-j2)I_2 - j10I_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)I_2 - (-j2)I_1 - (-j2)I_3 + 20\angle 90^\circ = 0$$

For mesh 3, $I_3 = 5$.

$$(8 + j8)I_1 + j2I_2 = j50$$
$$j2I_1 + (4 - j4)I_2 = -j20 - j10$$

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 32(1+j)(1-j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22^\circ$$

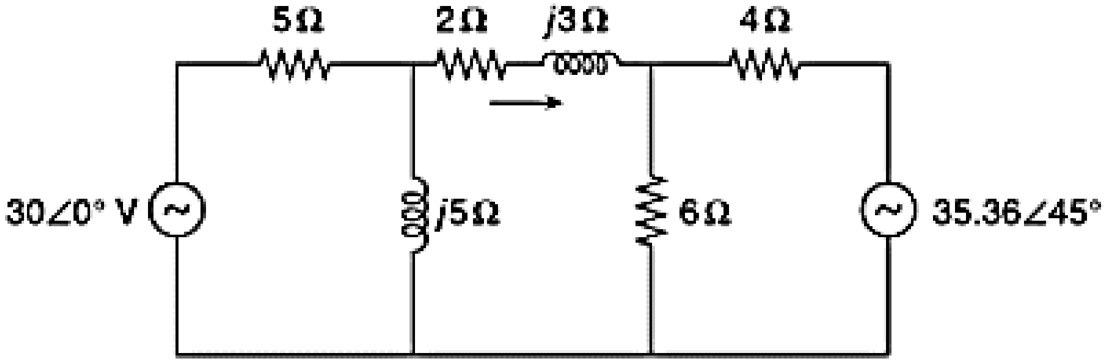
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

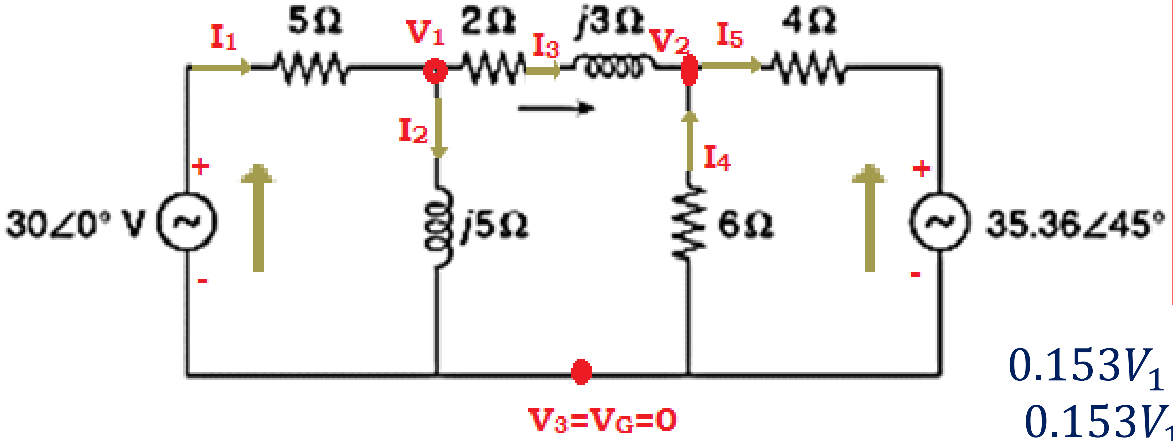
The desired current is

$$I_o = -I_2 = 6.12\angle 144.78^\circ \text{ A}$$

Mesh Analysis-Examples

3. Find the Node voltages using node analysis





KCL at node-1

$$I_1 = I_2 + I_3$$

$$\frac{30\angle 0^0 - V_1}{5} = \left(\frac{V_1}{j5}\right) + \frac{V_1 - V_2}{2 + j3}$$

$$6 - 0.2V_1 = -j0.2V_1 + \frac{V_1 - 2}{3.6\angle 56.3^0}$$

$$6 - 0.2V_1 = -j0.2V_1 + 0.277\angle - 56.3^0(V_1 - V_2)$$

$$6 - 0.2V_1 = -j0.2V_1 + 0.153V_1 - j0.23V_1 - 0.153V_2 + j0.23V_2$$

$$6 = 0.2V_1 - j0.2V_1 + 0.153V_1 - j0.23V_1 - 0.153V_2 + j0.23V_2$$

$$\mathbf{(0.353 - j0.43)V_1 + (-0.153 + j0.23)V_2 = 6 --- (1)}$$

KCL at node-2

$$I_3 + I_4 = I_5$$

$$\frac{V_1 - V_2}{2 + j3} + \left(-\frac{V_2}{6}\right) = (V_2 - 35.36\angle 45^0)/4$$

$$0.153V_1 - j0.23V_1 - 0.153V_2 + j0.23V_2 - 0.166V_2 = 0.25V_2 - 8.84\angle 45$$

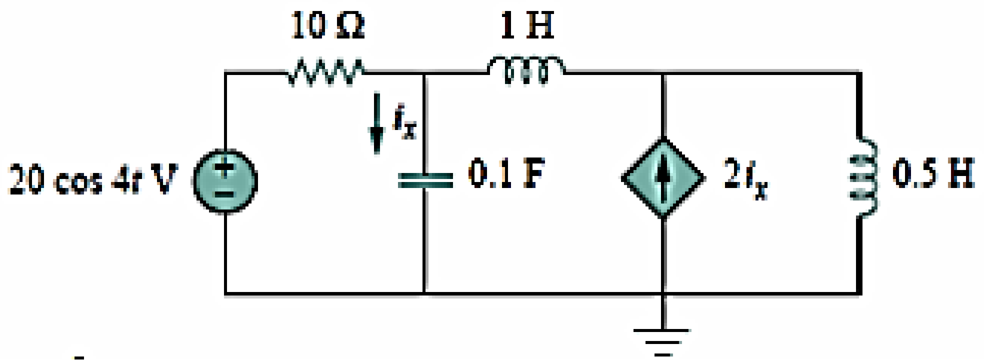
$$0.153V_1 - j0.23V_1 - 0.153V_2 + j0.23V_2 - 0.166V_2 - 0.25V_2 = 8.84\angle 45$$

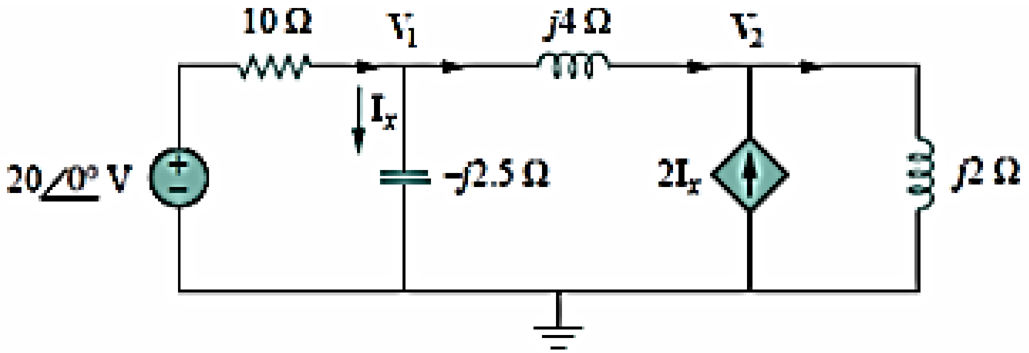
$$\mathbf{(0.153 - j0.23)V_1 + (-0.569 + j0.23)V_2 = 8.84\angle 45 --- (2)}$$

$$\textit{Solve } (1) \textit{ and } (2) \textit{ to find } V_1 \textit{ and } V_2$$

Problems

4. Find I_X using node analysis.





Transform to frequency domain.

$$20\cos 4t \Longrightarrow 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$

$$1\text{ H} \Longrightarrow j\omega L = j4$$

$$0.5\text{ H} \Longrightarrow j\omega L = j2$$

$$0.1\text{ F} \Longrightarrow \frac{1}{j\omega C} = -j2.5$$

Applying KCL at node 1,

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

or

$$(1 + j1.5)V_1 + j2.5V_2 = 20$$

At node 2,

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$$

But $I_x = V_1/-j2.5$. Substituting this gives

$$\frac{2V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$$

By simplifying, we get

$$11V_1 + 15V_2 = 0$$

Problems

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

The current $\mathbf{I}_x$ is given by

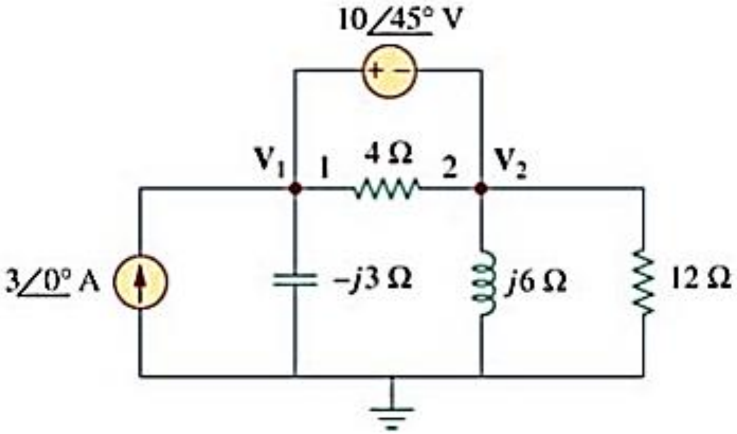
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

Transforming this to the time domain,

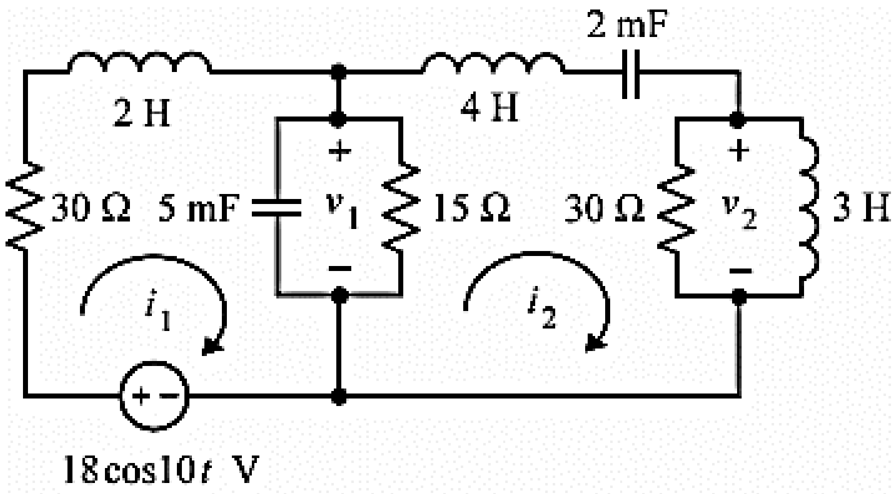
$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

Practice problems

1. Find node voltages



2. Find node voltages and mesh currents for the electrical circuit shown in figure.

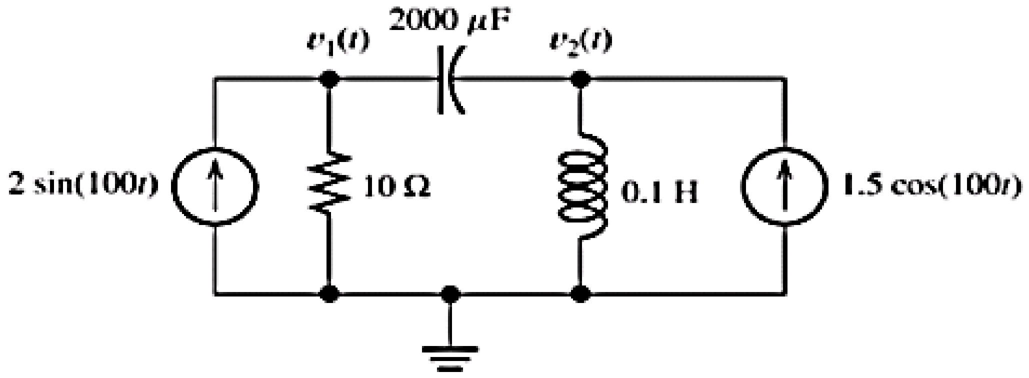


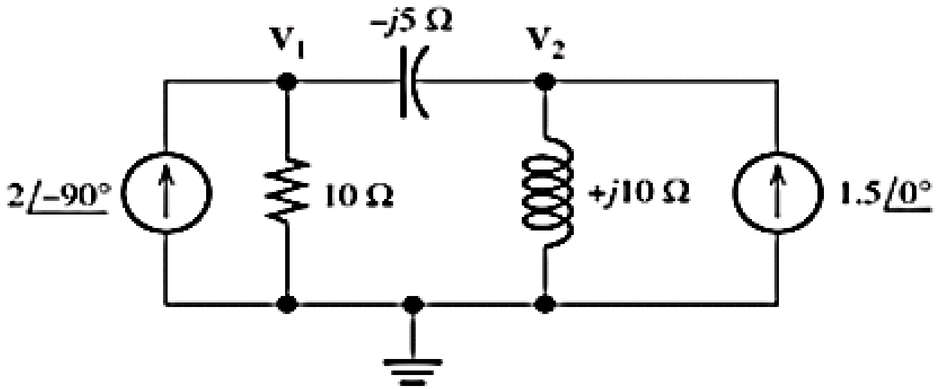
$V_1 = 3.318\angle -39.3^\circ$ V and $V_2 = 4.452\angle -12.7^\circ$ V

$I_1 = 0.4319\angle -25.9^\circ$ A and $I_2 = 0.2099\angle -57.7^\circ$ A

Practice problems

3. Find node voltages





$$v_1 = 16.1\cos(100t + 29.74^\circ)\ \text{V}$$

Network Theorems

Theorems:

Theorems are statements that can be demonstrated to be true by some accepted mathematical arguments and functions. Generally theorems are general principles. The process of showing a theorem to be correct is called a proof.

- Proved theorems can be used to analyze the given system, and theorems helps to analyze the complex systems easily.
- In electrical system most popular theorems are

1. Thevenin's Theorem
2. Norton's Theorem
3. Superposition Theorem
4. Reciprocity theorem
5. Millman's Theorem and
6. Maximum Power Transfer Theorem

Thevenin's Theorem

1. **Thevenin's Theorem**

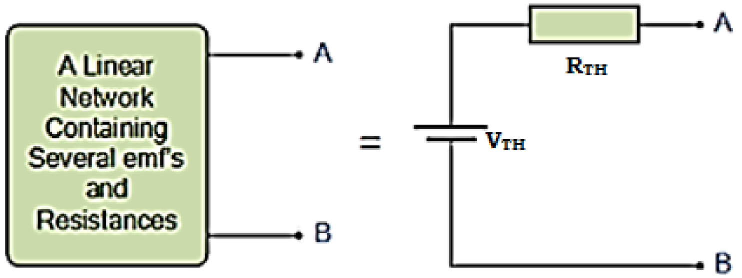
Statement: Any active linear bilateral complex electrical network between open circuited load terminals can be replaced by a single practical voltage source between the same open circuited load terminals.

A practical voltage source is a series combination of ideal voltages source and a resistor (DC circuit)/Impedance(AC Circuits).

The voltage source being equal to the voltage measured between the open circuited load terminals, denoted as V_{TH} or V_{OC} and Resistor/Impedance being equal to the equivalent Resistance / Impedance measured between open circuited load terminals by replacing all independent sources by their internal impedances, denoted as R_{TH} or Z_{TH}.

Internal Impedance of an ideal voltage sources is zero, Hence replace it by short circuit.

Internal Impedance of an ideal current sources is infinity , hence replace it by open circuit.



Thevenin's Theorem

Procedure to obtain Thevenin's equivalent circuit.

Step-1: Identify the load element, remove the load element and name the load terminals.

Step-2: Find the open circuit voltage using any network analysis technique.

Step-3: Find the equivalent resistance/Impedance between the open circuited terminals by replacing all independent sources by their internal impedance.

Step-4: Replace the given circuit between the open circuited load terminals by the Thevenin's equivalent circuit.

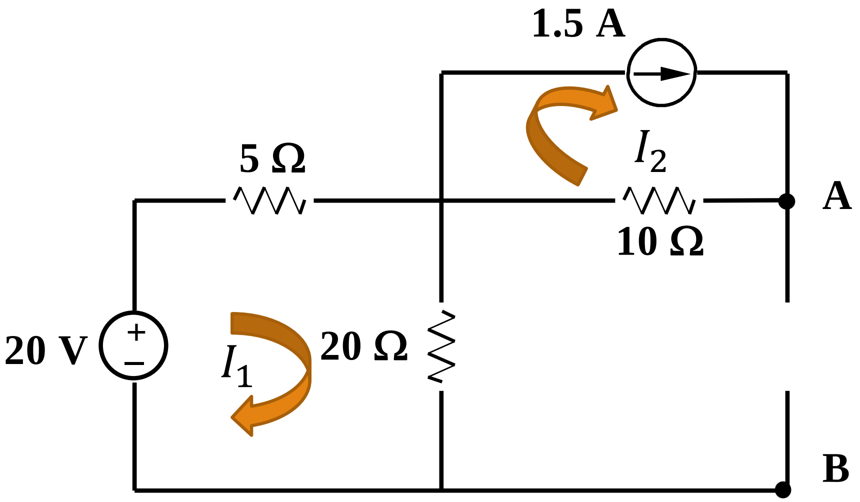
Step-5: Connect the load element between the load terminals and find the required load quantity using current division or voltage division formula.

NOTE: For the circuits with dependent sources, find R_{TH} using the ratio=V_{OC}/I_{SC}

Thevenin's Theorem-Examples

Example:

1. Obtain the Thevenin's equivalent circuit between the terminals A and B.



Step-2. To find V_{TH}

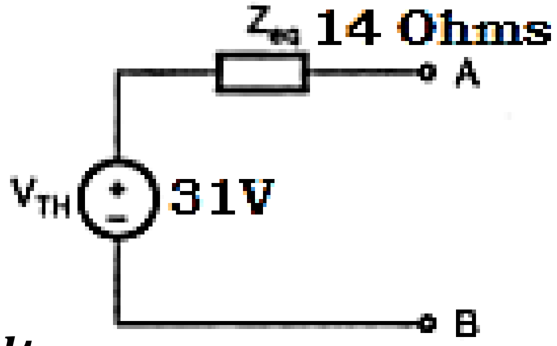
$$V_{AB} = V_{OC} = V_{TH} = V_{10} + V_{20}$$
$$V_{TH} = 10(I_2) + 20(I_1) --- (1)$$

Apply KVL at mesh-1

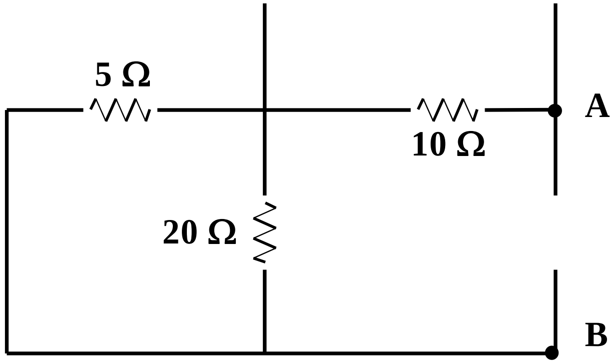
$$5I_1 + 20I_1 = 20$$
$$25I_1 = 20$$
$$I_1 = 0.8A.$$

At mesh-2

$$I_2 = 1.5A$$



$$V_{TH} = 10(1.5) + 20(0.8) \Rightarrow \mathbf{31\ Volts}$$

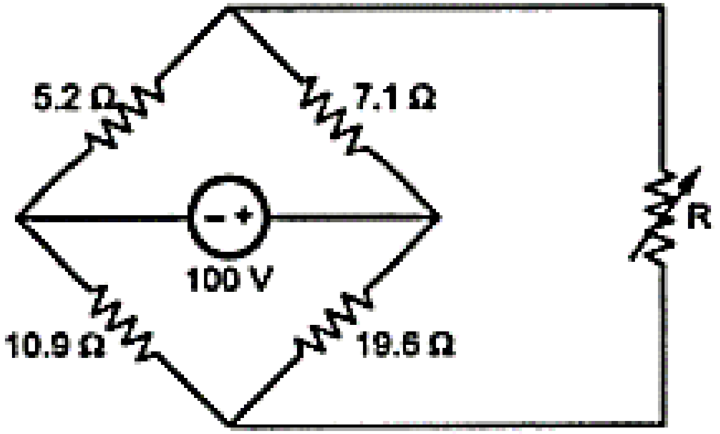


Step-3: To find R_{TH}

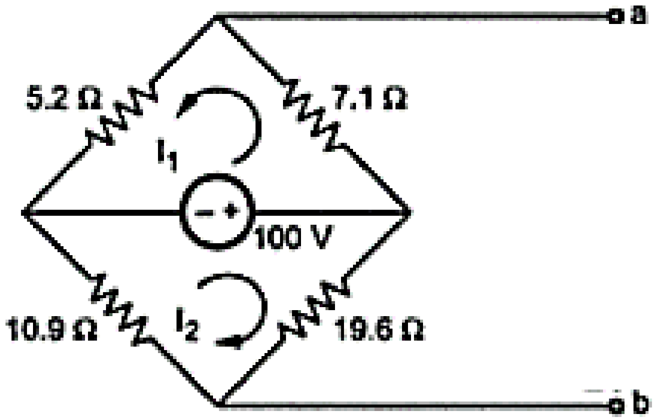
$$R_{TH} = R_{AB} = R_{eq} = 20||5 + 10$$
$$R_{TH} = \mathbf{14\ Ohms}$$

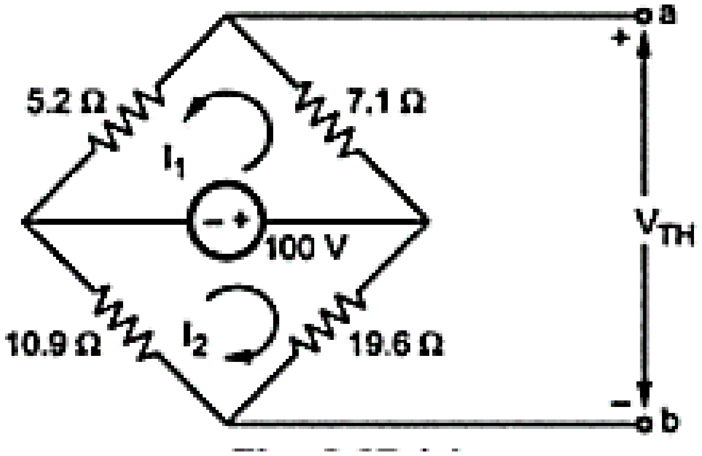
Thevenin's Theorem-Examples

2. Obtain the Thevenin's equivalent circuit to find the current through R of 10 ohms



Remove the load element and name the load terminals.





$$V_{TH} = V_{5.2} + V_{10.9} \Rightarrow 5.2(I_1) + 10.9(-I_2)$$

$$V_{TH} = V_{7.1} + V_{19.6} \Rightarrow 7.1(-I_1) + 19.6(I_2)$$

$$V_{TH} = V_{5.2} - 100 + V_{19.6} \Rightarrow 5.2(I_1) - 100 + 19.6(I_2)$$

$$V_{TH} = V_{7.1} + 100 + V_{10.9} \Rightarrow 7.1\ (-I_1) + 100 + 10.9(-I_2)$$

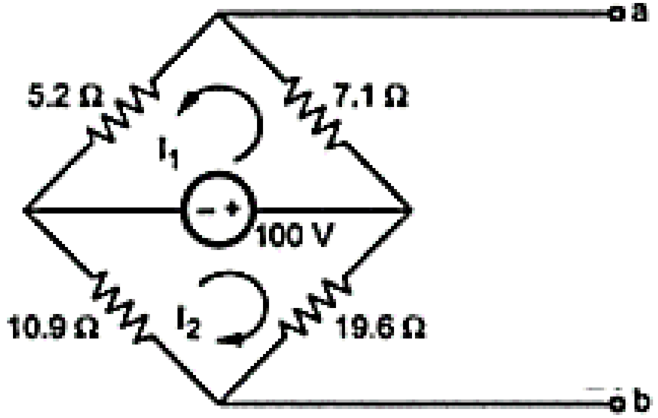
Apply KVL at mesh-1; $12.3I_1 = 100; I_1 = 8.13A$

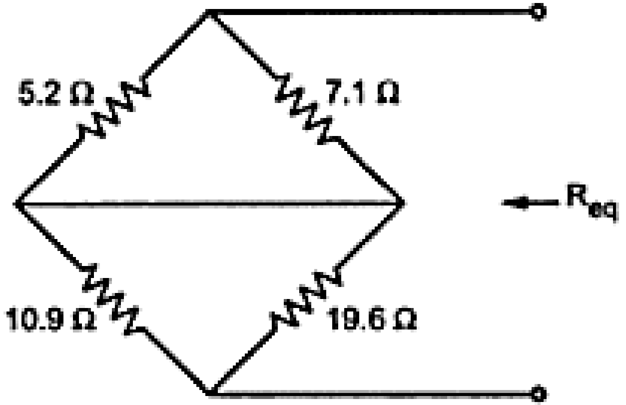
Apply KVL at mesh-2; $30.5I_2 = 100; I_2 = 3.28\ A.$

$$\boldsymbol{V_{TH} = 6.63V}$$

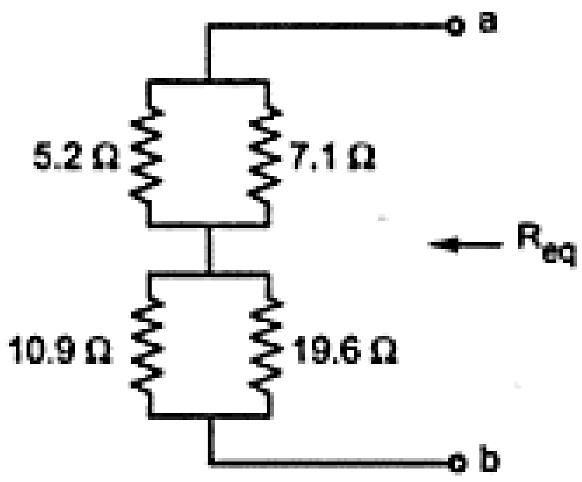
Thevenin's Theorem-Examples

To find R_{TH}



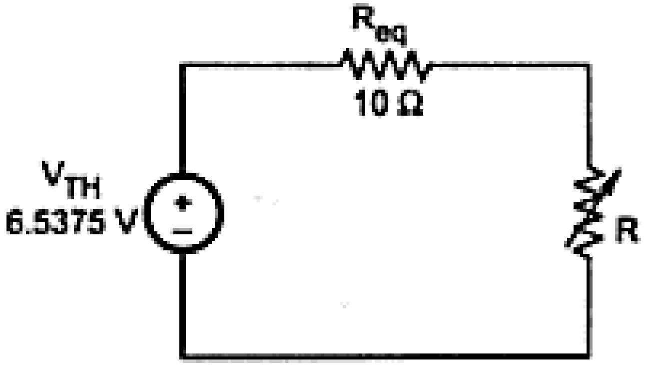


Re arranging the resistors



$$R_{TH} = (5.2||7.1) + (10.9||19.6)$$
$$R_{TH} = 10\ Ohms$$

Thevenin's Equivalent circuit

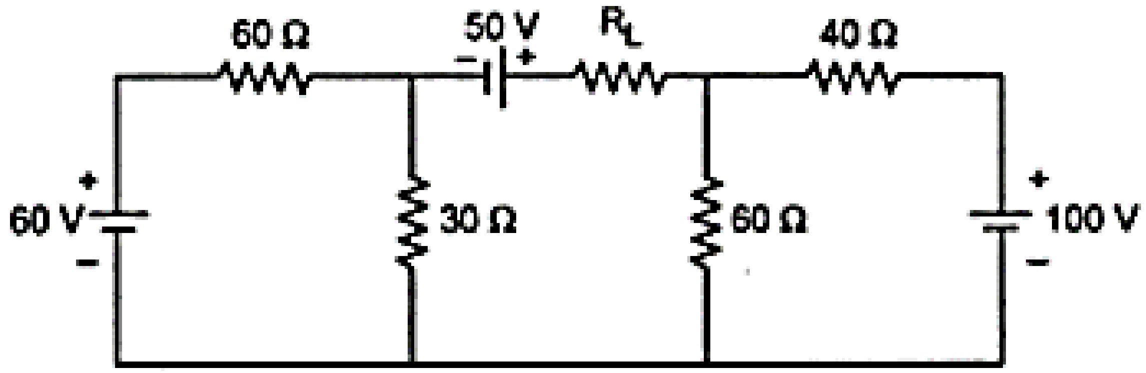


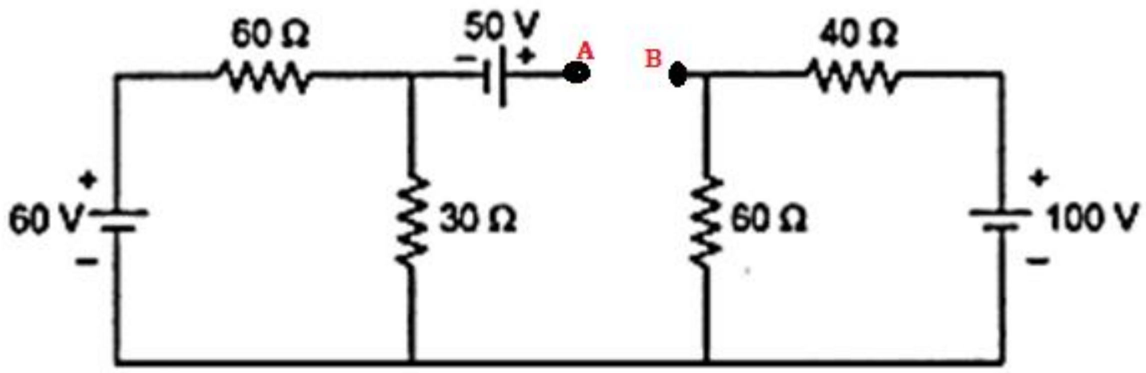
Given R=10 Ohms

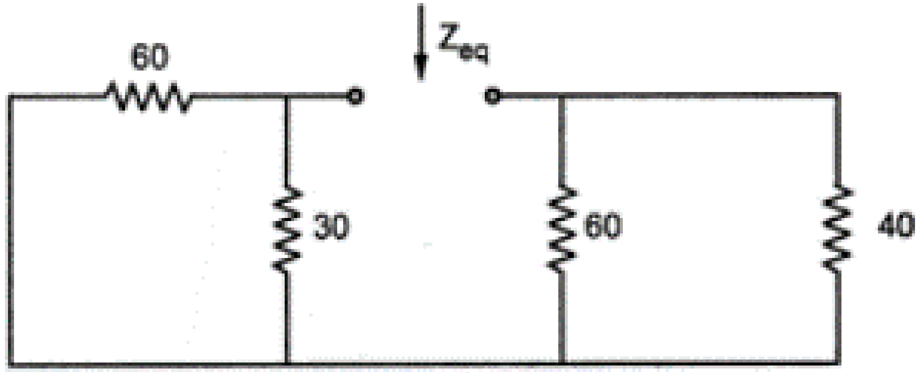
$$I_{10} = \frac{V_{TH}}{R_{eq} + R} \Rightarrow 0.33A.$$

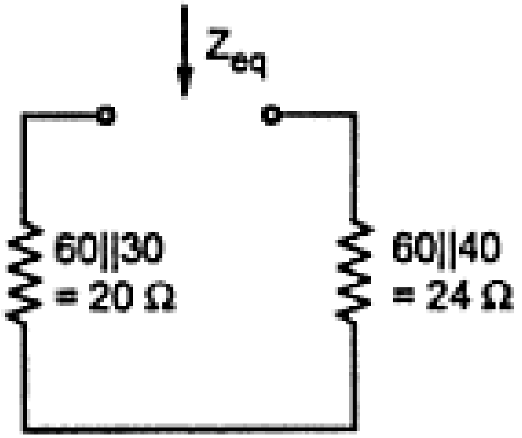
Thevenin's Theorem-Examples

3. Obtain the Thevenin's equivalent circuit and find the current through RL of 20 Ohms









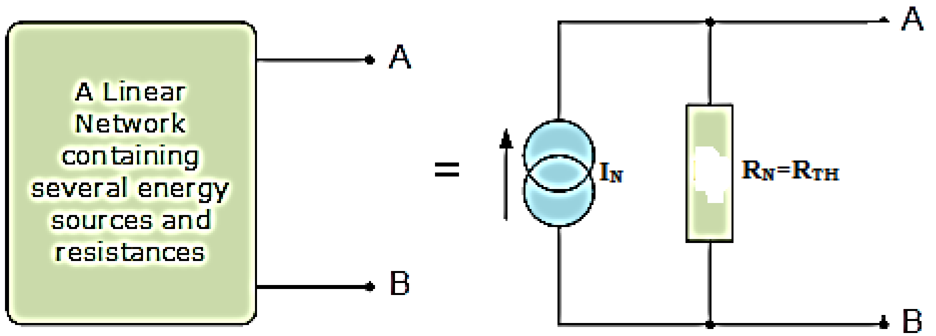
Norton's Theorem

1. Norton's Theorem

Statement: Any active linear complex bilateral electrical network between open circuited load terminals can be replaced by a single practical current source between the same open circuited load terminals.

A practical current source is a parallel combination of ideal current source and a resistor (DC circuit)/Impedance(AC Circuits).

The Current source being equal to the **current measured through the short circuited load terminals,** denoted as I_N or I_{SC} and Resistor/Impedance being equal to the equivalent Resistance / Impedance measured between open circuited load terminals, denoted as R_N or Z_N



Norton’s Theorem

Procedure to obtain Norton’s equivalent circuit.

Step-1: Identify the load element, remove the load element, name the load terminals and short the load terminals.

Step-2: Find the short circuit current using any network analysis technique.

Step-3: Find the equivalent resistance/Impedance between the open circuited terminals by replacing all independent sources by their internal impedances. (NOTE: $R_{TH}=R_N$)

Step-4: Replace the given circuit between the open circuited load terminals by the Norton’s equivalent circuit.

Step-5: Connect the load element between the load terminals and find the required load quantity using current division or voltage division formula.

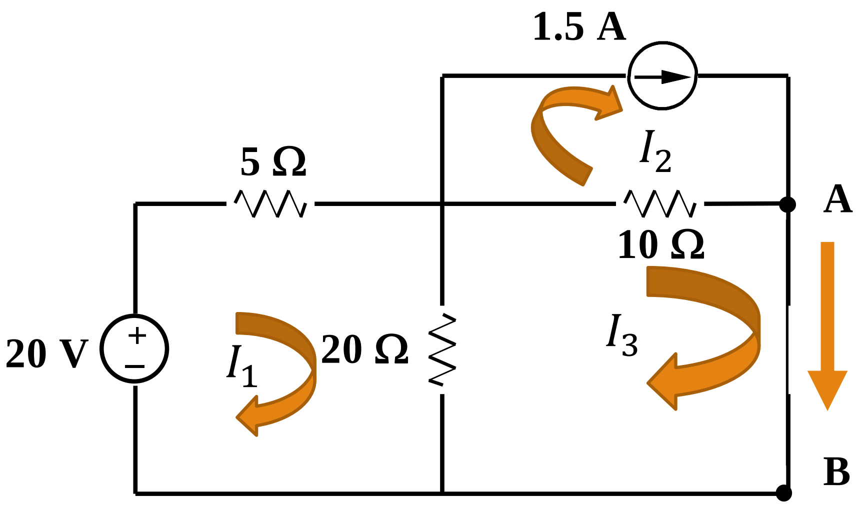
NOTE:

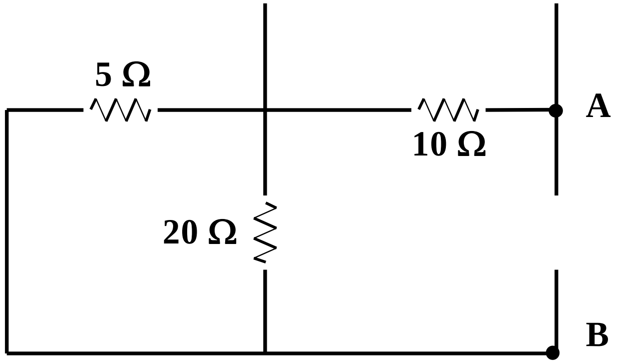
- **For the circuits with dependent sources, find R_N using the ratio=V_{OC}/I_{SC}; $R_{TH}=R_N$**
- **Thevenin’s Theorem is the dual of Norton’s Theorem**
- **Thevenin’s equivalent circuit can be converted into Norton’s Equivalent circuit and vice-versa using source transformation**

Norton's Theorem

Example:

2. Obtain the Norton's equivalent circuit between the terminals A and B.





To find I_N

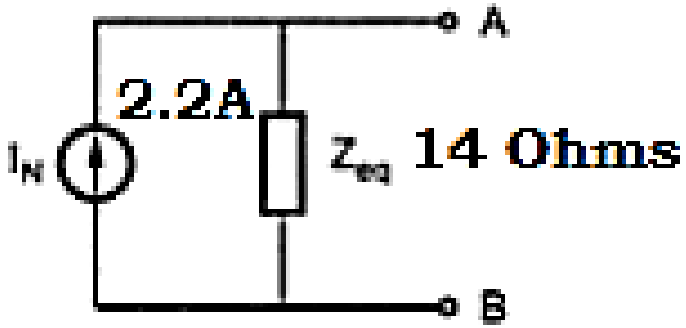
$$I_N = I_{AB} = I_{SC} = I_3$$

$$25I_1 - 20I_3 = 20 ---(1)$$
$$-20I_1 - 10I_2 + 30I_3 = 0 ---(2)$$
$$I_2 = 1.5A ---(3)$$
$$\mathbf{I_3 = 2.2.A}$$

$$\mathbf{I_N = I_{SC} = I_3 = 2.2A}$$

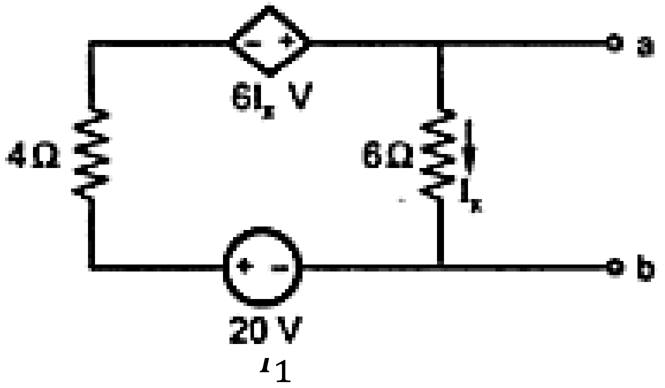
To find R_N

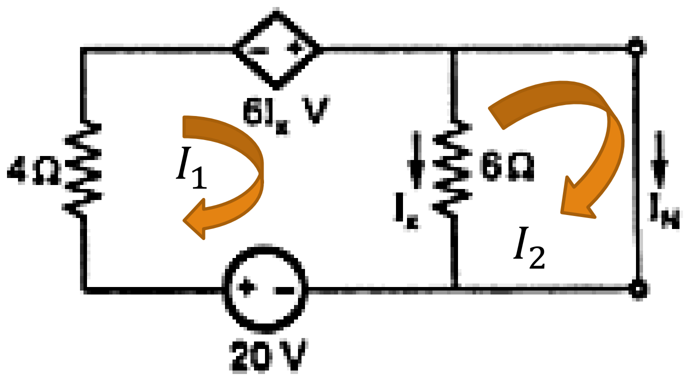
$$R_{AB} = R_{TH} = R_N = (5||20) + 10$$
$$\mathbf{R_N = 14\Omega}$$

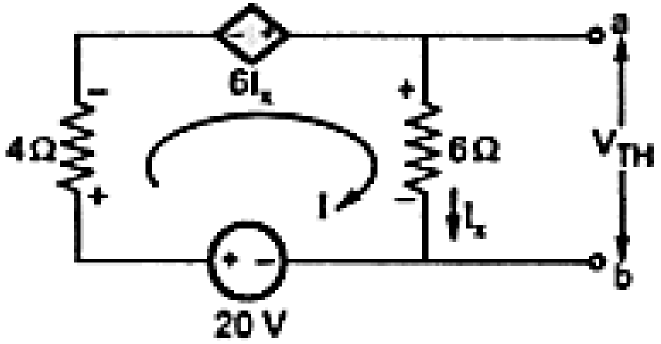


Norton's Theorem

3. Obtain the Thevenin's and Norton's equivalent circuit between the terminals A and B.







To find I_N

$$4I_1 - 6I_x + 6(I_1 - I_2) = 20 ---(1)$$
$$4I_1 = 20; I_1 = 5A.$$
$$-6I_1 + 6I_2 = 0 ---(2)$$
$$I_1 = I_2$$
$$I_x = I_1 - I_2$$

From (1), $\boldsymbol{I_1 = I_2 = 5A}$

$$\boldsymbol{I_{sc} = I_N = I_2 = 5A.}$$

To find R_N

$$R_N = \frac{V_{OC}}{I_{SC}}$$

To find V_{OC}

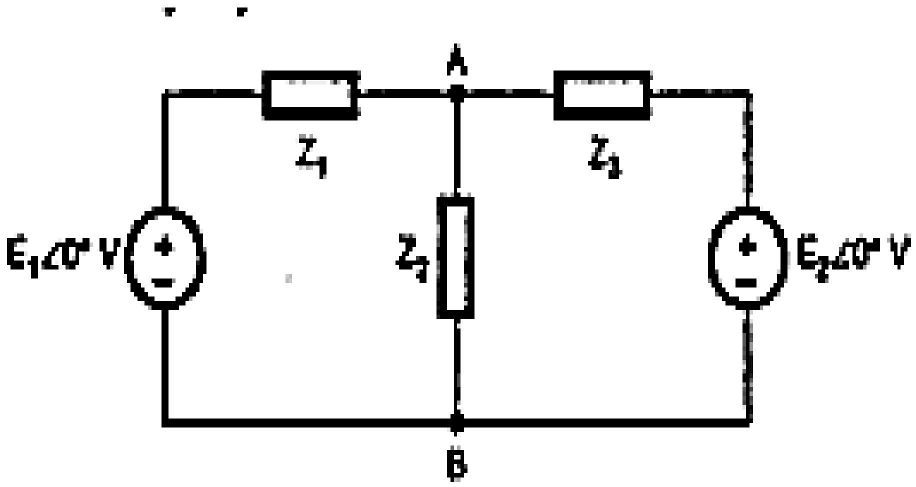
$$4I - 6I_x + 6I - 20 = 0$$
$$I_x = I$$
$$\boldsymbol{I = 5A.}$$
$$\boldsymbol{V_{OC} = 6 * 5 = 30V.}$$

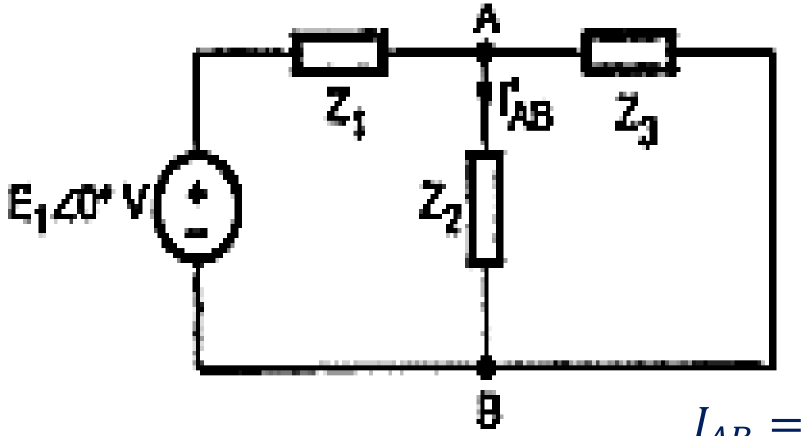
$$\boldsymbol{R_N = \frac{30}{5} = 6\ Ohms}$$

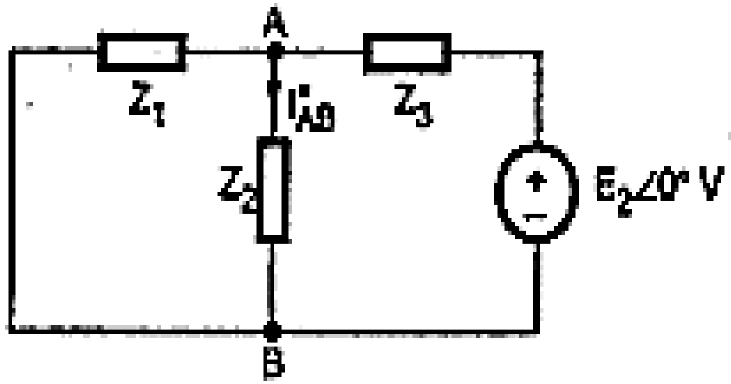
Superposition Theorem

3. Superposition Theorem

Statement: Any active linear complex **multisource** electrical network, the voltage across or current through any given element is equal to the algebraic sum of individual voltages or currents, produced independently across or in that element by each source acting independently, with all the remaining sources are replaced by their respective internal impedances.







$$I_{AB} = I'_{AB} + I''_{AB} \text{ and } V_{AB} = V'_{AB} + V''_{AB}$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

NOTE: Superposition theorem not applicable to find the power

$$P(I^2R) \neq P(I_1^2R) + P(I_2^2R)$$

$$P(I^2R) = P\left((I_1 + I_2)^2R\right) = P(I_1^2R + I_2^2R + 2I_1I_2R) \neq P(I_1^2R) + P(I_2^2R)$$

Superposition Theorem

Procedure to apply superposition theorem.

Step-1: Identify the load element and load quantity (either current or voltage)

Step-2: Consider only one source and set remaining sources equal to zero (replaced by their internal impedance) **(NOTE: Internal Impedance of the voltage source is zero, hence replaced by short circuit. Internal Impedance of the current source is infinity, hence replaced by open circuit.)**

Step-3: Find the required quantity and denote that quantity as I' or V' (for other source I'' or V'' and so on.)

Step-4: Repeat the steps 2 and 3 for all the sources.

Step-5: Find the resultant(Total) output using the following relation.

$$I_{AB} = I'_{AB} + I''_{AB} + \cdots.$$

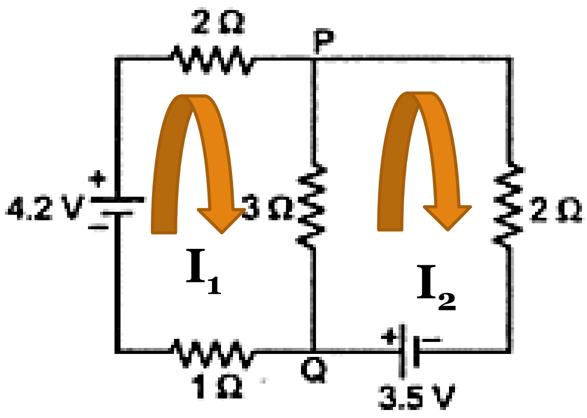
and

$$\mathrm{V_{AB}} = \mathrm{V'_{AB}} + \mathrm{V''_{AB}} + \cdots.$$

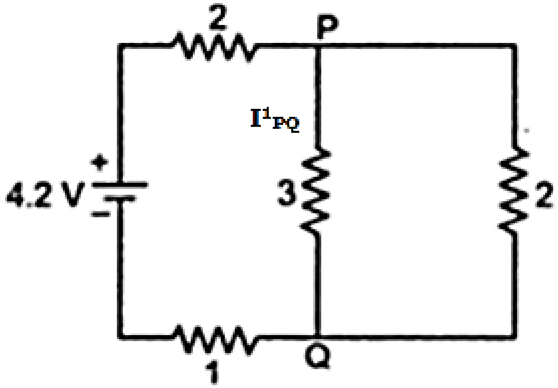
Superposition Theorem

Example:

4. Find the current through the branch PQ using superposition theorem.



Case (1): active 4.2V source and set 3.5V to zero.

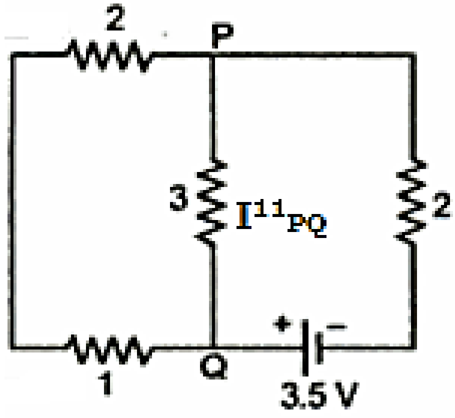


$$I'_{PQ} = \frac{I_T(R_2)}{R_1 + R_2}$$

$$I'_{PQ} = (4.2)/(3 + 3||2)\left(\frac{2}{3+2}\right)$$

$$I'_{PQ} = 0.4A$$

Case (2): active 3.5V source and set 4.2V to zero.



$$I''_{PQ} = \frac{-I_T(R_2)}{R_1 + R_2}$$

$$I''_{PQ} = -(3.5)/(2 + 3||3)\ \frac{(3)}{3+3}$$

$$I''_{PQ} = -0.5A$$

Cross Verification:

$$6I_1 - 3I_2 = 4.2 --- (1)$$

$$-3I_1 + 5I_2 = 3.5 --- (2)$$

$$I_1 = 1.5A$$

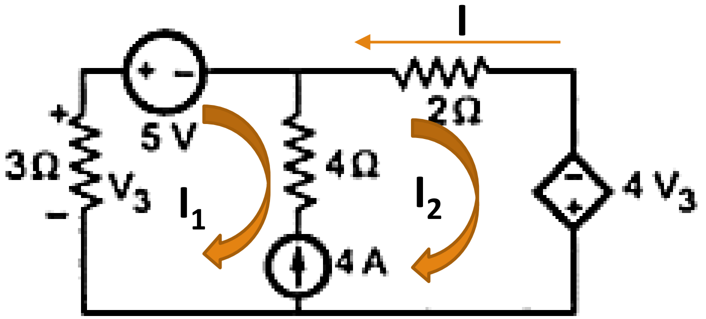
$$I_2 = 1.6A$$

$$I_{PQ} = I_1 - I_2 \Rightarrow -0.1A.$$

Resultant current

$$I_{PQ} = I'_{PQ} + I''_{PQ} = (0.4 - 0.5) \Rightarrow -0.1A.$$

Find the current through 2 Ohms resistor using superposition theorem and also verify the same.



$$I_2 - I_1 = 4 ---(1)$$

KVL at supermesh

$$5 + 2I_2 - 4V_3 + 3I_1 = 0 ---(3)$$

$$V_3 = -3I_1 ---(3)$$

$$5 + 2I_2 + 12I_1 + 3I_1 = 0$$

$$15I_1 + 2I_2 = -5 ---(4)$$

Solve (1) and (4) we get.

$$I_2 = 3.23A.$$

From the circuit.,

$$\boldsymbol{I = -I_2 = -3.23A}$$

Case(1): Activate 5V source and set 4A source to zero

Control variable, V_3

$$V_3 = 3I' ---(1)$$

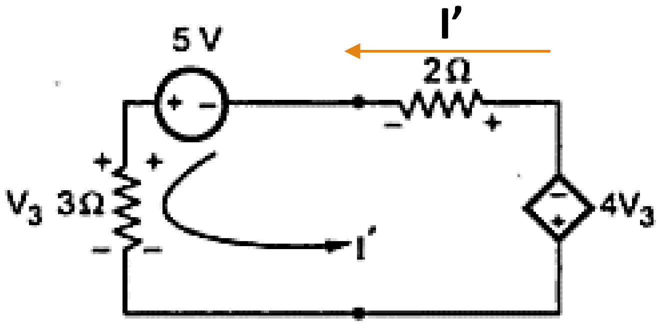
KVL equation

$$3I' + 4V_3 + 2I' - 5 = 0$$

$$3I' + 4(3I') + 2I' - 5 = 0$$

$$17I' = 5$$

$$\boldsymbol{I' = 0.2941A}$$



Case(2): Activate 4A source and deactivate 5V source.

Control variable is V_3

$$V_3 = 3(-I_1) \Rightarrow -3I_1 ---(2)$$

4A, source is common to mesh 1 and 2

$$I_2 - I_1 = 4 ---(3)$$

Super mesh KVL equation

$$2I_2 - 4V_3 + 3I_1 = 0$$

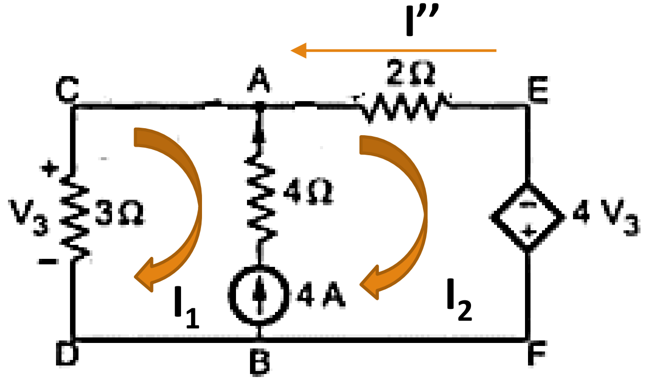
$$2I_2 - 4(-3I_1) + 3I_1 = 0$$

$$15I_1 + 2I_2 = 0 --(4)$$

$$I_1 = -0.47A$$

$$I_2 = 3.53A.$$

$$\boldsymbol{I'' = -I_2 = -3.53A}.$$



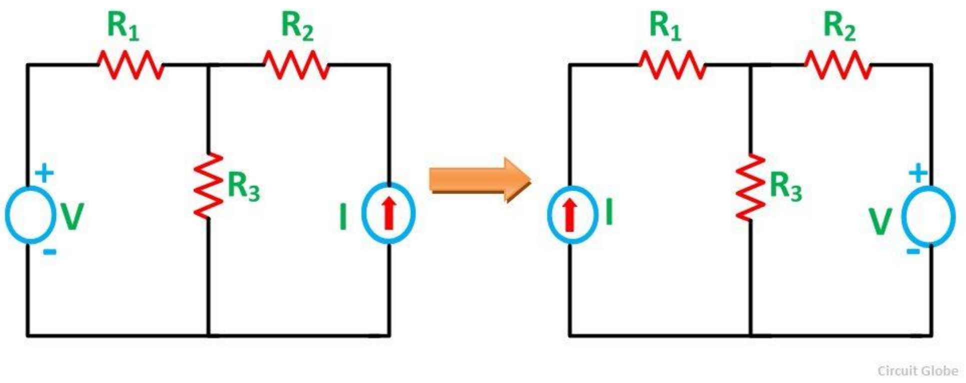
According to the superposition theorem, I=I'+I''=>-3.23A

Reciprocity Theorem

4. Reciprocity Theorem

Statement: Any active linear **single source** electrical network, the ratio of response to excitation remains same even after interchanging their position.

If the response is voltage, excitation is current and vice-versa.

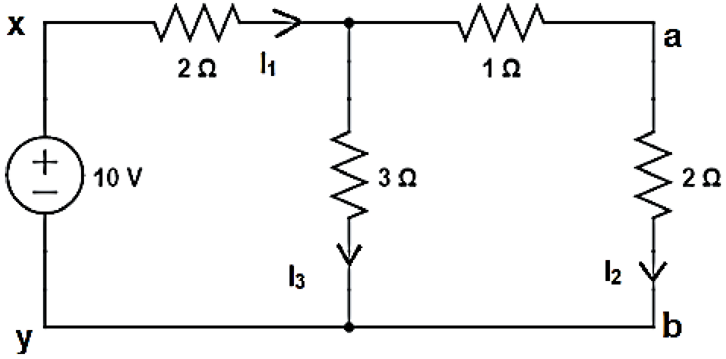


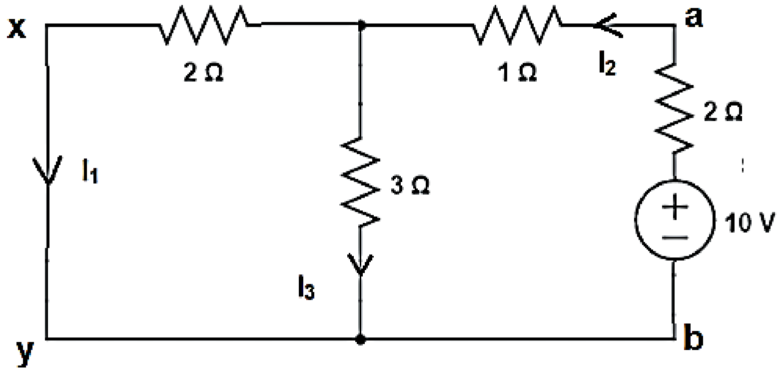
Procedure:

Step 1 – Firstly, select the branches between which reciprocity has to be established.
Step 2 – The current in the branch is obtained using any conventional network analysis method.
Step 3 – The voltage source is interchanged between the branch which is selected.
Step 4 – The current in the branch where the voltage source was existing earlier is calculated.
Step 5 – Now, it is seen that the current obtained in the previous connection, i.e., in step 2 and the current which is calculated when the source is interchanged, i.e., in step 4 are identical to each other.

Reciprocity Theorem

5. Verify the reciprocity theorem for the electrical circuit shown in figure by finding I_2.





$$I_2 = I_T \frac{3}{3+3}$$

$$I_2 = 10/(2+3||3)(\frac{3}{3+3})$$

$$I_2 = 1.43A$$

Apply Reciprocity theorem,

$$I_1 = I_T \left(\frac{3}{2+3}\right)$$

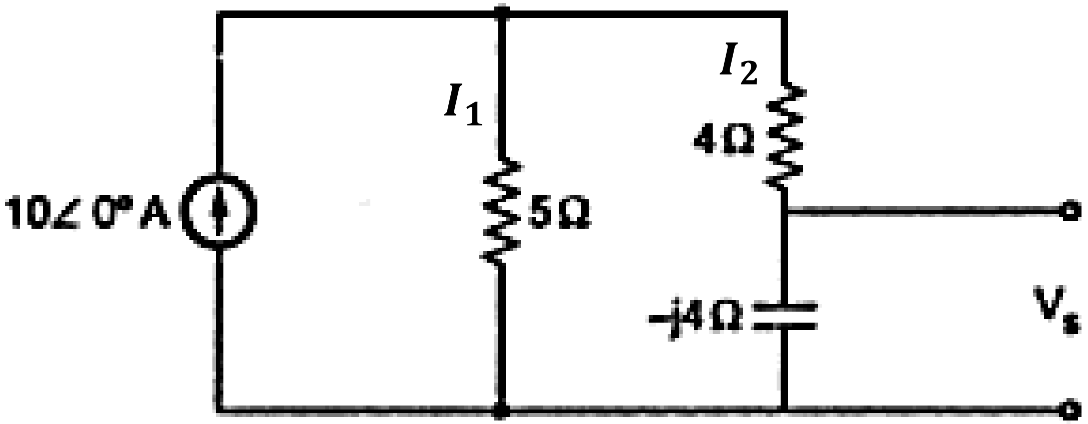
$$I_1 = 10/(3+2||3)(\frac{3}{5})$$

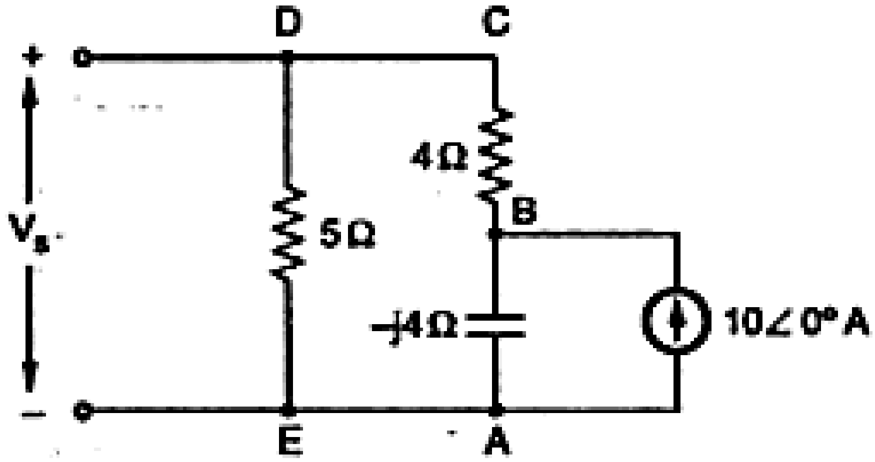
$$I_1 = 1.43A$$

If I1 and I2 are equal, then the given system is reciprocal in nature.

Reciprocity Theorem

6. Verify the reciprocity theorem for the electrical circuit shown in figure by finding V_S.





Current division formula

$$i_2 = 10\angle 0^0 \frac{(5)}{5+4-j4}$$

$$V_s = I_2 * (-j4)$$

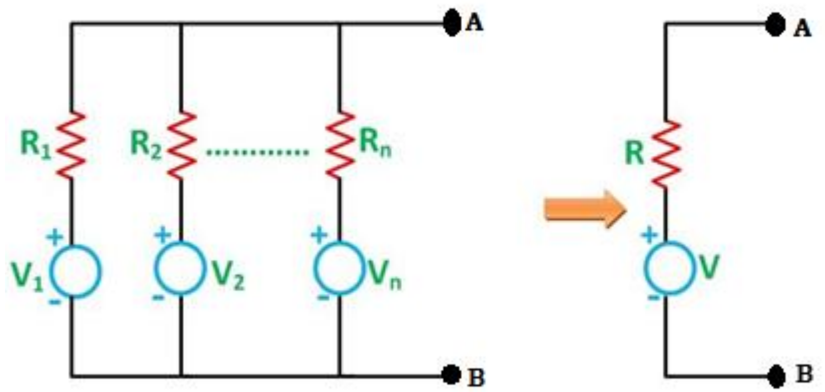
Millman's Theorem

5. Millman's Theorem

Statement: Any active linear electrical network of two or more practical voltages sources are connected between the two terminals can be replaced by a single practical voltage source between the same two terminals.

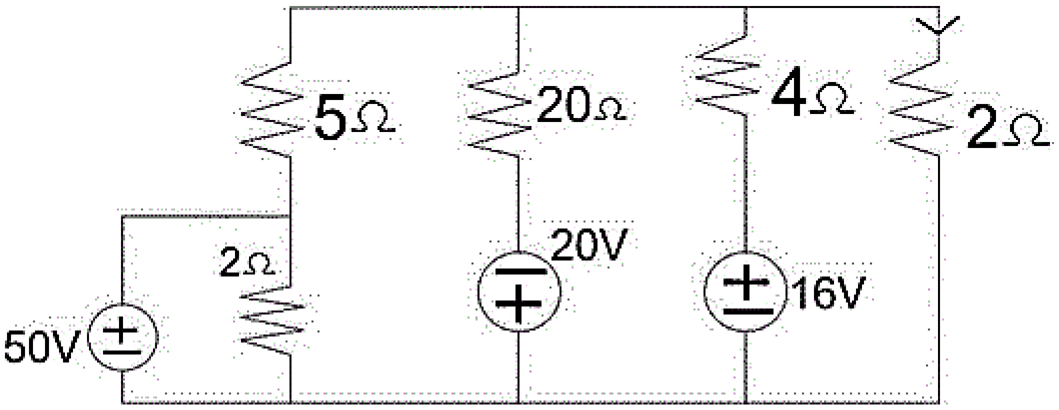
Resultant practical voltage source consisting of an ideal voltage source of **V** volts, connected in series with the single resistor/impedance of **R** Ohms.

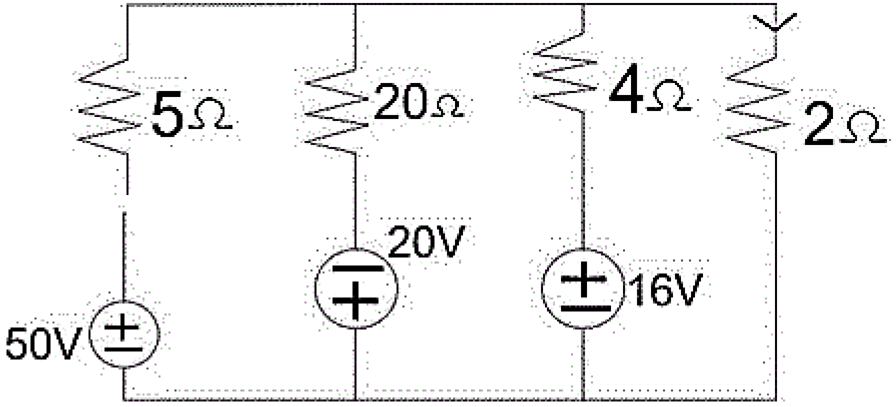
Where, $\boldsymbol{V} = \sum_{i=1}^{n} \frac{V_i G_i}{G_i}$ **and** $\boldsymbol{R} = \sum_{i=1}^{n} \frac{1}{G_i}$ $(G_i = \frac{1}{R_i}$ *and n is the number of ptactical voltage sources*$)$



Millman's Theorem-Example

7. Find the current through 2 Ohms resistor using Millman's Theorem





$$V = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}; R = \frac{1}{G_! + G_2 + G_3}$$

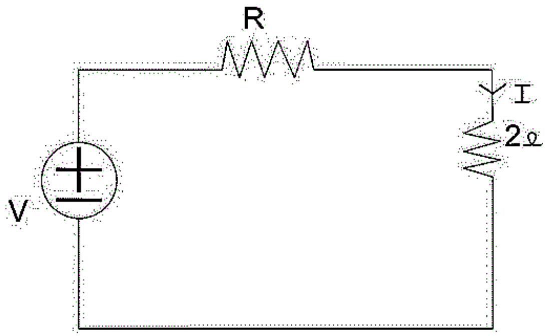
$$V_1 = 50V, V_2 = -20V \text{ and } V_3 = 16V$$

$$R_1 = 5\Omega, R_2 20\Omega \text{ and } R_3 = 4\ \Omega$$

$$\therefore G_1 = \frac{1}{R_1} \Rightarrow 0.2\ S, G_2 = \frac{1}{R_2} \Rightarrow 0.05\ S \text{ and } G_3 = \frac{1}{R_3} \Rightarrow 0.25\ S$$

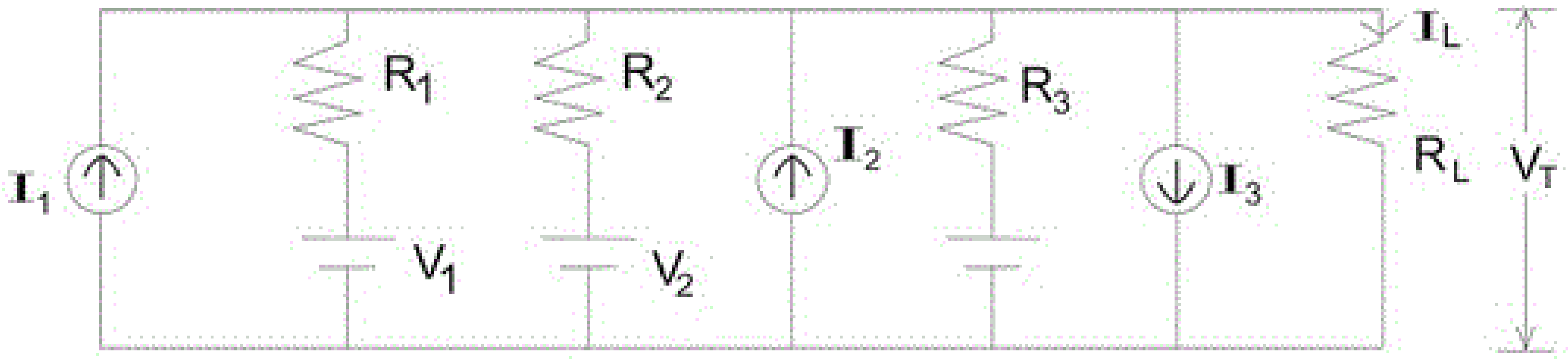
$V = 26\ Volts$ and $R = 2\Omega$

Millman's Equivalent circuit



$$I = \frac{26}{2+2}$$

$$I = 6.5\ A$$



V1=10, V2=5V, V3=15V, I1=2A, I2=3A, I3=4A

Maximum Power Transfer Theorem

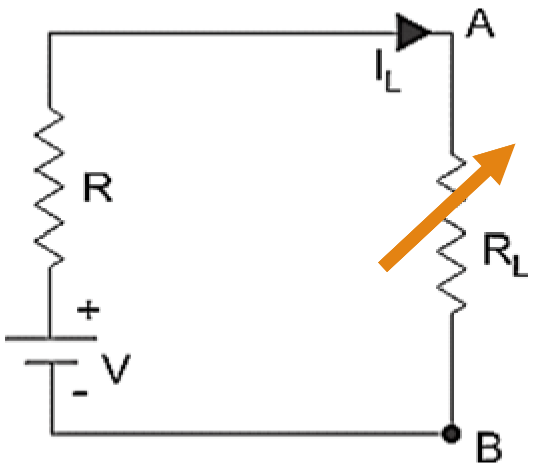
6. Maximum Power Transfer Theorem

- As the name implies, it evaluates the condition(Resistance or Impedance) to be satisfied to transfer maximum power from source to the load.
- Defined under different cases, depending on the type of circuit and load.
- Case(1): DC network - *Resistive Load* - **Variable Resistive Load**
- Case(2): AC network - *Resistive Load* - **Variable Resistive Load**
- Case(3): AC Network - *Complex Load* - **Variable Resistive and Fixed Reactance Load**
- Case(4): AC Network - *Complex Load* - **Fixed Resistive and Variable Reactance Load**
- Case(5): AC Network - *Complex Load* - **Both Resistive and Reactance are variable**

Maximum Power Transfer Theorem

Case(1): DC network - *Resistive Load* - Variable Resistive Load

Consider a DC electrical circuit with variable resistive load shown in figure. Where, R is the network resistance, R_L is the load resistance and V is the applied voltage.



Power delivered to the load resistor is given by

$$P = I_L^2 R_L \text{ Watts} ---(1)$$

From the circuit,

$$I_L = \frac{V}{R+R_L} ---(2); \therefore P = \frac{V^2 R_L}{(R+R_L)^2} ---(3)$$

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dR_L} = 0$$

Differentiate equation (3) w.r.t RL and find RL

$$\therefore \mathbf{R_L = R}$$

Statement: In any DC electrical network with variable Resistive load, the maximum power will be transferred from source to the load if the load resistance is equal to the network resistance.

$$\mathbf{i.e., R_L = R.}$$

$$P = \frac{V^2 R_L}{(R + R_L)^2}$$

$$\frac{dp}{dR_L} = \frac{(R + R_L)^2 . V^2 . 1 . -V^2 R_L . 2(R + R_L) . 1}{Dr^2} = 0$$

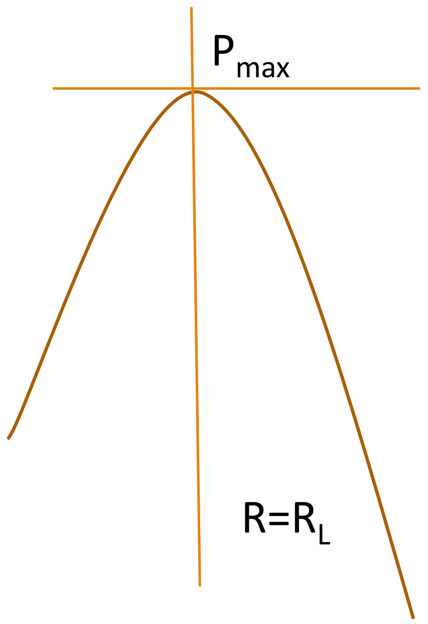
$$(R + R_L)^2 . V^2 - V^2 R_L 2(R + R_L) = 0$$

$$(R + R_L)^2 - 2RR_L - 2R_L^2 = 0$$

$$R^2 + R_L^2 + 2RR_L - 2RR_L - 2R_L^2 = 0$$

$$R^2 - R_L^2 = 0$$

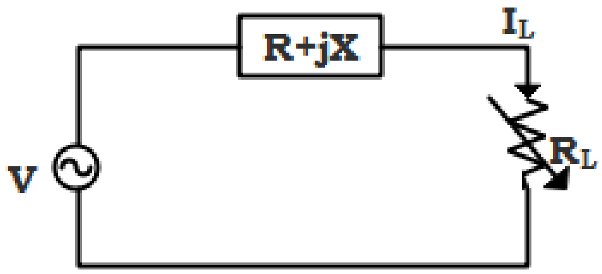
$$\boldsymbol{R = R_L}$$



Maximum Power Transfer Theorem

Case(2): AC network - *Resistive Load* - Variable Resistive Load

Consider an AC electrical circuit with variable resistive load shown in figure. Where, R+jX is the network Impedance, R_L is the load resistance and V is the applied voltage.



Power delivered to the load resistor is given by

$$P = I_L^2 R_L \text{ Watts} ---(1)$$

From the circuit,

$$I_L = \frac{V}{R+jX+R_L} \Rightarrow \frac{V}{\sqrt{(R+R_L)^2+X^2}} ---(2); \therefore P = \frac{V^2 R_L}{(R+R_L)^2+X^2} ---(3)$$

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dR_L} = 0$$

Differentiate equation (3) w.r.t RL and find RL

$$\therefore \mathbf{R_L} = \sqrt{R^2 + X^2}$$

Statement: In any AC electrical network with variable Resistive load, the maximum power will be transferred from source to the load if the load resistance is equal to the magnitude of the network Impedance. $i.e., \mathbf{R_L} = \sqrt{R^2 + X^2}$.

$$P = \frac{V^2 R_L}{(R + R_L)^2 + X^2}$$

$$(R + R_L)^2 + X^2 . V^2 . 1 - V^2 R_L . 2(R + R_L) . 1 . = 0$$

$$(R + R_L)^2 + X^2 - 2RR_L - 2R_L^2 = 0$$

$$R^2 + R_L^2 + 2RR_L + X^2 - 2RR_L - 2R_L^2 = 0$$

$$R^2 - R_L^2 + X^2 = 0$$

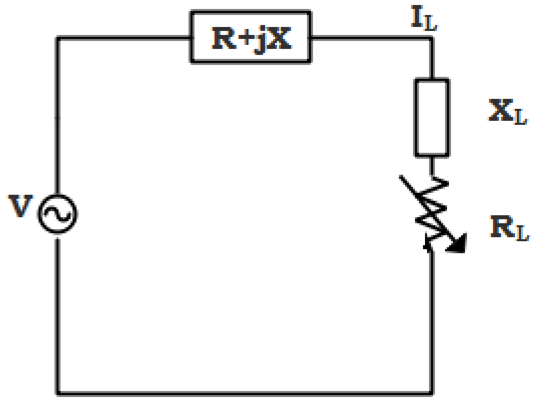
$$R_L^2 = R^2 + X^2$$

$$R_L = \sqrt{R^2 + X^2}$$

Maximum Power Transfer Theorem

Case(3): AC network – *Complex Load* – Fixed Reactance and Variable Resistive Load

Consider an AC electrical circuit with variable resistive load shown in figure. Where, R+jX is the network Impedance, R_L is the load resistance, X_L is the fixed load reactance and V is the applied voltage.



Power delivered to the load resistor is given by

$$P = I_L^2 R_L \text{ Watts} ---(1)$$

From the circuit,

$$I_L = \frac{V}{R+jX+R_L+jX_L} \Rightarrow \frac{V}{\sqrt{(R+R_L)^2+(X+X_L)^2}} ---(2); \therefore P = \frac{V^2 R_L}{(R+R_L)^2+(X+X_L)^2} ---(3)$$

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dR_L} = 0$$

Differentiate equation (3) w.r.t RL and find RL

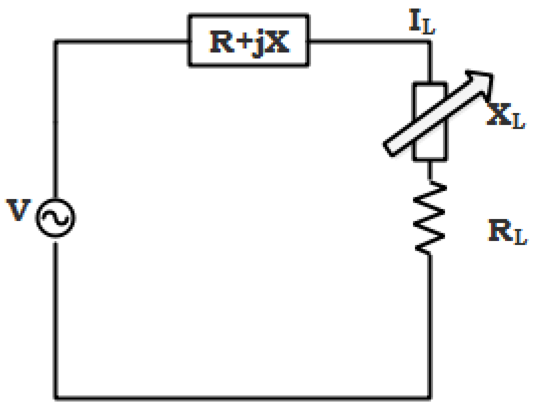
$$\therefore \mathbf{R_L} = \sqrt{\boldsymbol{R^2 + (X + X_L)^2}}$$

Statement: In any AC electrical network with fixed reactance load and variable Resistive load, the maximum power will be transferred from source to the load if the load resistance is equal to the magnitude of the network Impedance along with load reactance. $i.e., \mathbf{R_L} = \sqrt{\boldsymbol{R^2 + (X + X_L)^2}}$.

Maximum Power Transfer Theorem

Case(4): AC network – *Complex Load* – Fixed Resistive and Variable Reactance Load

Consider an AC electrical circuit with variable resistive load shown in figure. Where, R+jX is the network Impedance, R_L is the load resistance, X_L is the fixed load reactance and V is the applied voltage.



Power delivered to the load resistor is given by

$$P = I_L^2 R_L \text{ Watts} ---(1)$$

From the circuit,

$$I_L = \frac{V}{R+jX+R_L+jX_L} \Rightarrow \frac{V}{\sqrt{(R+R_L)^2+(X+X_L)^2}} ---(2); \therefore P = \frac{V^2 R_L}{(R+R_L)^2+(X+X_L)^2} ---(3)$$

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dX_L} = 0$$

Differentiate equation (3) w.r.t XL and find XL

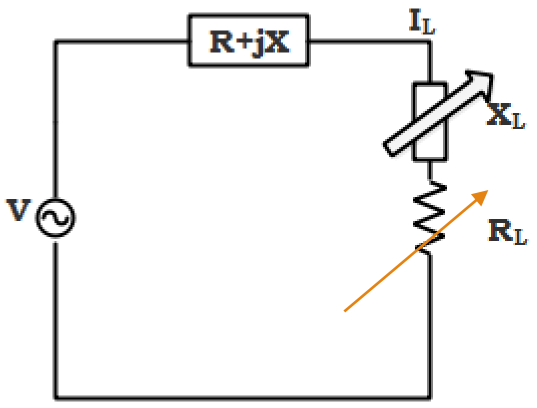
$$\therefore \boldsymbol{X_L = -X}$$

Statement: In any AC electrical network with fixed resistive load and variable reactance load, the maximum power will be transferred from source to the load if the load reactance is equal to the conjugate of the network reactance. $i.e., \boldsymbol{X_L = -X}$.

Maximum Power Transfer Theorem

Case(5): AC network – *Complex Load* – Both Resistive Reactance are variable

Consider an AC electrical circuit with variable resistive load shown in figure. Where, R+jX is the network Impedance, R_L is the load resistance, X_L is the load reactance and V is the applied voltage.



Power delivered to the load resistor is given by $P = I_L^2 R_L$ Watts ---(1)

From the circuit, $I_L = \frac{V}{R+jX+R_L+jX_L} \Rightarrow \frac{V}{\sqrt{(R+R_L)^2+(X+X_L)^2}}$ ---(2); $\therefore P = \frac{V^2 R_L}{(R+R_L)^2+(X+X_L)^2}$ ---(3)

Case(i): Consider X_L

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dX_L} = 0$$

Differentiate equation (3) w.r.t XL and find RL $\therefore \boldsymbol{X_L = -X}$

Case(i): Consider R_L

As per the maxima theorem, P is maximum when its derivative with respect to RL is equal to zero.

$$\text{i.e.,} \frac{dP_{max}}{dR_L} = 0$$

Differentiate equation (3) w.r.t RL and find RL $\therefore \mathbf{R_L} = \sqrt{\boldsymbol{R^2 + (X + X_L)^2}}; \boldsymbol{R_L = R}$

Statement: In any AC electrical network with variable resistive load and variable reactance load, the maximum power will be transferred from source to the load if the load Impedance is equal to the complex conjugate of the network Impedance. $i.e., \boldsymbol{R_L + jX_L = R - jX}$.

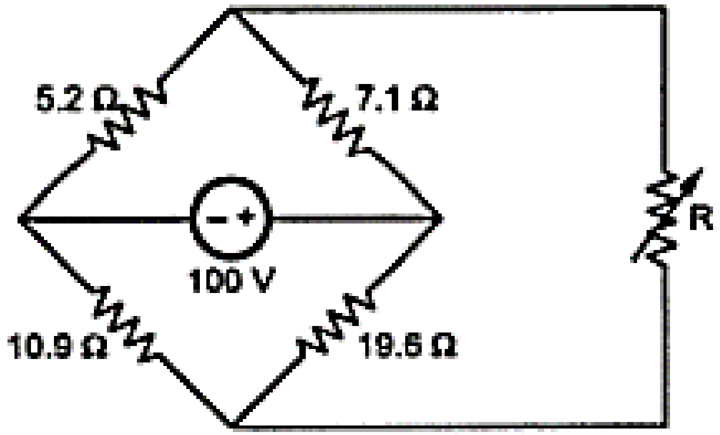
Maximum Power Transfer Theorem

Procedure to solve problems on maximum power transfer theorem

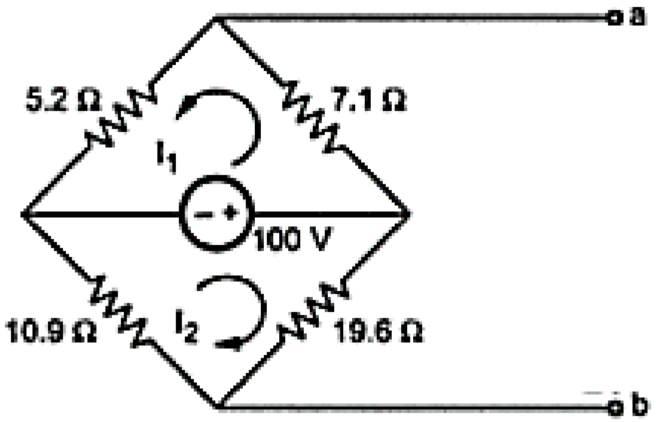
1. Identify the load element, Remove the load element and name the load terminals as A and B.
2. Obtain the Thevenin's equivalent circuit between the terminals A and B.
3. Reconnect the load element between the terminals
4. Apply the maximum power transfer theorem and find the load element required to transfer maximum power.

Maximum Power Transfer Theorem-Examples

1. Find the value of R that will receive maximum power and determine the maximum power



Remove the load element and name the load terminals.



Obtain the Thevenin's equivalent circuit between the terminals a and b.

To find V_{TH}

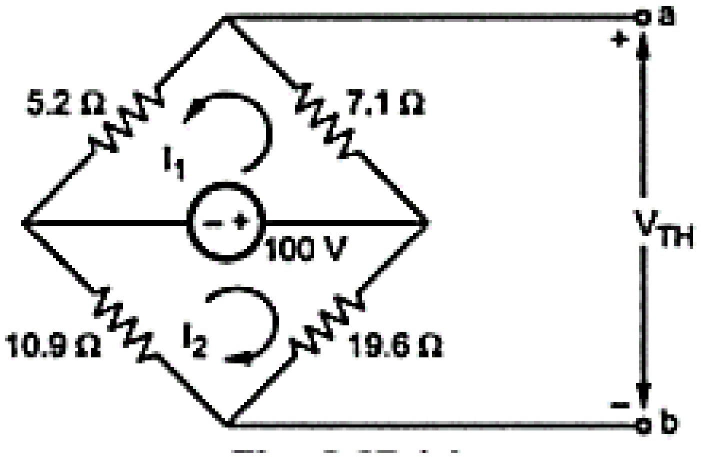
$$V_{TH} = V_{7.1} + V_{19.6}$$

$$V_{TH} = 7.1(-I_1) + 19.6(I_2)$$

KVL at Mesh-1 $12.3I_1 = 100; \boldsymbol{I_1 = 8.13A}$

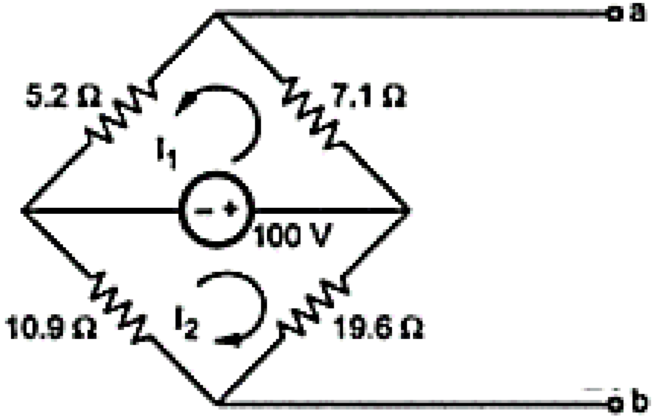
KVL at Mesh-2 $20.5I_2 = 100; \boldsymbol{I_2 = 3.2786A}$

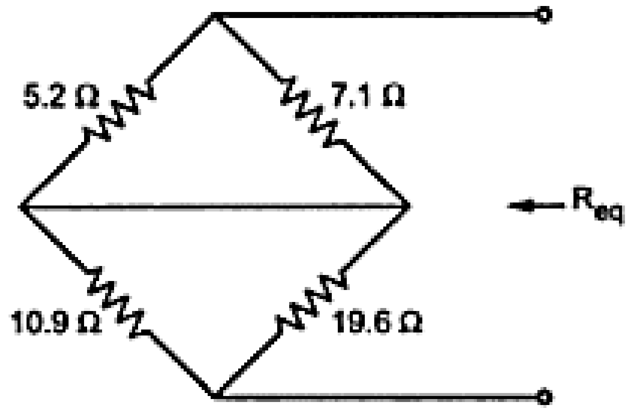
$$\boldsymbol{V_{TH} = 7.1(-I_1) + 19.6(I_2) \Rightarrow 6.5375\ Volts}$$



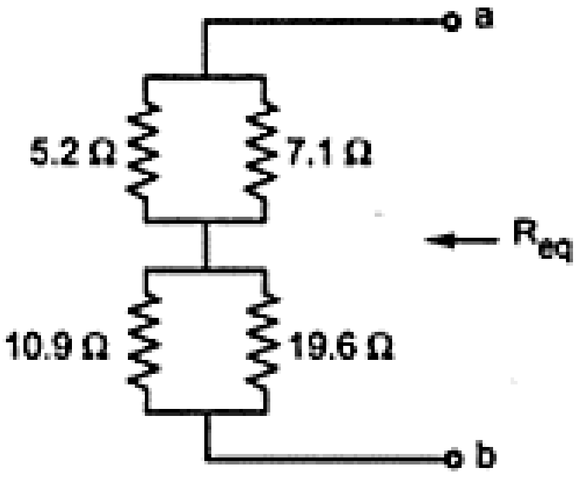
Maximum Power Transfer Theorem-Examples

To find R_{TH}



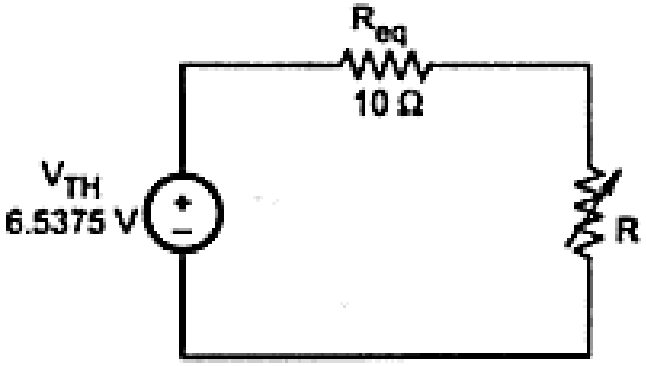


Re arranging the resistors



$$R_{TH} = (5.2||7.1) + (10.9||19.6)$$
$$R_{TH} = 10\ Ohms$$

Thevenin's Equivalent circuit

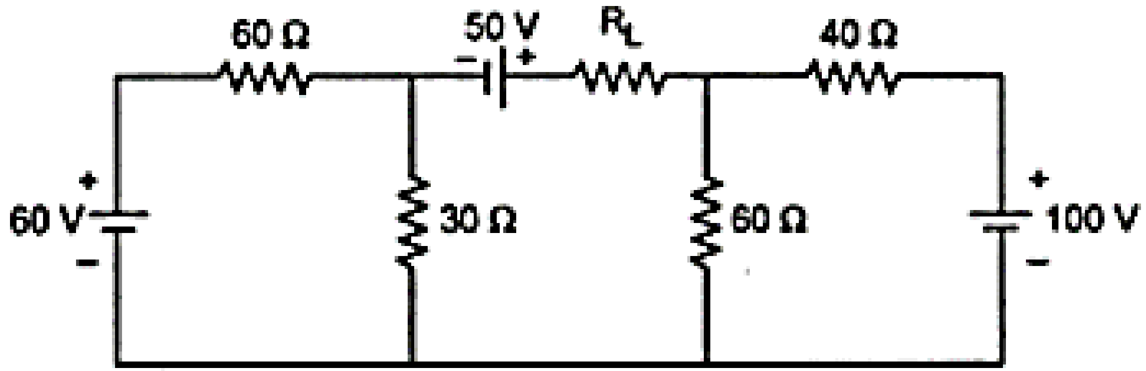


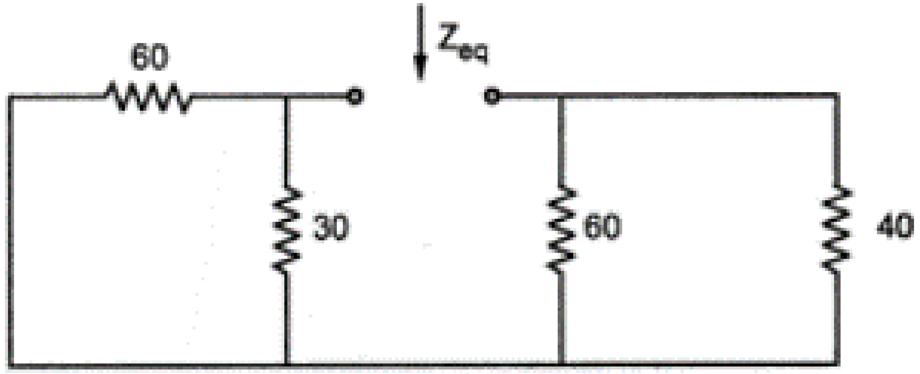
Apply MPT theorem
As per the statement, R=Req
Therefore, R=10 Ohms.
Maximum Power

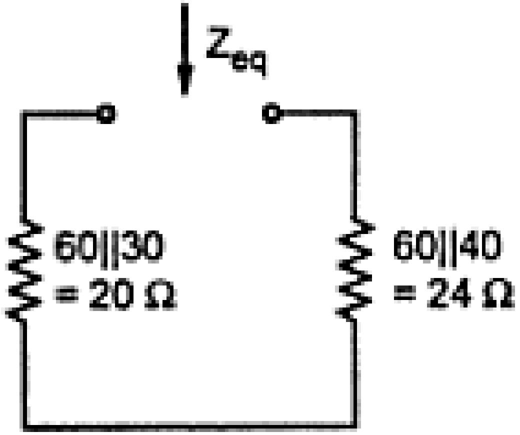
$$P_{\max} = \frac{V_{TH}^2}{4R} \Rightarrow \mathbf{1.068\,Watts}$$

Maximum Power Transfer Theorem-Examples

2. Find the value of RL that will receive maximum power and determine the maximum power





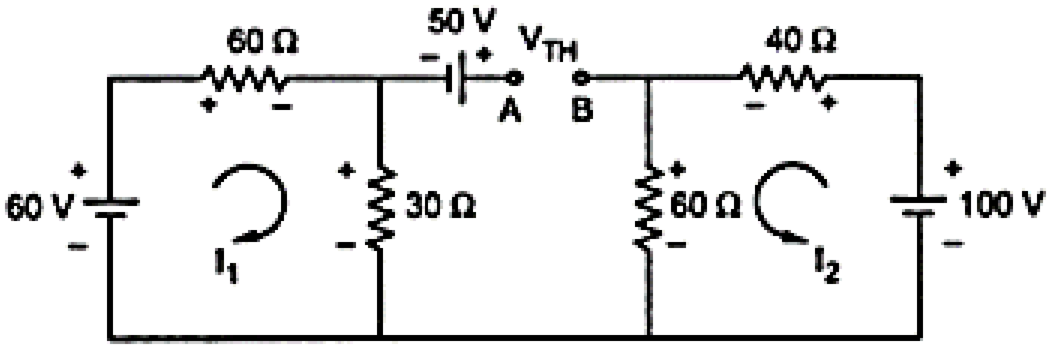


$\therefore Z_{eq} = 20 + 24 = 44\ \Omega$

$\therefore R_L = 44\ \Omega$ for P_{max}

To find P_{max}, find Thevenin's voltage $V_{TH.}$

Maximum Power Transfer Theorem-Examples



Applying KVL to the two loops,

$$-60I_1 - 30I_1 + 60 = 0$$

$$\therefore \quad I_1 = 0.6667\ A$$

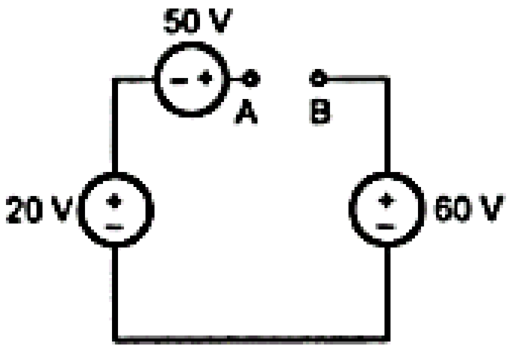
$$-40I_2 - 60I_2 + 100 = 0$$

$$\therefore \quad I_2 = 1A$$

$$\therefore \quad \text{Drop across } 30\ \Omega = 30 \times I_1 = 20\ V$$

and $\quad$ drop across $60\ \Omega = 60 \times I_2 = 60\ V$

Tracing path from A to B we get the voltages as shown in the Fig.



$$\therefore \quad V_{AB} = 50 + 20 - 60 = 10\ V$$

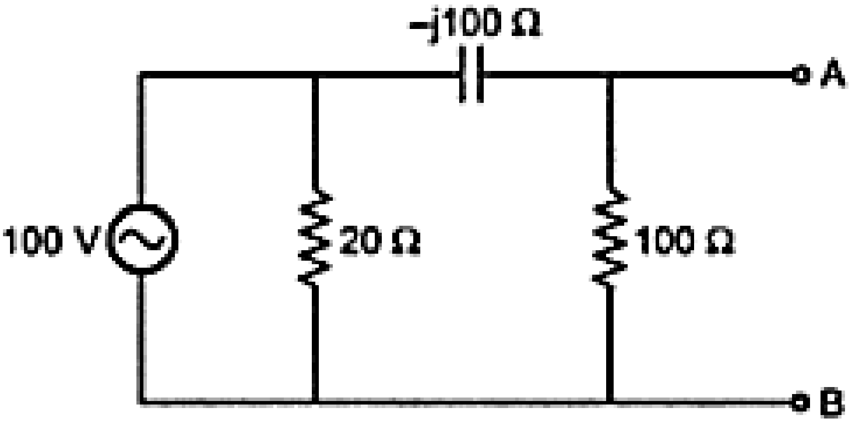
$$\therefore \quad V_{TH} = 10\ V$$

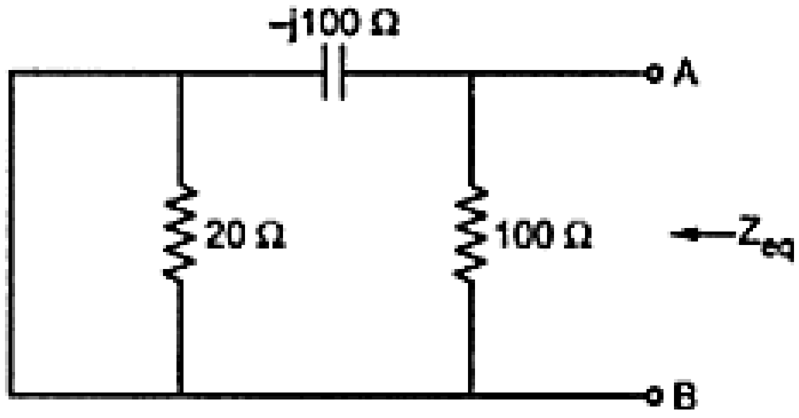
$$\therefore \quad P_{max} = \frac{(V_{TH})^2}{4R_L}$$

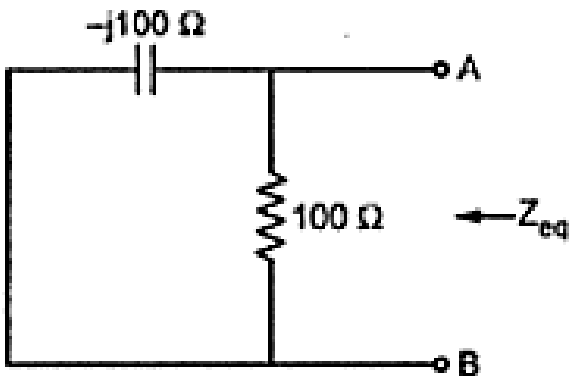
$$= \frac{(10)^2}{4 \times 44} = 0.5681\ W$$

Maximum Power Transfer Theorem-Examples

3. Find the load element and its value that will be connected between the terminals A and B for receive maximum power and determine the maximum power







As there is direct short across 20 Ω, it becomes redundant.

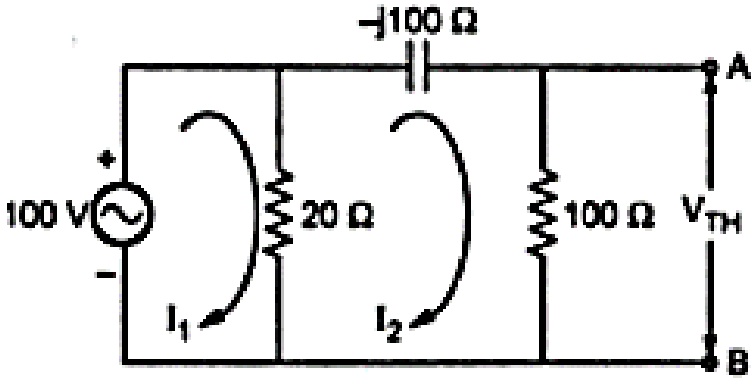
$$Z_{eq} = (100) \parallel (-j100) = \frac{100 \times -j100}{100 - j100} = \frac{10^4 \angle -90^\circ}{141.421 \angle -45^\circ}$$

$$= 70.7106 \angle -45^\circ = 50 - j50 \ \Omega$$

$$\therefore \quad Z_L = Z_{eq}^* = 50 + j\,50 \ \Omega$$

Maximum Power Transfer Theorem-Examples

To find P_{max}, find V_{TH}.



$$-20I_1 + 20I_2 + 100 = 0$$

$$\therefore \quad I_1 - I_2 = 5\ A \quad \text{... (1)}$$

$$-I_2(-j100 + 100) - 20I_2 + 20I_1 = 0$$

$$\therefore \quad 20I_1 - [120 - j100]I_2 = 0 \quad \text{... (2)}$$

Multiply equation (1) by 20 and subtract from equation (2),

$$[-100 + j\,100]I_2 = -100$$

$$\therefore \quad I_2 = \frac{-100}{-100 + j100} = \frac{+100\angle 180°}{141.421\angle 135°} = +0.7071\ \angle +45°\ A$$

$$\therefore \quad V_{TH} = I_2 \times 100 = +70.71\angle +45°\ V$$

$$\therefore \quad P_{max} = \frac{|[V_{TH}]|^2}{4\,R_L} = \frac{(70.71)^2}{4 \times 50} = 25\ W$$

Resonance

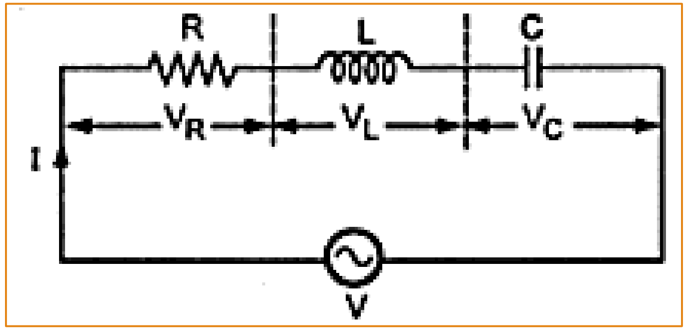
Definition:

- A phenomenon in which applied voltage and resulting current are in phase.
- An A.C. circuit is said to be resonance if it exhibits unity power factor condition- Applied voltage is in phase with the resulting current.
- **Two types**
    - Series Resonance
    - Parallel Resonance
- **Applications:**

Communication - radio receiver has ability to select the desired frequency signal, transmitted by station.(**Selection of required frequency components and rejecting the unwanted signals**).

Series Resonance

In electrical system, the basic elements Resistor, Inductor and Capacitors are connected in series and the circuit is excited by an A.C. source which undergo resonance based on certain conditions is called series resonance.



Definition:

- Applied voltage is in phase with the resulting current
- Power factor is unity
- Net imaginary part is equal to zero.
- Inductive reactance is equal to the capacitive reactance.
- Net impedance is equal to only resistive
- Resulting current is maximum and net impedance is minimum.

Realization:

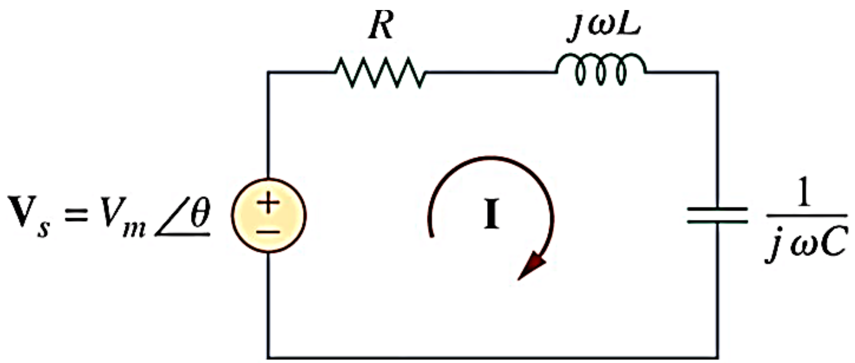
- Varying the frequency of the source with C and L are fixed - **frequency tuning**
- Varying C with L and frequency are fixed - **Capacitive tuning**
- Varying L with C and Frequency are fixed - **Inductive tuning**

Series Resonance

1. Frequency tuning

Varying the frequency to achieve, Inductive reactance and capacitive reactance. Hence net imaginary part get cancelled out and applied voltage is inphase with the resulting current.

Consider an RLC series electrical circuit.



Net Impedance, $Z_T = R + j(X_L \sim X_C) ---(1)$

$$|Z_T| = \sqrt{R^2 + (X_L \sim X_C)^2} ---(2)$$

$$I = \frac{V_s}{|Z_T|} ---(3)$$

By varying the frequency, at some particular frequency

$$\boldsymbol{X_L = X_C} ---(\mathbf{4})$$

$\therefore \boldsymbol{I_{max} = \frac{V}{R}}$ ***Amperes*** (Voltage is inphase with the current) $---(5)$

Peak power delivered at the load is

$$\boldsymbol{P_{Peak} = \frac{V^2}{R}\ Watts} ---(6)$$

Series Resonance

Important Parameters:

1. Resonant frequency:

The frequency at which inductive reactance is equal to the capacitive reactance is called resonant frequency.

$$\textbf{i.e., } X_L = X_C$$

We know that, $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

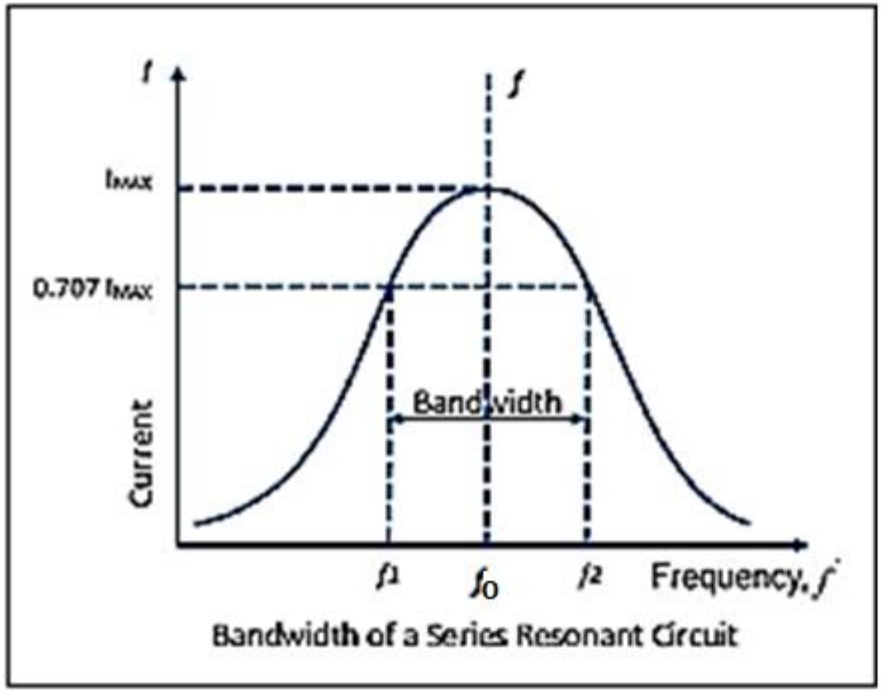
$$\therefore \omega L = \frac{1}{\omega C} --- (7)$$

$$\omega_O = \frac{1}{\sqrt{LC}} \text{ rad /sec or } f_O = \frac{1}{2\pi\sqrt{LC}} \text{ Herts} --- (8)$$

where, ω_O is the resonant frequency in radians per second and f_O is the resonant frequency in Hertz.

2. Frequency Response:

The response of magnitude of current vs frequency is the frequency response.



Bandwidth of a Series Resonant Circuit

Where, f_1 and f_2 are half power frequencies/Corner frequencies/cutoff frequencies. f_1-lower cut off frequency and f_2-upper cu off frequency,

Series Resonance

3. Bandwidth

The difference between the two half power frequencies is called Bandwidth.

i.e., $\mathbf{B.W.(\beta) = f_2 - f_1 \text{ or } \omega_2 - \omega_1} ---(9)$

4. Quality factor

The ratio of resonant frequency to the bandwidth is called quality factor.

i.e., $Q = \frac{\omega_O}{\beta} ---(10)$

Also defined as the ratio of the energy stored in the oscillating resonator to the energy dissipated per cycle.

i.e., $Q = 2\pi \frac{Energy\ Stored}{Average\ Energy\ dissipated\ per\ cycle} ---(11)$

Derivation of Q.

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{I^2}{2}Rt}$$

$$\boldsymbol{Q = \frac{\omega_O L}{R} ---(12)}$$

Or

$$Q = 2\pi \frac{\frac{1}{2}CV^2}{\frac{I^2}{2}Rt}$$

$$Q = 2\pi \frac{\frac{1}{2}C\left(\frac{I}{\omega C}\right)^2}{\frac{I^2}{2}Rt}$$

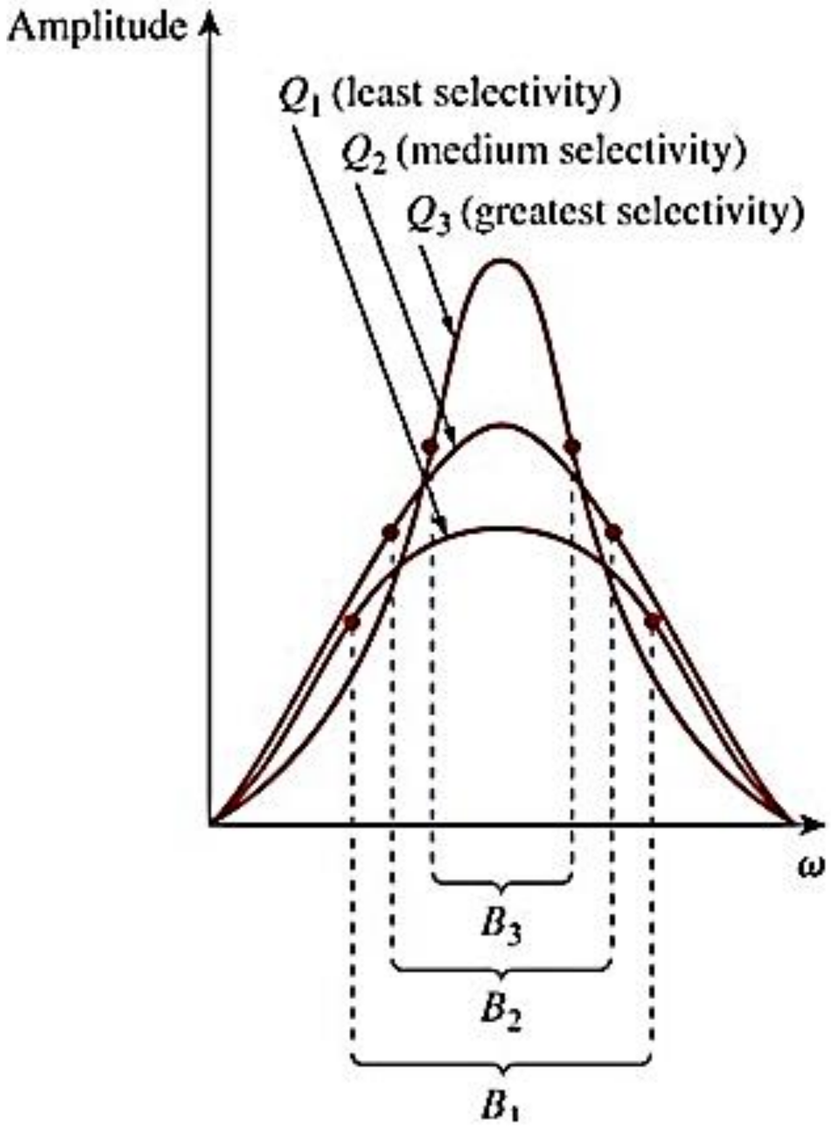
$$\boldsymbol{Q = \frac{1}{\omega_O CR} ---(13)}$$

or

$$\boldsymbol{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Series Resonance

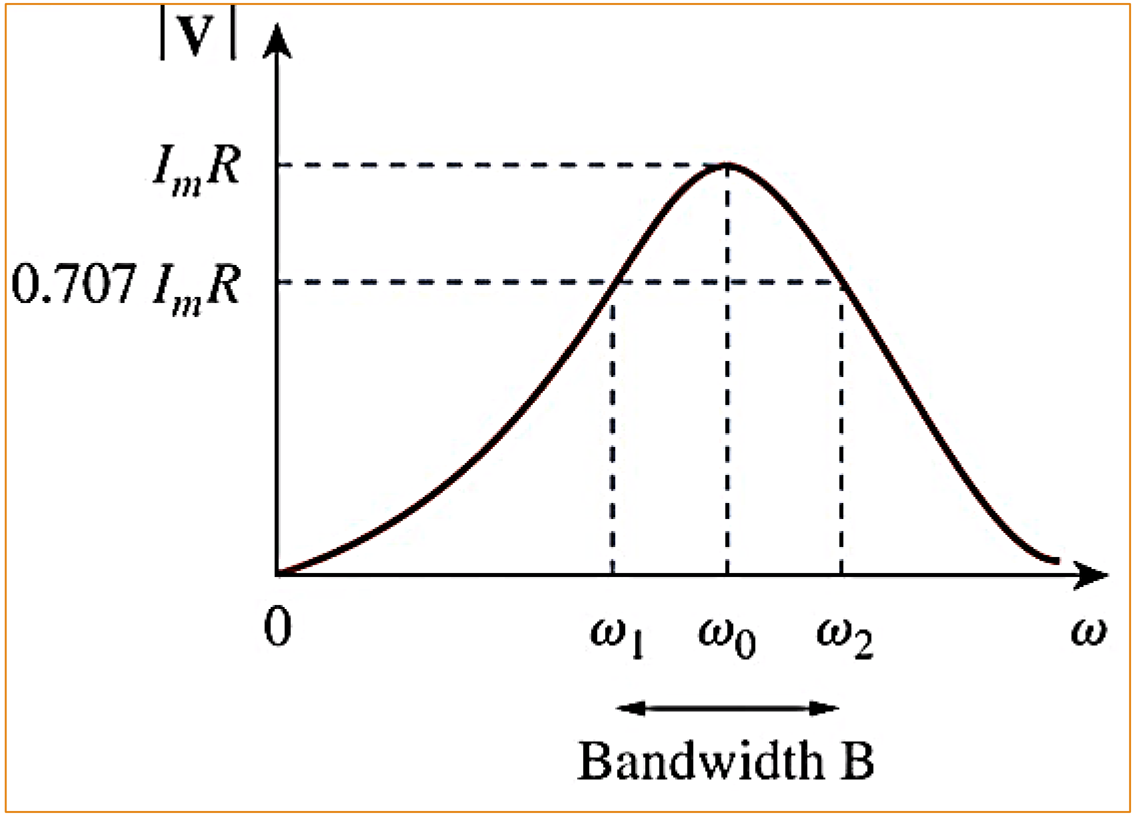
- Sharpness of the response could be measured by Quality factor.
- If Q>10 (called as high Q circuit)
- If Q≤10 (Called low Q circuit)
- Higher the Q, lower the Bandwidth and higher the selectivity.
- lower the Q, higher the bandwidth and higher the selectivity.



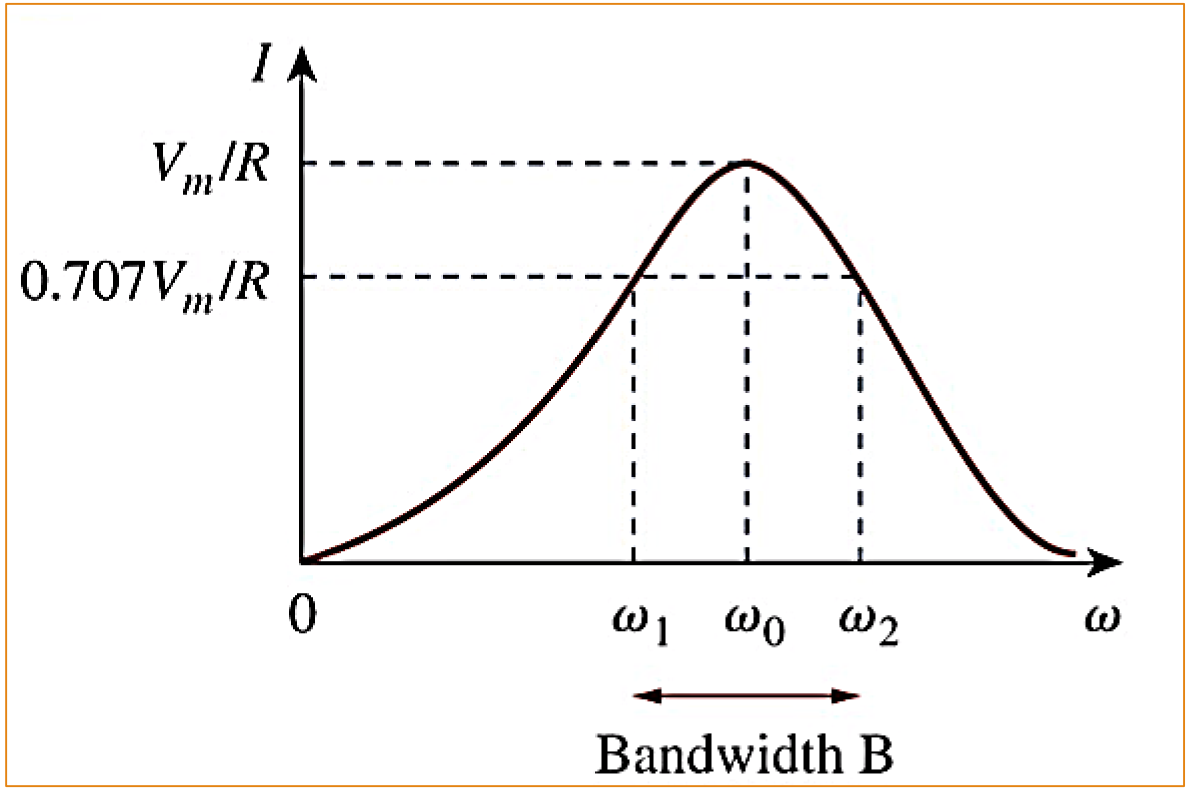
Series Resonance

Properties of Series Resonance:

1. Voltage Response curve

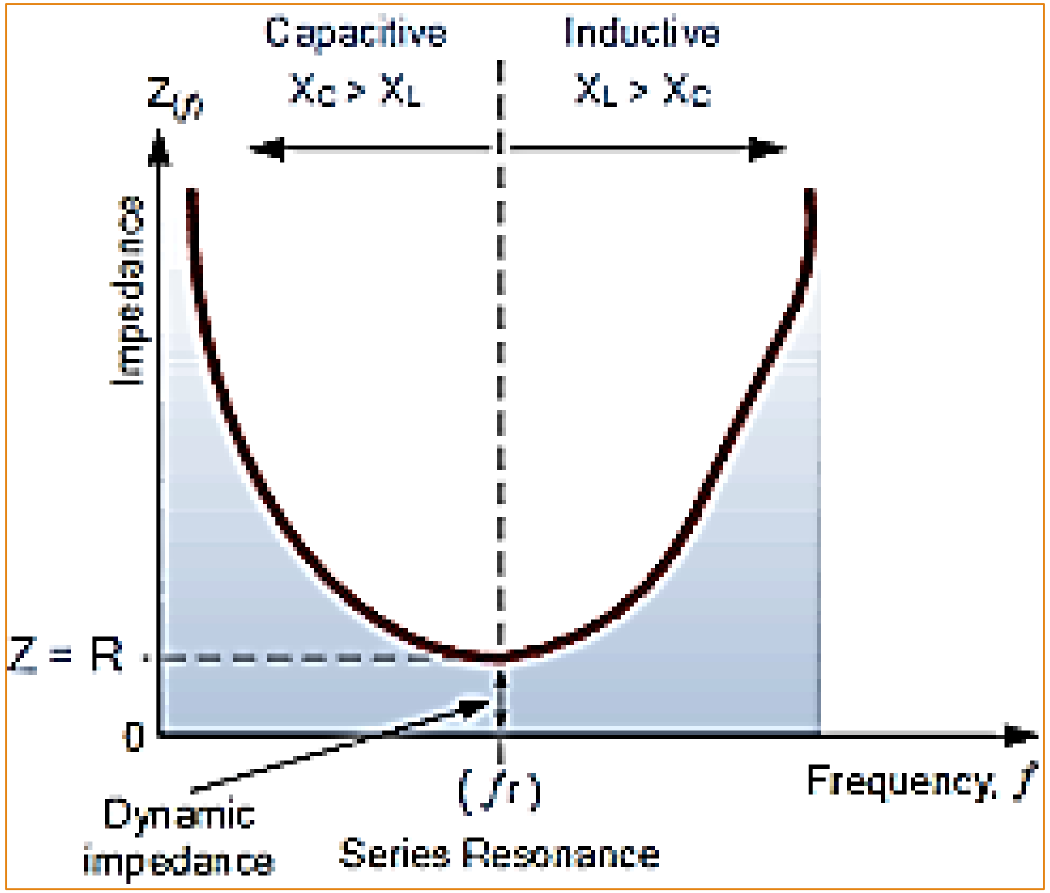


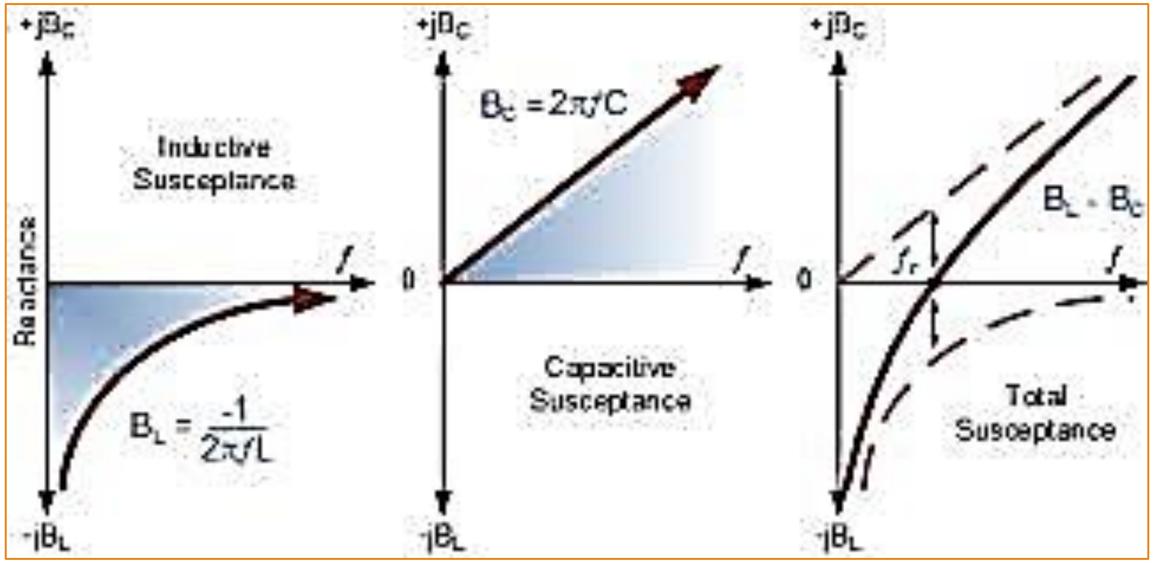
2. Current Response curve

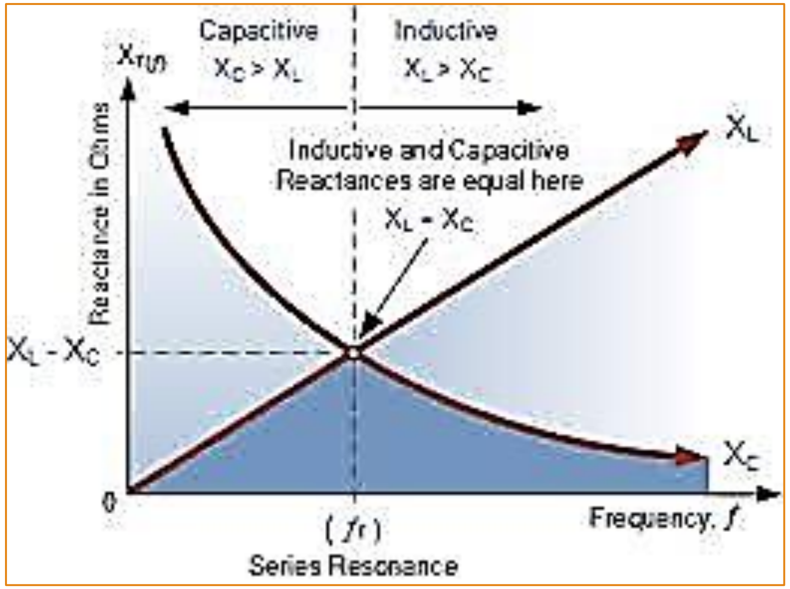


Series Resonance

3. Impedance Curve

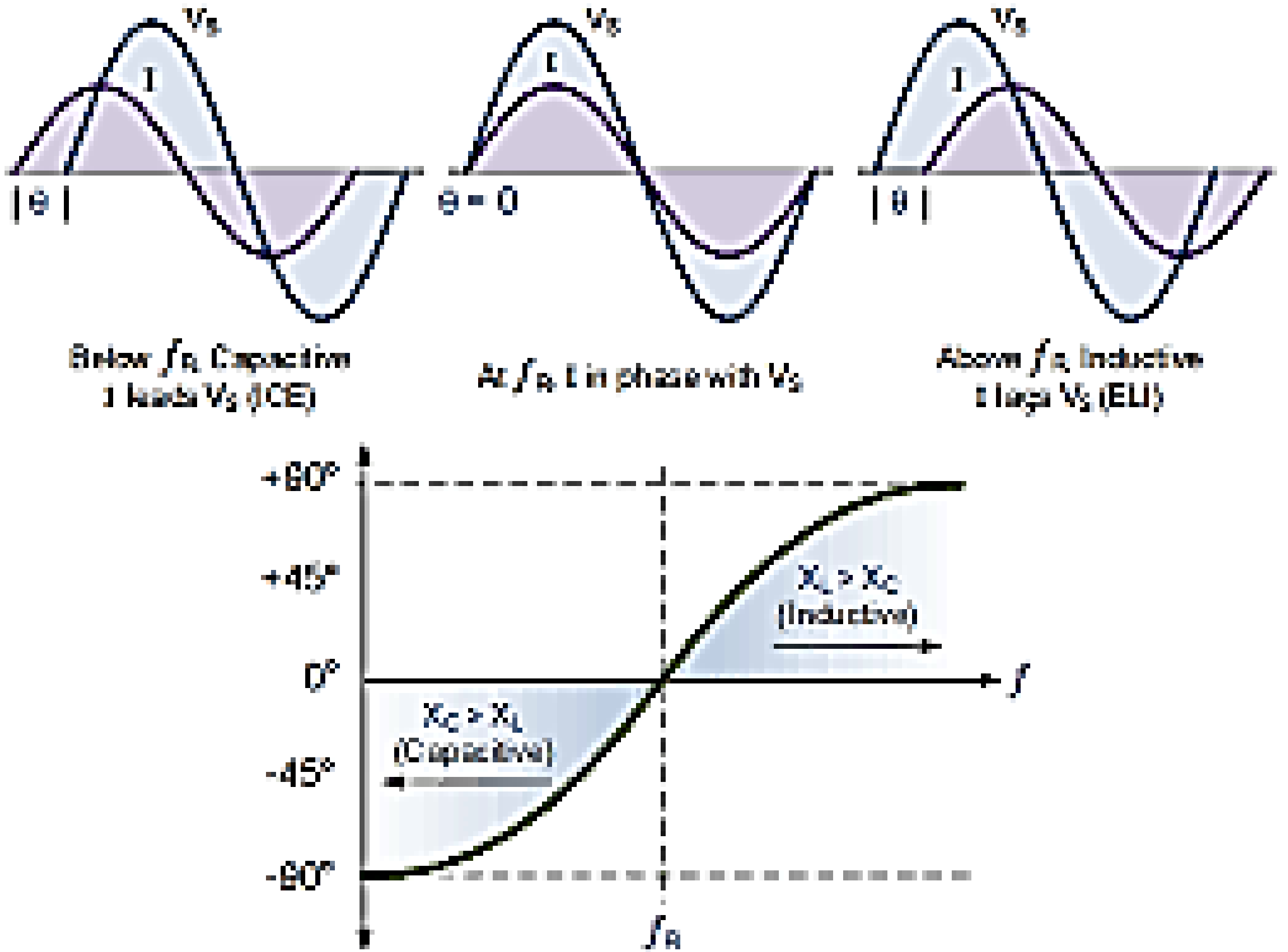






Series Resonance

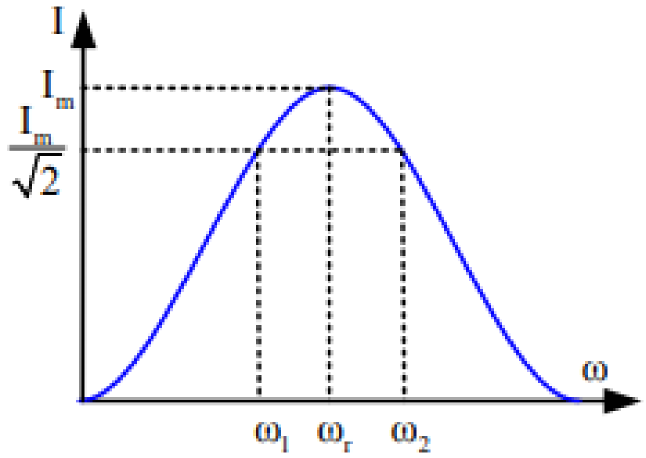
4. Phase Curve



Series Resonance

Derivation of Half power frequencies

Refer the frequency response of series RLC circuit shown in figure.



At resonance frequency f_r $Z = R$ and current is I_m

At half power frequencies f_1 and f_2 the current is $\frac{I_m}{\sqrt{2}}$

$$Z = \sqrt{2}R$$

$$Z = R + jX_L - JX_C = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\begin{aligned}\sqrt{R^2 + (X_L - X_C)^2} &= \sqrt{2}R \\ R^2 + (X_L - X_C)^2 &= 2R^2 \\ (X_L - X_C)^2 &= R^2 \\ X_L - X_C &= R\end{aligned}$$

At frequency ω_1 the circuit impedance $X_C > X_L$

$$\begin{aligned}X_C - X_L &= R \\ \frac{1}{\omega_1 C} - \omega_1 L &= R \\ \frac{1 - \omega_1^2 LC}{\omega_1 C} &= R \\ R\omega_1 C - 1 + \omega_1^2 LC &= 0 \\ \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} &= 0\end{aligned}$$

$$a = 1, \quad b = \frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_1

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Series Resonance

At frequency ω_2 the circuit impedance $X_L > X_C$

$$\begin{aligned} X_L - X_C &= R \\ \omega_2 L - \frac{1}{\omega_2 C} &= R \\ \frac{\omega_2^2 LC - 1}{\omega_2 C} &= R \\ \omega_2^2 LC - R\omega_2 C - 1 &= 0 \\ \omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} &= 0 \end{aligned}$$

$$a = 1, \quad b = -\frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_2

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Series Resonance

- **Relation between resonant frequency and half power frequencies.**
- **Show that the resonant frequency is the geometrical mean of half power frequencies.**

We know that at half power frequencies,

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

Adding the above two equations

$$\therefore \quad (\omega_2 + \omega_1) L - \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right)\frac{1}{C} = 0$$

$$\therefore \quad (\omega_2 + \omega_1) L - \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C} = 0$$

$$\therefore \quad (\omega_1 + \omega_2) L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C}$$

$$\therefore \quad \omega_1 . \omega_2 = \frac{1}{LC}$$

But from condition of resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore \quad \omega_1 . \omega_2 = \omega_0^2$$

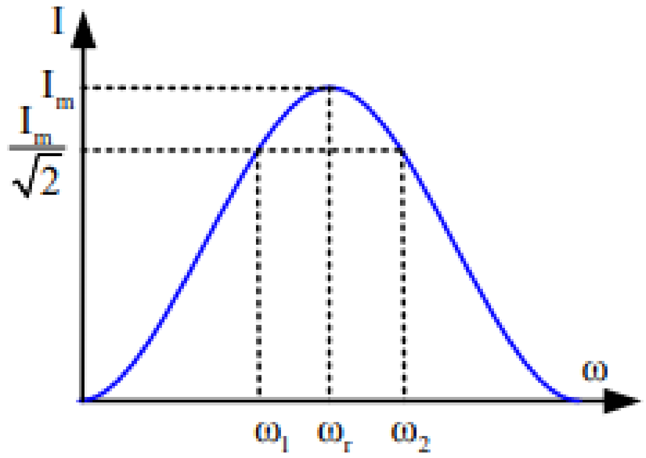
i.e. $f_1 \cdot f_2 = f_0^2$

$$\boxed{f_0 = \sqrt{f_1 \cdot f_2}}$$

Series Resonance

Derivation of Half power frequencies

Refer the frequency response of series RLC circuit shown in figure.



At resonance frequency f_r $Z = R$ and current is I_m

At half power frequencies f_1 and f_2 the current is $\frac{I_m}{\sqrt{2}}$

$$Z = \sqrt{2}R$$

$$Z = R + jX_L - JX_C = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\begin{aligned} \sqrt{R^2 + (X_L - X_C)^2} &= \sqrt{2}R \\ R^2 + (X_L - X_C)^2 &= 2R^2 \\ (X_L - X_C)^2 &= R^2 \\ X_L - X_C &= R \end{aligned}$$

At frequency ω_1 the circuit impedance $X_C > X_L$

$$\begin{aligned} X_C - X_L &= R \\ \frac{1}{\omega_1 C} - \omega_1 L &= R \\ \frac{1 - \omega_1^2 LC}{\omega_1 C} &= R \\ R\omega_1 C - 1 + \omega_1^2 LC &= 0 \\ \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} &= 0 \end{aligned}$$

$$a = 1, \quad b = \frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_1

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Series Resonance

At frequency ω_2 the circuit impedance $X_L > X_C$

$$\begin{aligned} X_L - X_C &= R \\ \omega_2 L - \frac{1}{\omega_2 C} &= R \\ \frac{\omega_2^2 LC - 1}{\omega_2 C} &= R \\ \omega_2^2 LC - R\omega_2 C - 1 &= 0 \\ \omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} &= 0 \end{aligned}$$

$$a = 1, \quad b = -\frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_2

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Series Resonance

- **Relation between resonant frequency and half power frequencies.**
- **Show that the resonant frequency is the geometrical mean of half power frequencies.**

We know that at half power frequencies,

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

Adding the above two equations

$$\therefore \quad (\omega_2 + \omega_1) L - \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right)\frac{1}{C} = 0$$

$$\therefore \quad (\omega_2 + \omega_1) L - \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C} = 0$$

$$\therefore \quad (\omega_1 + \omega_2) L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C}$$

$$\therefore \quad \omega_1 . \omega_2 = \frac{1}{LC}$$

But from condition of resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore \quad \omega_1 . \omega_2 = \omega_0^2$$

i.e. $f_1 \cdot f_2 = f_0^2$

$$\boxed{f_0 = \sqrt{f_1 \cdot f_2}}$$

Series Resonance

- **Derivation of Bandwidth**

We know that at half power frequencies,

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

Subtracting the above two equations

$$\therefore \quad (\omega_2 - \omega_1) L + \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{1}{C} = 2R$$

$$\therefore \quad (\omega_2 - \omega_1) + \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right)\frac{1}{LC} = \frac{2R}{L}$$

We know that

$$\omega_1 \omega_2 = \frac{1}{LC}$$

$$\therefore \quad (\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L}$$

$$\therefore \quad (\omega_2 - \omega_1) = \frac{R}{L}$$

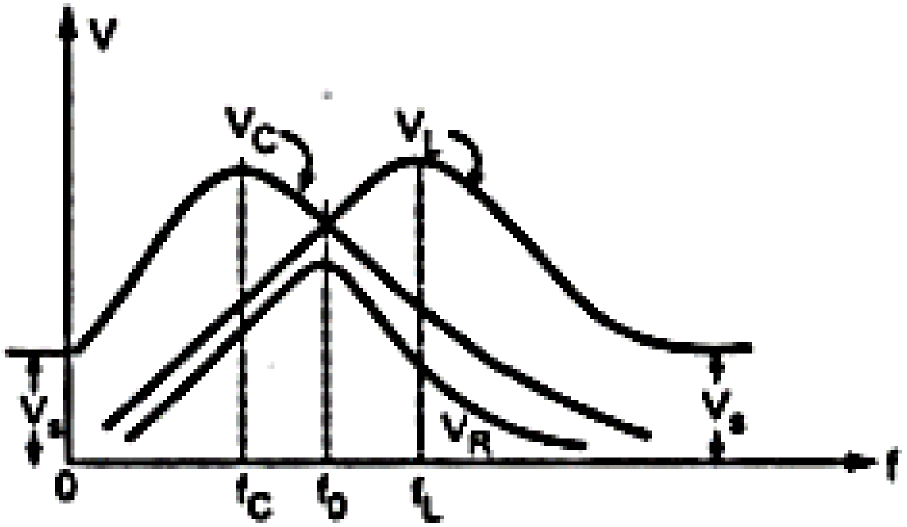
$$\text{i.e.} \quad (f_2 - f_1) = \frac{R}{2\pi L}$$

$$\therefore \quad \boxed{\text{Bandwidth} = (f_2 - f_1) = \frac{R}{2\pi L}}$$

Series Resonance

- **Derivation of frequencies at which maximum voltage across the L and C.**

Variation of V_R, V_C and V_L with frequency is as shown in Fig.



it is clear that, voltage across C and voltage across L is not maximum at resonant frequency. At resonant frequency f_0, the voltages V_C and V_L are equal in magnitude but opposite in phase. The voltage V_C is maximum at frequency f_C which is less than f_0 and the voltage V_L is maximum at frequency f_L which is greater than f_0.

Series Resonance

Consider that voltage across capacitor is V_C and it is given by,

$$V_C = I\left(\frac{1}{\omega C}\right) \qquad \text{but } I = \frac{V}{Z}$$

$$\therefore \qquad V_C = \frac{V}{\omega C\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \qquad \ldots (5)$$

To find frequency at which V_C is maximum, we have to differentiate V_C with respect to ω and equate it to zero. But first removing radical sign by squaring expression. Then equating $\frac{d V_C^2}{d\omega} = 0$; since when V_C^2 is maximum, V_C is maximum. By squaring equation (5), we have,

$$V_C^2 = \frac{V^2}{\omega^2 C^2\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}$$

$$\therefore \qquad V_C^2 = \frac{V^2}{\omega^2 C^2\left[R^2+\frac{(\omega^2 LC-1)^2}{\omega^2 C^2}\right]}$$

Series Resonance

$$\therefore \qquad V_C^2 = \frac{V^2}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

Now, differentiating V_C^2 with respect to ω and equating to zero, we have,

$$\frac{d V_C^2}{d\omega} = \frac{V^2 [2\omega R^2 C^2 + 2(\omega^2 LC - 1)(2\omega LC)]}{[\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2]^2} = 0$$

Then equating only numerator terms to zero, we have,

$$V^2 [2\omega R^2 C^2 + 2(\omega^2 LC - 1)(2\omega LC)] = 0$$

But V is input voltage which can not be zero.

$$\therefore \qquad 2\omega R^2 C^2 + 2(2\omega LC)(\omega^2 LC - 1) = 0$$

$$2\omega R^2 C^2 + 4\omega^3 L^2 C^2 - 4\omega LC = 0$$

$$4\omega^3 L^2 C^2 = 4\omega LC - 2\omega R^2 C^2$$

$$\therefore \qquad \omega^2 = \frac{4\omega LC}{4\omega L^2 C^2} - \frac{2\omega R^2 C^2}{4\omega L^2 C^2}$$

$$\therefore \qquad \omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\therefore \qquad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \quad \text{rad/sec}$$

Therefore, the frequency f_C at which capacitor voltage V_C is maximum, is given by,

$$\therefore \qquad \boxed{f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}} \qquad \ldots (6)$$

From equation (6),

$$f_C = \frac{1}{2\pi\sqrt{LC}} \cdot \sqrt{1 - \frac{R^2 C}{2L}}$$

$$\therefore \qquad \boxed{f_C = f_0 \cdot \sqrt{1 - \frac{R^2 C}{2L}}} \qquad \ldots (7)$$

Series Resonance

Similarly let us calculate the frequency at which the voltage across the inductance is at its maximum.

The voltage across inductor is V_L and is given by,

$$V_L = I.(\omega L) \quad \text{but } I = \frac{V}{Z}$$

$$\therefore \quad V_L = \frac{V(\omega L)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \ldots (8)$$

Squaring equation (8),

$$V_L^2 = \frac{V^2 \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\therefore \quad V_L^2 = \frac{V^2 \omega^2 L^2}{\frac{R^2 C^2 \omega^2 + (\omega^2 LC - 1)^2}{\omega^2 C^2}}$$

$$\therefore \quad V_L^2 = \frac{V^2 \cdot \omega^4 L^2 C^2}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

By differentiating V_L^2 with respect to ω and equating only numerator term to zero, we have,

$$2\omega^2 LC - \omega^2 R^2 C^2 - 2 = 0$$

$$\therefore \quad \omega^2 (2LC - R^2 C^2) = 2$$

$$\therefore \quad \omega^2 = \frac{2}{2LC - R^2 C^2}$$

$$\therefore \quad \omega^2 = \frac{1}{LC - \frac{R^2 C^2}{2}}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC - \frac{R^2 C^2}{2}}} \text{ rad/sec}$$

Therefore, the frequency f_L at which inductor voltage V_L is maximum is given by,

$$\therefore \quad \boxed{f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2 C^2}{2}}}} \quad \ldots (9)$$

Series Resonance

Resonance by varying circuit inductance

Consider a series RLC circuit as shown in Figure is become resonant by varying inductance of the circuit.

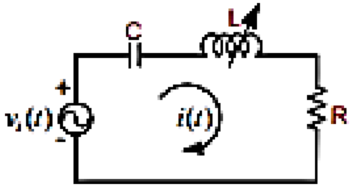


Figure : Resonance by varying inductance

Let L_1 is the inductance at ω

$$X_C - X_L = R$$
$$\frac{1}{\omega C} - \omega L_1 = R$$
$$L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega}$$

Let L_2 is the inductance at ω

$$X_L - X_C = R$$
$$\omega L_2 - \frac{1}{\omega C} = R$$
$$L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega}$$

Resonance by varying circuit capacitance

Consider a series RLC circuit as shown in Figure is become resonant by varying capacitance of the circuit.

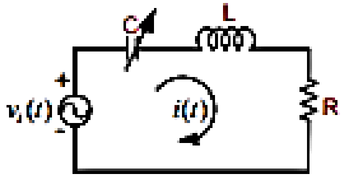


Figure: Resonance by varying capacitance

Let C_1 is the capacitance at ω_1

$$X_C - X_L = R \Rightarrow \frac{1}{\omega_1 C_1} - \omega_1 L = R$$
$$\frac{1}{\omega_1 C_1} = R + \omega_1 L$$
$$C_1 = \frac{1}{\omega_1^2 L + \omega_1 R}$$

Let C_2 is the capacitance at ω_2

$$X_L - X_C = R \Rightarrow \omega_2 L - \frac{1}{\omega_2 C_2} = R$$
$$\frac{1}{\omega_2 C_2} = \omega_2 L - R$$
$$C_2 = \frac{1}{\omega_2^2 L - \omega_2 R}$$

Series Resonance

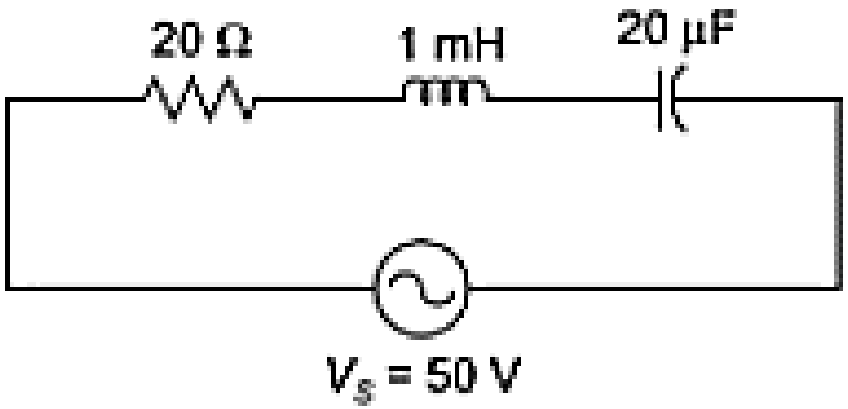
Following are the important properties of the series resonant circuit.

1) Under resonance, applied a.c. voltage and resulting a.c. current are in phase.
2) Under resonance, the series resonance circuit shows unity power factor condition.
3) Under resonance, the total reactance of the circuit becomes zero. The impedance of the circuit becomes purely resistive, thus the voltage and current are in phase.
4) The impedance under resonance is of minimum value as compared to the impedance at any frequency other than the resonant frequency.
5) Under resonance, the current in the circuit is of maximum value (as impedance is minimum) and hence power in the circuit is maximum under resonance.
6) Under resonance only, the series resonant circuit acts as voltage amplifier with the quality factor of the circuit i.e. Q_0 acting as amplification or the magnification factor.
7) Under resonance, energy stored by L and C is of equal value, hence quality factor of the circuit is nothing but the quality factor of L and C at resonating frequency.
8) The quality factor of the circuit decides selectivity of the circuit. Its required value must be large enough. It decides how much the resonant circuit is selective.
9) The impedance under resonance is capacitive in nature while above resonance it is inductive in nature. In general, the series resonant circuit is used when high power output is required at a perticular frequency and impedance requirement is lower. Basically in series resonant circuit under resonance impedance is minimum (ideally zero), hence it is used in m-derived filters (to be discussed latter) to increase attenuation to infinity suddenly.

Series Resonance-Examples

Series Resonance- Problems

1. Determine the resonant frequency for the circuit shown in figure. Also find the current at resonance and the voltage across each element and the impedance at resonance.



Solution:

Given data:

R=20 Ohms, L=1mH and C=20 uF, Vs=50V.

To find:

$f_o, I_{max}, V_R, V_L, V_C, Z_{resonance}$

$$f_o = \frac{1}{2\pi\sqrt{LC}} --(1); \boldsymbol{f_o = 1.125K\ Hz.}$$

$$\boldsymbol{Z_T = R = 20\ Ohms; I_{max} = \frac{V}{R} = 2.5A.}$$

$$V_R = I_{\text{max}} R \Rightarrow 50V$$

$$V_L = Ima_{xx} X_L \Rightarrow 2.5\text{x}2\text{x}3.14\text{x}1.125\text{x}10^3\text{x}1\text{x}10^{-3}$$

$$\boldsymbol{V_L = 17.66\ Volts}$$

$$V_C = Ima_{xx}(-X_C) \Rightarrow \frac{2.5}{2x3.14x1.125x10^3x20x10^{-6}}$$

$$\boldsymbol{V_C = -17.66\text{V}}$$

Series Resonance- Problems

2. A series RLC circuit has a capacitance of 0.5microfarads and a resistance of 10 ohms. Find the value of the inductance that will produce a resonant frequency of 5000Hz. Also calculate the maximum energy stored in the inductor at resonance. Assume the supply voltage to be 220V.

Given data:

R=10 Ohms, C=0.5uF, f_o=5KHz, Vs=220V.

To find:

L, E_{max}

Solution:

$$f_o = \frac{1}{2x3.14\sqrt{LC}} \Rightarrow 5000 = \frac{1}{2x3.14\sqrt{Lx0.5x10^{-6}}} => $$

L=2.0285mH

$$E_{max} = \frac{1}{2} L\, I^2{}_{max}$$

$$E_{max} = 0.49\ Joules.$$

NOTE: Instatanteneous maximum energy

$$E_{max} = \frac{1}{2} L\, I^2{}_{max} = \frac{1}{2} L\left(\sqrt{2} I_{max}\right)^2$$

$$E_{max} = 0.98\ Joules.$$

Series Resonance- Problems

3. A series RLC circuit with R=20 Ohms and L=1H results in a leading phase angle of 40° at a frequency of 50Hz. At what frequency will the circuit be resonant?

Given data:

R=20 Ohms, L=1H, φ=40°, f=50Hz

To find:

C, f_o

Solution:

$$Z_T = R + j(X_C \sim X_L)$$

Leading phase angle(I leads V)- Capacitor: XC>XL

NOTE: Lagging phase angle (V leads I/ I lags V)-Inductor: XL>XC

$$\varphi = \tan^{-1}((X_C - X_L)/R)$$

$tan\varphi = \frac{X_C - X_L}{R} \Rightarrow 0.839;\ X_C = \frac{1}{2x3.14x50xC};\ X_L = 2x3.14x50x1$

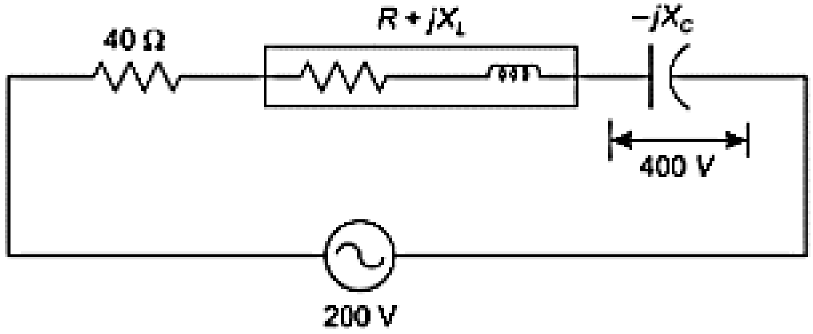
$\boldsymbol{C = 9.618}$ **uF**, $\boldsymbol{f_o = 51.32 Hz}$.

Series Resonance- Problems

4. A 40 Ohms resistor is in series with a coil, a capacitor and a 200V variable frequency supply as shown in figure. At a frequency of 250Hz, a maximum current of 0.8A flows through the circuit and voltage across capacitor is 400V. Determine

a) The Capacitance of the capacitor. And

b) The resistance of the coil



NOTE:

Coil=series combination of L and R(coil resistance)

Given data:

R= 40 Ohms, Vs=200V, Imax=0.8A, fo=250Hz, VC=400V

To find:

C, L, R(Coil resistance)

Series Resonance- Problems

Given data:

R= 40 Ohms, Vs=200V, Imax=0.8A, fo=250Hz, VC=400V

To find:

C, L, R(Coil resistance)

Solution:

$$V_C = Imax\ x\ X_C$$

$$400V = 0.8x \frac{1}{2x3.14xf_oxC}; \boldsymbol{C = 1.27uF}$$

At resonance $X_C = X_L$

$\frac{1}{2\pi f_o C} = 2\pi f_o L$; **L=0.318H**

At resonance Z_T=40+R

Imax=V/Z_T;

$$0.8 = \frac{200}{40 + R}; \boldsymbol{R = 210\ Ohms}.$$

Series Resonance- Problems

5. A series circuit has a resonance frequency of 150KHz, a bandwidth of 60KHz and Q=4. Determine the cut off frequencies.

Given data:

fo=150Khz, BW=60KHz, Q=4.

To find:

f1, f2

NOTE:

Q>5

$$f_1 = f_o - \frac{BW}{2} \; and \; f_2 = f_o + \frac{BW}{2}$$

Solution:

$$f_o = \sqrt{f_1 f_2} --- (1)$$

$$BW = f_2 - f_1 --- (2)$$

$$f_1^2 + 60000 f_1 - 225x10^8 = 0$$

$$f_1 = 122.97KHz \; and \; -182.95KHz, ; f_1 = \mathbf{122.97KHz}.$$

$$f_2 = BW + f_1 = \mathbf{182.97KHZ}.$$

Series Resonance- Problems

6. A Series RLC circuit has R=20 Ohms, L=0.02 H and C=0.06 micro Farads, Vs=200V. Find

a) Resonant frequency
b) Circuit impedance and current under resonant condition
c) Maximum value of the voltage across the L and f_L
d) Maximum vale of the voltage across the C and f_C.

$V_C max = I.X_C = \frac{V}{Z_T}\left(\frac{1}{2\pi f_C C}\right); Z_T = \sqrt{R + (X_C - X_L)^2}; X_C = \frac{1}{2\pi f_C C}$ and $X_L = 2\pi f_C L$.

$$\boldsymbol{V_{Cmax} = 2887.17\ Volts}$$

$$\boldsymbol{f_c = 4592.88 Hz.}$$

$V_L max = I.X_L = \frac{V}{Z_T}(2\pi f_L L); Z_T = \sqrt{R + (X_C - X_L)^2}; X_C = \frac{1}{2\pi f_L C}$ and $X_L = 2\pi f_L L$

$$\boldsymbol{V_{Lmax} = 2887.16 Volts}$$

$$\boldsymbol{f_L = 4595.17 Hz.}$$

Parallel Resonance - Introduction

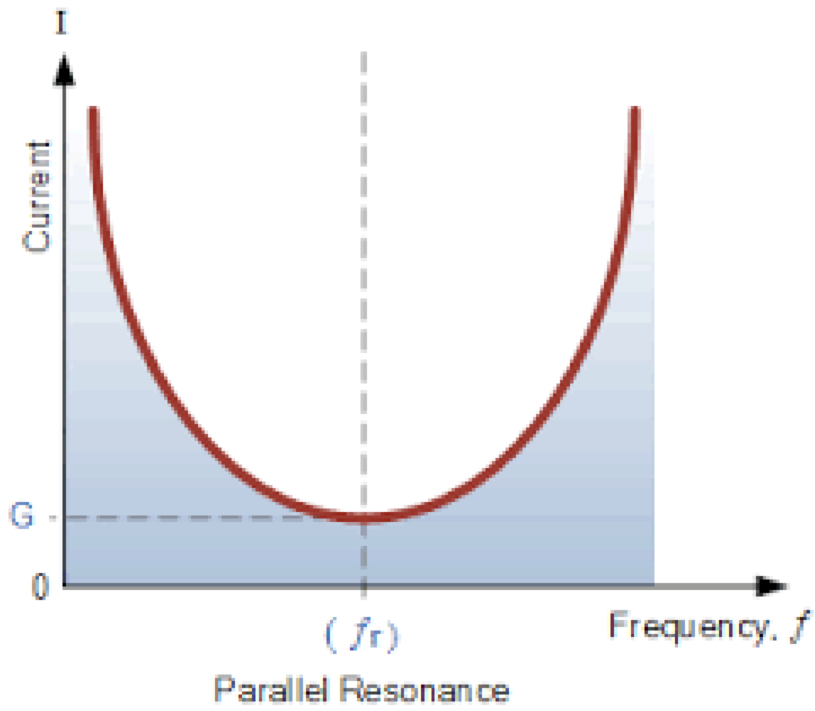
- Electrical elements, such as R, L and C and its combinations are connected in parallel called Parallel Circuits.
- Similar to the series circuit, parallel circuits also exhibits the resonance condition, when the circuit is excited by an AC source.
- In parallel circuit, the resonance is a phenomenon at which
    1. Voltage and currents are inphase
    2. Net susceptance is equal to zero (Imaginary part of admittance)
    3. Power factor is unity
    4. Maximum impedance and minimum current

NOTE:

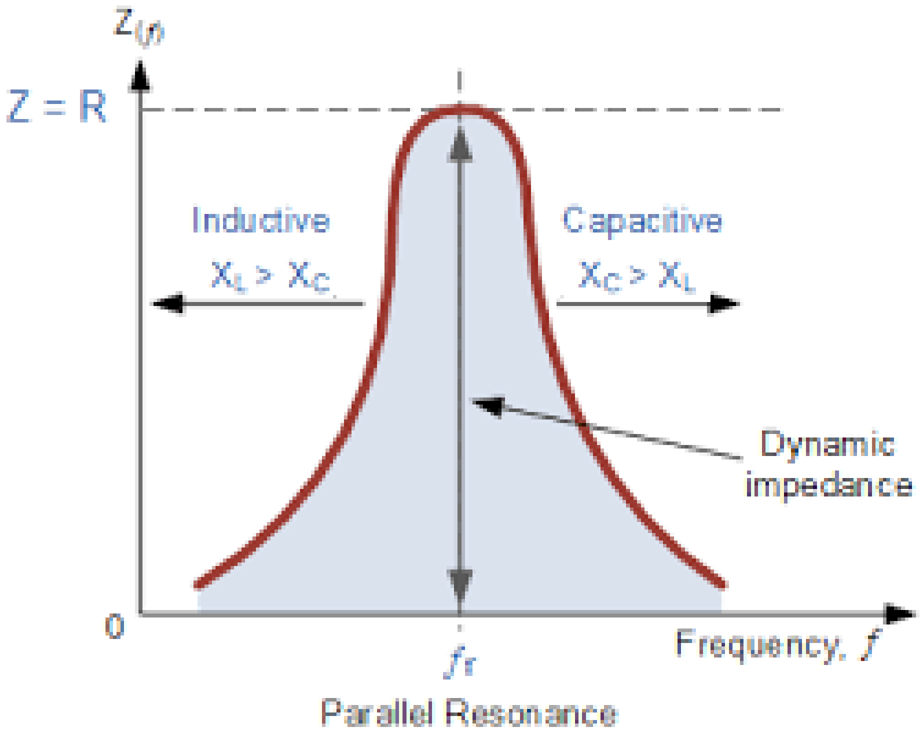
- ➢ **Current magnification circuit/ Anti-resonance circuit.**
- ➢ **There is no general circuit - parallel connection of circuit elements- Infinite circuits.**
- ➢ **No general formula/expression for resonant frequency- differs from one circuit to another circuit.**

Parallel Resonance - Introduction

Frequency response



Impedance response



Parallel Resonance - Introduction

- Bandwidth:

$$f_2 - f_1 = \frac{f_o}{Q}$$

- Relation between resonant frequency to the half power frequencies:

$$f_o = \sqrt{f_1 f_2}$$

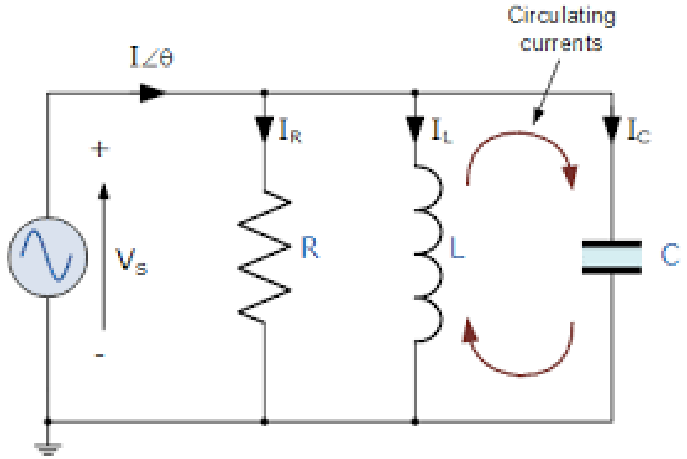
- Quality factor(Current Magnification factor):

$$Q = \frac{R}{\omega_o L} = \omega_o C R = \frac{1}{R}\sqrt{\frac{C}{L}}$$

Parallel Resonance - Introduction

- Example:

1. RLC parallel circuit:



Consider the electrical elements R, L and C are connected in parallel, where, I is the current supplied to the circuit, I_R, I_L and I_C are the current through R, L and C respectively.

$$Z_T = R||jX_L||jX_C$$

$$Y_T = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y_T = \frac{1}{R} + \frac{1}{j}\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

At resonance net susceptance is equal to zero

$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

$$\frac{1}{2\pi f_o L} = 2\pi f_o C$$

$$\boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

$$Y_D = \frac{1}{R}$$

$$Z_D = R$$

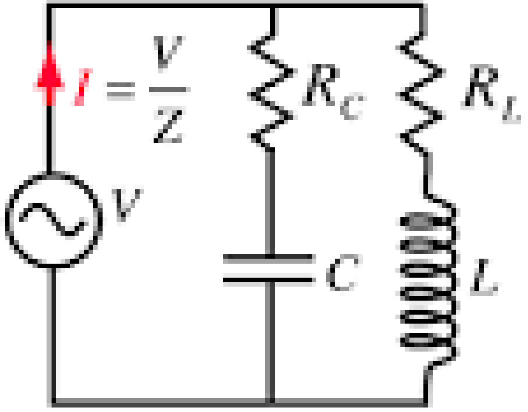
$$I_{min} = \frac{V_S}{Z_D}$$

Parallel Resonance - Introduction

- Example:

2. RL-RC parallel circuit:

Consider the electrical elements R, L and C are connected in parallel, where, I is the current supplied to the circuit, I_L and I_C are the current through the branches of Inductor and Capacitor respectively. R_L and R_C are the resistors connected in series with the Inductor and Capacitor respectively.



Parallel Resonance

$$Y_T = \frac{1}{Z_L} + \frac{1}{Z_C} \Rightarrow \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

Rationalize the denominator

$$Y_T = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$Y_T = \frac{R_L}{R_L^2 + X_L^2} + \frac{-jX_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + \frac{jX_C}{R_C^2 + X_C^2}$$

$$Y_T = \left\{\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right\} + \left\{\frac{jX_C}{R_C^2 + X_C^2} - \frac{jX_L}{R_L^2 + X_L^2}\right\}$$

$$Y_D = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}$$

$$Z_D = \frac{1}{Y_D}$$

Parallel Resonance - Introduction

At resonance net susceptance is equal to zero

$$\frac{X_C}{R_C^2+X_C^2}-\frac{X_L}{R_L^2+X_L^2}=0 ---(1)$$

We know that, $X_C=\frac{1}{\omega_o C}; X_L=\omega_o L$

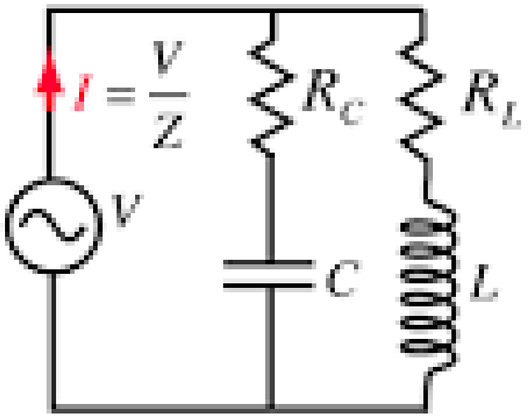
$$\frac{X_C}{R_C^2+X_C^2}=\frac{X_L}{R_L^2+X_L^2}$$

$$\frac{\frac{1}{\omega_o C}}{R_C^2+\left(\frac{1}{\omega_o C}\right)^2}=\frac{\omega_o L}{R_L^2+(\omega_o L)^2}$$

$$\frac{1}{\omega_o C}[R_L^2+(\omega_o L)^2]=\omega_o L[R_C^2+\left(\frac{1}{\omega_o C}\right)^2]$$

$$\frac{R_L^2}{\omega_o C}+\frac{\omega_o L^2}{C}=\omega_o L R_C^2+\frac{L}{\omega_o C^2}$$

$$\frac{1}{\omega_o}\left[\frac{R_L^2}{C}-\frac{L}{C^2}\right]=\omega_o[-\frac{L^2}{C}+LR_C^2]$$



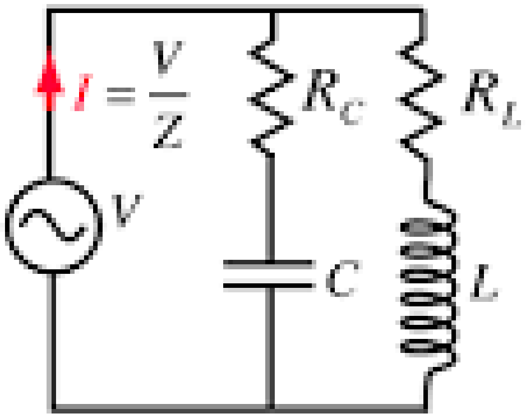
Parallel Resonance

Parallel Resonance - Introduction

$$\frac{1}{\omega_o}\left[\frac{R_L^2}{C}-\frac{L}{C^2}\right]=\omega_o\left[-\frac{L^2}{C}+LR_C^2\right]$$

$$\omega_o^2=\frac{\left[\frac{R_L^2}{C}-\frac{L}{C^2}\right]}{\left[-\frac{L^2}{C}+LR_C^2\right]}$$

$$\boldsymbol{\omega_o=\sqrt{\frac{\left[\frac{R_L^2}{C}-\frac{L}{C^2}\right]}{\left[LR_C^2-\frac{L^2}{C}\right]}}}\ ;$$

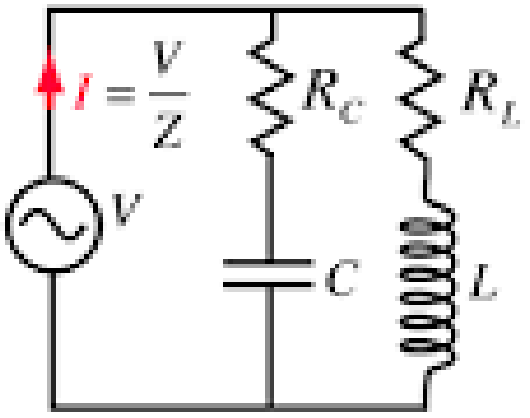


Parallel Resonance

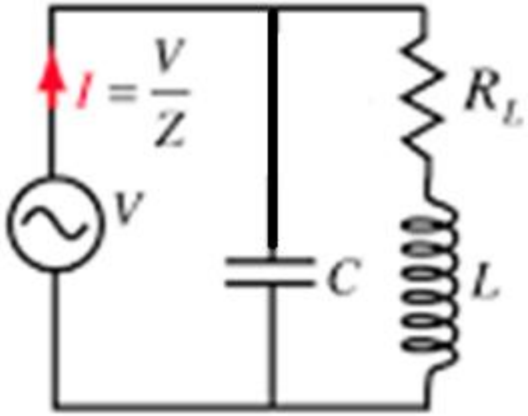
$$\boldsymbol{R_C=R_L=\sqrt{\frac{L}{C}}}$$

Parallel Resonance - Introduction

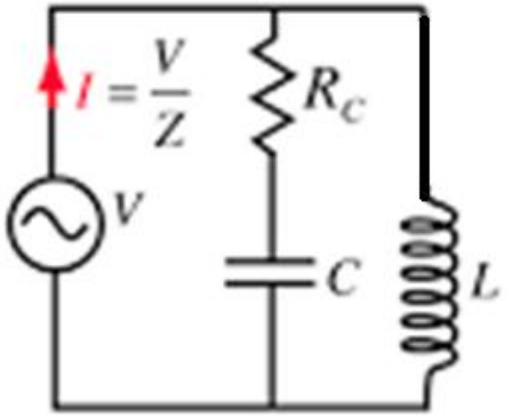
Summary:



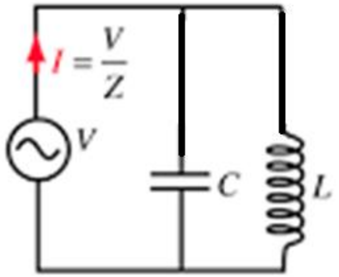
Parallel Resonance



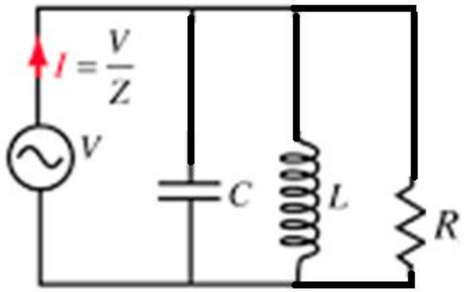
Parallel Resonance



Parallel Resonance



Parallel Resonance



Parallel Resonance

$$\omega_o = \sqrt{\frac{\left[\frac{R_L^2}{C} - \frac{L}{C^2}\right]}{\left[LR_C^2 - \frac{L^2}{C}\right]}} \;;$$

$R_C = 0$

$R_L = 0$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

R does not affect the resonant frequency

Parallel Resonance - Problems

A coil of inductance 10 H and 10 Ω resistance is connected in parallel with 100 pF capacitor. The combination is applied with a voltage of 100 V. Find resonant frequency and current at resonance.

Solution : $R_L = 10\ \Omega$, $L = 10$ H, $C = 100$ pF

Frequency of resonance is given by,

$$f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$= \frac{1}{2\times\pi}\sqrt{\frac{1}{(10\times100\times10^{-12})} - \frac{(10)^2}{(10)^2}}$$

$$= \mathbf{5.033\ kHz}$$

At resonant frequency, the impedance of parallel resonant circuit is given by,

$$Z_{ar} = \frac{L}{C R_L}$$

$$= \frac{10}{100\times10^{-12}\times10}$$

$$= 10\times10^{9}\ \Omega$$

This shows that impedance is very high at resonance.

The current at resonance is given by,

$$I_0 = \frac{V}{Z_{ar}}$$

$$= \frac{100}{10\times10^{9}}$$

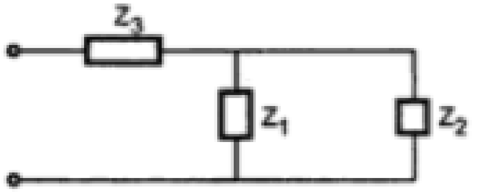
$$= \mathbf{10\ nA}$$

Thus the current is at its minimum value as the impedance is maximum.

Parallel Resonance - Problems

Two impedances $Z_1 = 20 + j\,10$ and $Z_2 = 10 - j\,30$ are connected in parallel and this combination is connected in series with $Z_3 = 30 + j\,X$. Find the value of X which will produce resonance.

Solution : From given information, given circuit is as shown in Fig



Total impedance is given by,

$$Z = Z_3 + (Z_1 \| Z_2)$$

$$= (30 + j\,X) + \frac{(20 + j\,10)(10 - j\,30)}{(20 + j\,10) + (10 - j\,30)}$$

$$= (30 + jX) + \frac{(200 + j\,100 - j\,600 + 300)}{(30 - j\,20)}$$

$$= (30 + jX) + \frac{(500 - j\,500)}{(30 - j\,20)}$$

$$= (30 + jX) + \frac{(500)(1 - j\,1)(30 + j\,20)}{(30)^2 + (20)^2}$$

$$= (30 + jX) + \frac{500}{1300}[30 - j\,30 + j\,20 + 20] = 30 + jX + \frac{5}{13}[50 - j\,10]$$

$$= \left[30 + \frac{250}{13}\right] + j\left[X - \frac{50}{13}\right]$$

The circuit shown in Fig. will resonate, if imaginary part is zero,

$$\therefore \quad X - \frac{50}{13} = 0$$

$$\therefore \quad X = \frac{50}{13}$$

$$\therefore \quad X = 3.846\ \Omega$$

Transient Response Analysis - Introduction

Time Response Analysis

The study of behaviour(Output/Response) of a system with respect to time is called Time Response analysis

Time response is divided into two parts

1. **Transient part**

    The response /Output before reaching the steady state or final value.

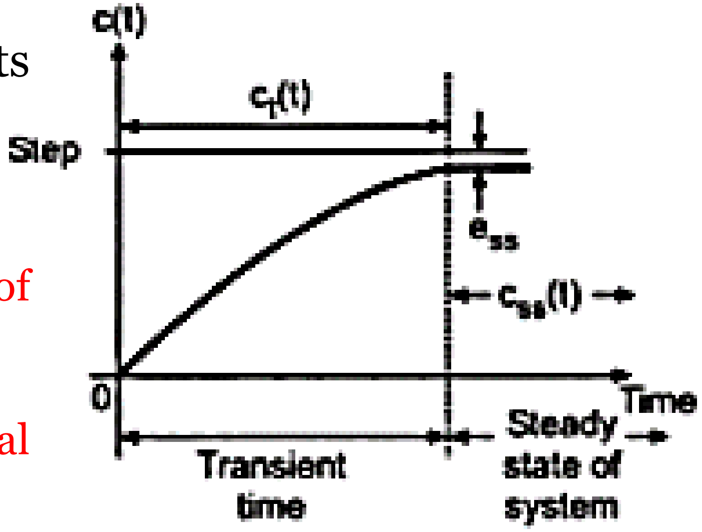
2. Steady State Part

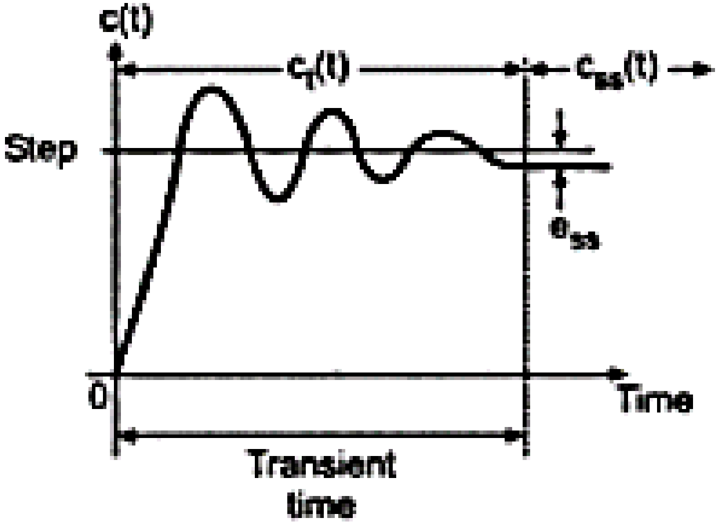
    The time response or part of the response after vanishing the transient part.

NOTE: Transients due to energy storage elements present in the system and its Initial values.

Natural Response:
Stored energy released to the resistive part of the network

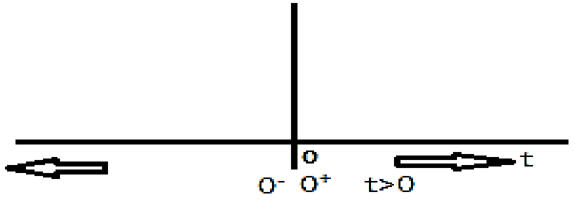
Forced Response:
An external energy supplied to the electrical network





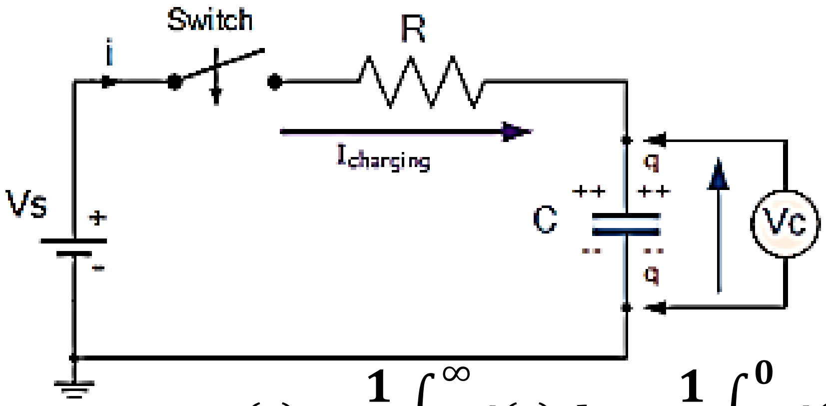
Transient Response Analysis - Introduction

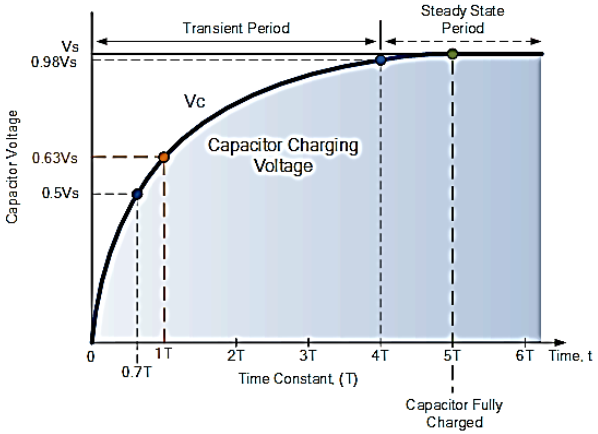
- Energy storage elements in the electrical system- **Inductor** and **Capacitor**.
- Initial conditions are evaluated at the time instants **t=0$^-$, t=0$^+$ and t>0** (before, just and after switching action respectively)



Voltage across the capacitor cannot change instantaneously

Capacitor charging



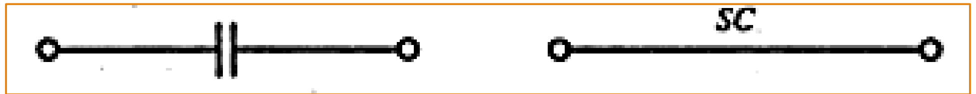


$$v_C(t) = \frac{1}{C}\int_{-\infty}^{\infty} i(t)dt \Rightarrow \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{\infty} i(t)dt$$

$$v_C(t) = v_C(o^-) + \frac{1}{C}\int_{0}^{\infty} i(t)dt$$

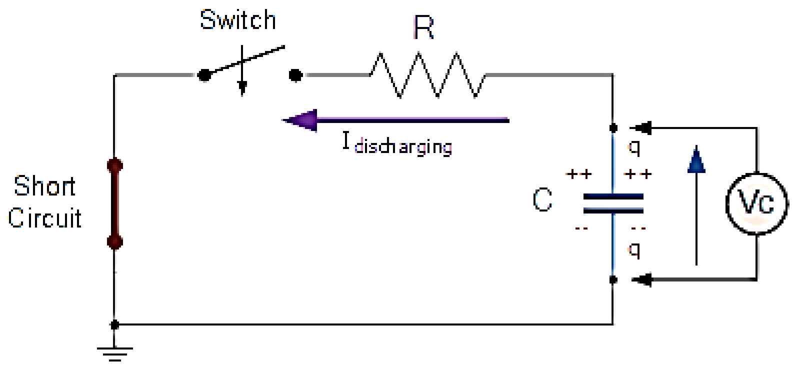
$$for\ uncharged\ Capacitor$$
$$v_C(o^-) = 0\ Votls$$
$$v_C(0^+) = 0\ Volts$$

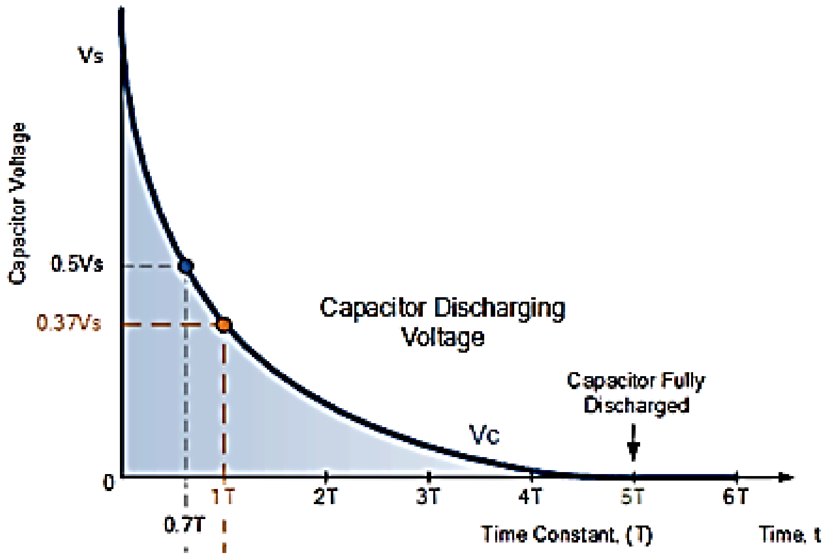
Equivalent Circuit



Transient Response Analysis - Introduction

Capacitor Discharging



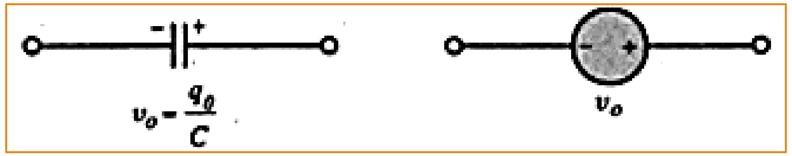


$$v_C(t) = \frac{1}{C}\int_{-\infty}^{\infty} i(t)dt \Rightarrow \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{\infty} i(t)dt$$

$$v_C(t) = v_C(o^-) + \frac{1}{C}\int_{0}^{\infty} i(t)dt$$

$$for\ charged\ Capacitor$$
$$v_C(o^-) = V_c\ Votls$$
$$v_C(0^+) = V_c\ Volts$$

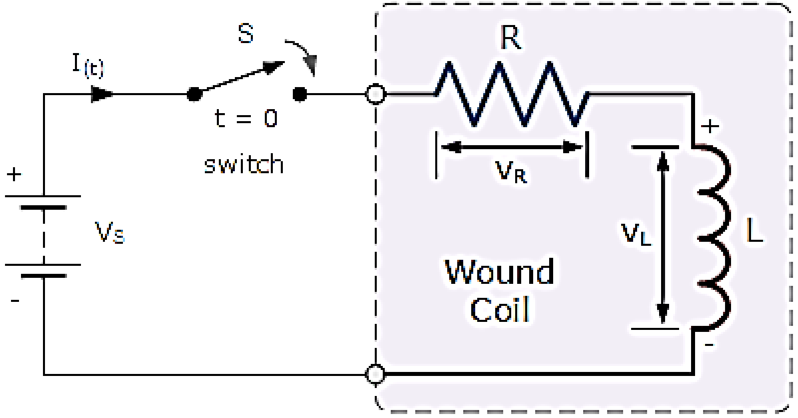
Equivalent Circuit

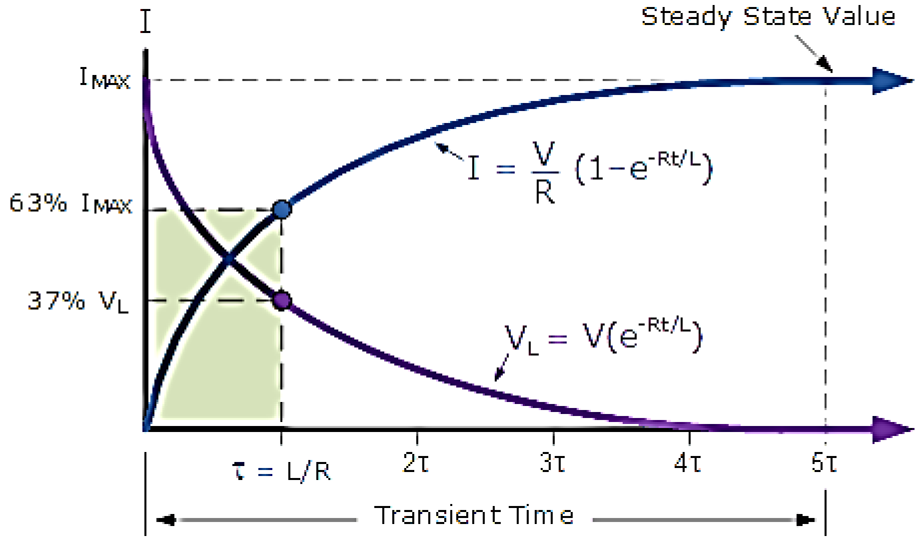


Transient Response Analysis - Introduction

Current through the inductors cannot change instantaneously

Inductor Charging and discharging





$$i_L(t) = \frac{1}{L}\int_{-\infty}^{\infty} v(t)dt \Rightarrow \frac{1}{L}\int_{-\infty}^{0} v(t)dt + \frac{1}{C}\int_{0}^{\infty} v(t)dt$$

$$i_L(t) = i_L(o^-) + \frac{1}{C}\int_{0}^{\infty} v(t)dt$$

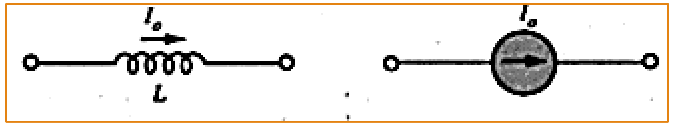
for uncharged Inductor

$$i_L(o^-) = 0 \ Amperes$$
$$i_L(0^+) = 0 \ \ Amperes$$

for charged Inductor

$$i_L(o^-) = I \ Amperes$$
$$i_L(0^+) = I \ Amperes$$

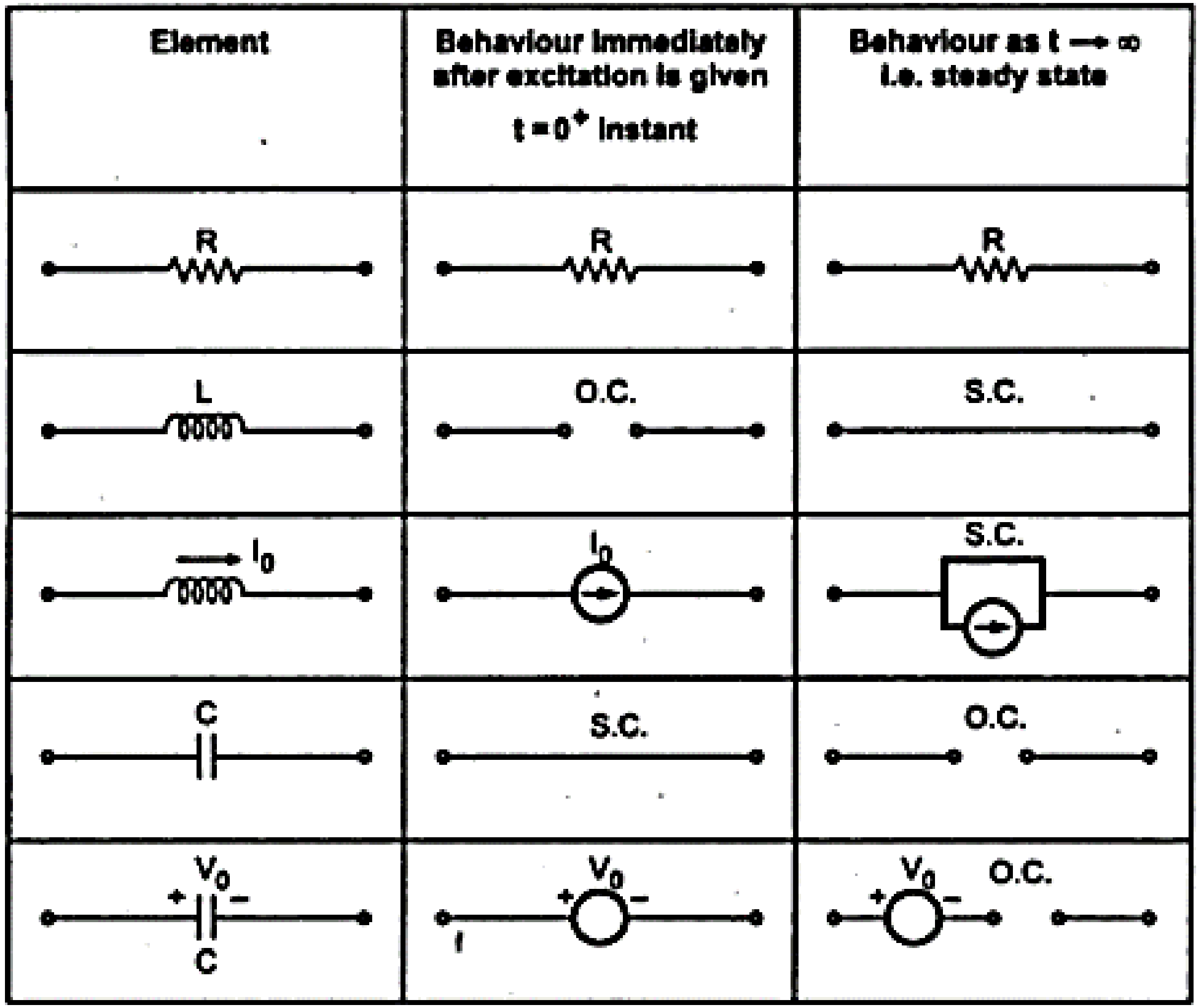
Equivalent Circuit



Transient Response Analysis - Introduction

Resistor- current and voltage across the resistor changes instantaneously

Summary



Transient Response Analysis - Introduction

Initial and final conditions of energy storage elements

Procedure:

1. Identify the energy storage elements and its state

NOTE: State-Uncharged/Charged state.

2. Draw the equivalent circuit at t=0- and find the current through the inductors and voltage across the capacitors. i.e., $i_L(0^-)$ and $v_C(0^-)$

NOTE: $i_L(0^-) = i_L(0^+)$ and $v_C(0^-)=v_C(0^+)$

3. Draw the equivalent circuit at t=0+, by replacing inductor by $i_L(0^+)$ Amperes, capacitor by $v_C(0^+)$ volts and resistors are kept as it is.

NOTE: Charged current value and charged voltage value of inductor and capacitor respectively would be called as steady state values.

NOTE: To find the steady state values

i) Current through the inductor is maximum at steady state and would be calculated by replacing the inductor by short circuit. i.e., $i_{SC}=i_L(0-) = i_L(0^+)$

ii) Voltage across the capacitor is maximum at steady state and would be calculated by replacing the capacitor by open circuit. i.e., $v_{OC}= v_C(0^-)=v_C(0^+)$.

4. Find the initial voltages and currents at t=0+.

4. Draw the equivalent circuit at t>0 and obtain system equations (KVL/KCL).

5. Find the derivatives of initial voltages and currents using the above initial conditions, $i.e., \frac{d}{dt}i(0^+)$, $\frac{d^2}{dt^2}i(0^+), \frac{d}{dt}v(0^+) and \frac{d^2}{dt^2}v(0^+)$.

Transient Response Analysis – Examples

1.For the circuit shown in figure, the switch K is closed at t=0, then find $i(0^+)$, $\frac{d}{dt}i(0^+)$ and $\frac{d^2}{dt^2}i(0^+)$.

Solution:

Step-1:

L is the energy storage element-Uncharged state

Step-2: $t=0^-$

$$i_L(0^-) = 0A$$

$$i_L(0^+) = i_L(0^-) = 0\ A.$$

Step-3:

Equivalent circuit at $t=0^+$

Step-4:

$$i(0^+) = 0A$$

Step-5: equivalent circuit at t>0

Step6: apply KVL

$$V = Ri(t) + \frac{Ldi(t)}{dt} --- (1)$$

Step7: at $t=0^+$

From equation (1)

$$V = Ri(t) + \frac{Ldi(t)}{dt}$$

$$\frac{d}{dt}i(0^+) = \frac{V}{L} - \frac{R}{L}i(0^+)$$

$$\boldsymbol{\frac{d}{dt}i(0^+) = \frac{V}{L}\ \text{A/sec}}$$

Differentiate Equation (1) w.r.t t

$$0 = \frac{Rdi(t)}{dt} + \frac{Ld^2i(t)}{dt^2} --- (2)$$

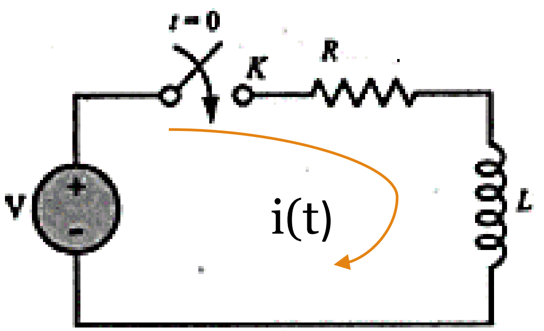
From equation (2).

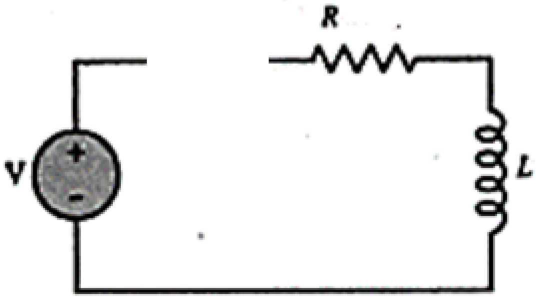
$$\frac{Ld^2i(t)}{dt^2} = -\frac{Rdi(t)}{dt}$$

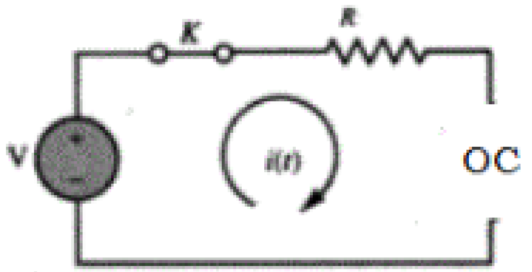
At $t=0^+$

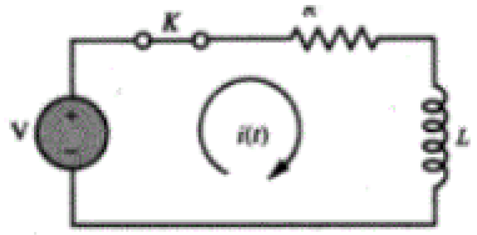
$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = -\frac{Rdi(0^+)}{Ldt}}$$

$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = -\frac{VR}{L^2}\ \text{A/sec}^2}$$



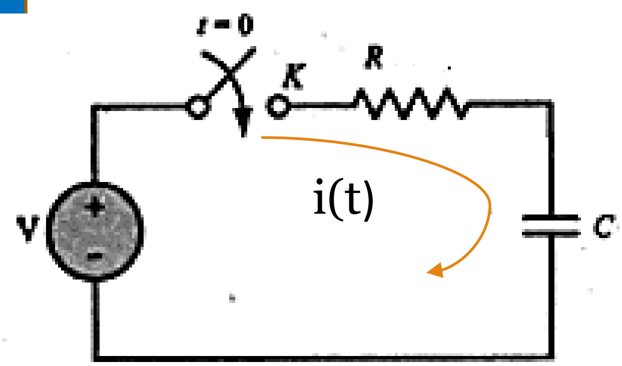






Transient Response Analysis – Examples

2.For the circuit shown in figure, the switch K is closed at t=0, then find i(0^+), $\frac{d}{dt}$i(0^+) and $\frac{d^2}{dt^2}i(0^+)$.



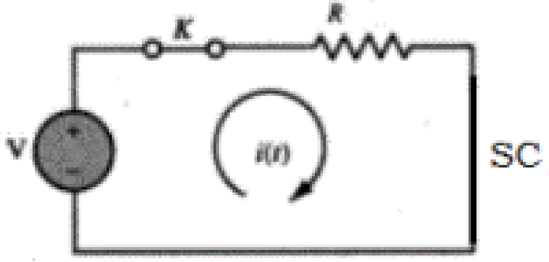
Solution:

C- Energy storage element-Uncharged state

$$v_C(0^-) = v_C(0^+) = 0\,Volts.$$

At t=0^+

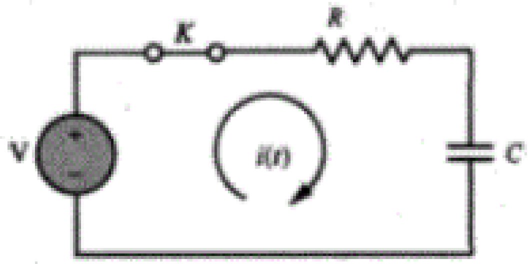
Equivalent circuit



Find **i(0^+)=V/R A.**

At t>0

Equivalent circuit



Apply KVL

$$V = Ri(t) + \frac{1}{C}\int i(t)dt \; ---(1)$$

D. (1) w.r.t

$$0 = \frac{Rdi(t)}{dt} + \frac{1}{C}i(t) \; ---(2)$$

At t=0+

$$\frac{d}{dt}i(0^+) = -\frac{1}{RC}i(0^+)$$

$$\boldsymbol{\frac{d}{dt}i(0^+) = -\frac{V}{R^2C}A/Sec}$$

D. (2) w.r.t

$$0 = \frac{Rd^2i(t)}{dt^2} + \frac{1}{C}\left(\frac{di(t)}{dt}\right) \; ---(3)$$

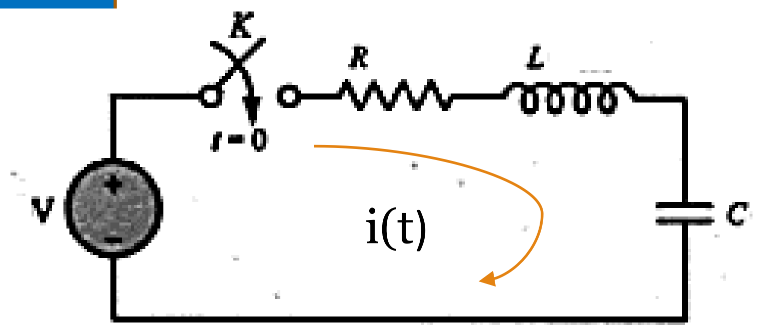
At t=0+

$$\frac{d^2}{dt^2}i(0^+) = -\frac{1}{RC}di(0^+)/\mathrm{dt}$$

$$\boldsymbol{\frac{d^2}{dt^2}i(0^+) = \frac{V^2}{R^3C^2}A/Sec^2}$$

Transient Response Analysis – Examples

3.For the circuit shown in figure, the switch K is closed at t=0, then find $i(0^+), \frac{d}{dt}i(0^+)$ and $\frac{d^2}{dt^2}i(0^+)$.

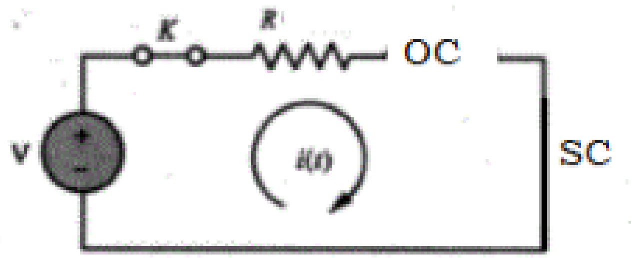


L, C storage elements- Uncharged state

At t=0-

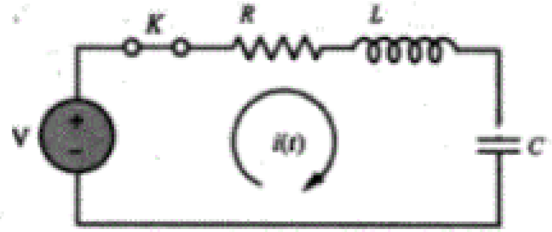
$$i_L(0^-) = i_L(0^+) = 0\ A$$
$$v_C(0^-) = v_C(0^+) = 0\ Volts$$

At t=0+



$$\boldsymbol{i(0^+) = 0\ A}$$

At t>0



$$KVL\ equation$$

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt --- (1)$$

At t=0+

$$V = Ri(0^+) + L\frac{di(0^+)}{dt} + v_C(0^+)$$

$$\boldsymbol{\frac{di(0^+)}{dt} = \frac{V}{L}\ A/sec}$$

D (2) w.r.t

$$0 = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}\ i(t) --- (2)$$

At t=0+

$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = \frac{VR}{L^2}\ A/sec^2}$$

Transient Response Analysis - Introduction

Initial and final conditions of energy storage elements

Procedure:

1. Identify the energy storage elements and its state

NOTE: State-Uncharged/Charged state.

2. Draw the equivalent circuit at t=0- and find the current through the inductors and voltage across the capacitors. i.e., $i_L(0^-)$ and $v_C(0^-)$

NOTE: $i_L(0^-) = i_L(0^+)$ and $v_C(0^-)=v_C(0^+)$

3. Draw the equivalent circuit at t=0+, by replacing inductor by $i_L(0^+)$ Amperes, capacitor by $v_C(0^+)$ volts and resistors are kept as it is.

NOTE: Charged current value and charged voltage value of inductor and capacitor respectively would be called as steady state values.

NOTE: To find the steady state values

i) Current through the inductor is maximum at steady state and would be calculated by replacing the inductor by short circuit. i.e., $i_{SC}=i_L(0-) = i_L(0^+)$

ii) Voltage across the capacitor is maximum at steady state and would be calculated by replacing the capacitor by open circuit. i.e., $v_{OC}= v_C(0^-)=v_C(0^+)$.

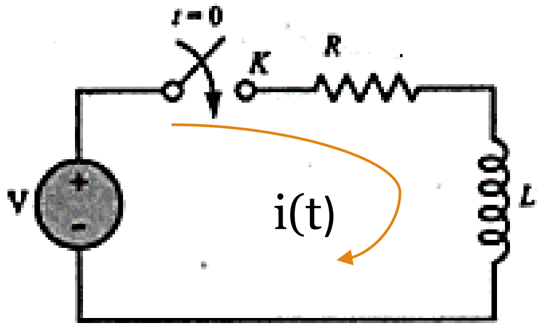
4. Find the initial voltages and currents at t=0+.

4. Draw the equivalent circuit at t>0 and obtain system equations (KVL/KCL).

5. Find the derivatives of initial voltages and currents using the above initial conditions, $i.e., \frac{d}{dt}i(0^+)$, $\frac{d^2}{dt^2}i(0^+), \frac{d}{dt}v(0^+) and \frac{d^2}{dt^2}v(0^+)$.

Transient Response Analysis – Examples

1.For the circuit shown in figure, the switch K is closed at t=0, then find $i(0^+)$, $\frac{d}{dt}i(0^+)$ and $\frac{d^2}{dt^2}i(0^+)$.



Solution:

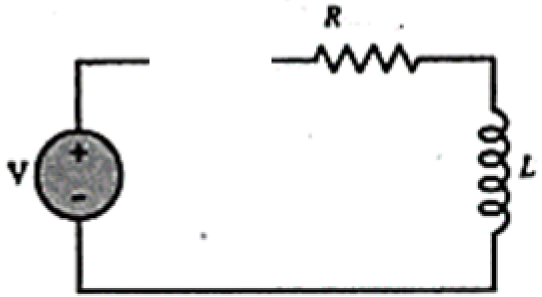
Step-1:

L is the energy storage element-Uncharged state

Step-2: t=0-

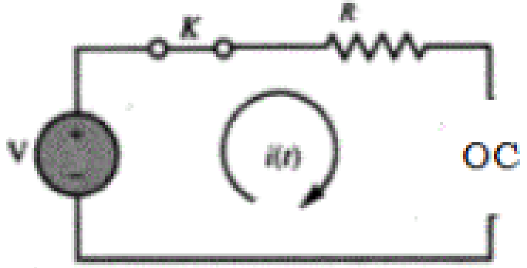
$$i_L(0^-) = 0A$$

$$i_L(0^+) = i_L(0^-) = 0\ A.$$



Step-3:

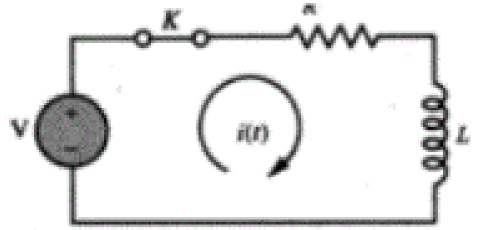
Equivalent circuit at t=0+



Step-4:

$$i(0^+) = 0A$$

Step-5: equivalent circuit at t>0



Step6: apply KVL

$$V = Ri(t) + \frac{Ldi(t)}{dt} --- (1)$$

Step7: at t=0+

From equation (1)

$$V = Ri(t) + \frac{Ldi(t)}{dt}$$

$$\frac{d}{dt}i(0^+) = \frac{V}{L} - \frac{R}{L}i(0^+)$$

$$\boldsymbol{\frac{d}{dt}i(0^+) = \frac{V}{L}\ \mathbf{A/sec}}$$

Differentiate Equation (1) w.r.t t

$$0 = \frac{Rdi(t)}{dt} + \frac{Ld^2i(t)}{dt^2} --- (2)$$

From equation (2).

$$\frac{Ld^2i(t)}{dt^2} = -\frac{Rdi(t)}{dt}$$

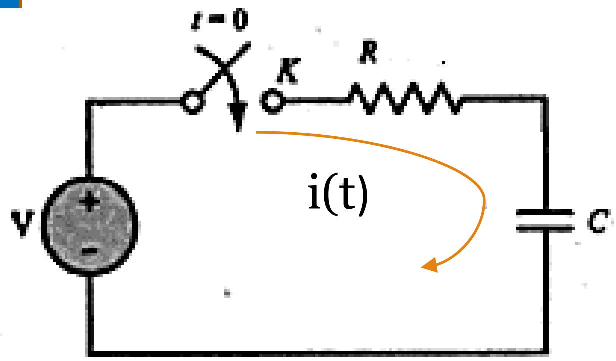
At t=0+

$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = -\frac{Rdi(0^+)}{Ldt}}$$

$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = -\frac{VR}{L^2}\ \mathbf{A/sec^2}}$$

Transient Response Analysis – Examples

2.For the circuit shown in figure, the switch K is closed at t=0, then find $i(0^+)$, $\frac{d}{dt}i(0^+)$ and $\frac{d^2}{dt^2}i(0^+)$.



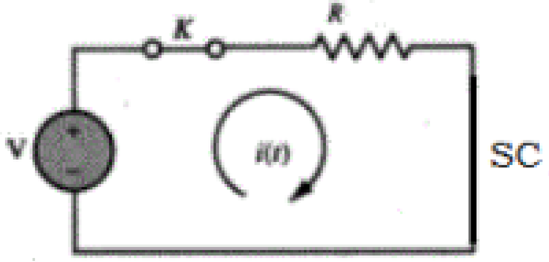
Solution:

C- Energy storage element-Uncharged state

$$v_C(0^-) = v_C(0^+) = 0\, Volts.$$

At t=0$^+$

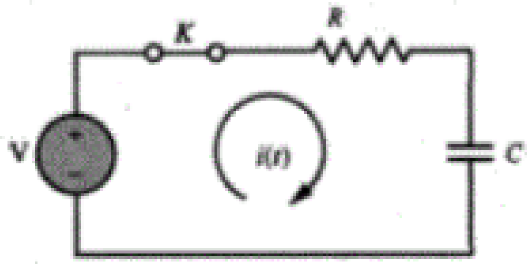
Equivalent circuit



Find **i(0$^+$)=V/R A.**

At t>0

Equivalent circuit



Apply KVL

$$V = Ri(t) + \frac{1}{C}\int i(t)dt \text{ ---(1)}$$

D. (1) w.r.t

$$0 = \frac{Rdi(t)}{dt} + \frac{1}{C}i(t) \text{ ---(2)}$$

At t=0+

$$\frac{d}{dt}i(0^+) = -\frac{1}{RC}i(0^+)$$

$$\boldsymbol{\frac{d}{dt}i(0^+) = -\frac{V}{R^2C}A/Sec}$$

D. (2) w.r.t

$$0 = \frac{Rd^2i(t)}{dt^2} + \frac{1}{C}\left(\frac{di(t)}{dt}\right) \text{ ---(3)}$$

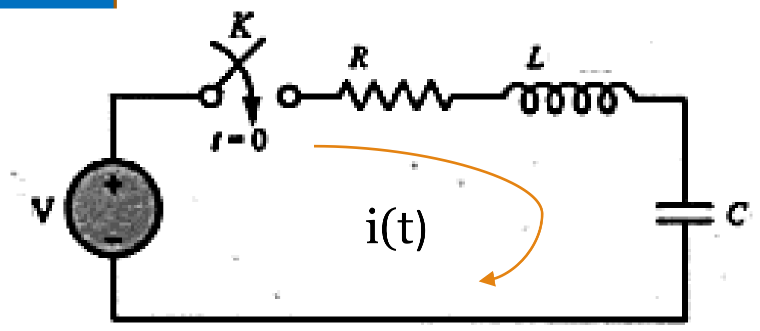
At t=0+

$$\frac{d^2}{dt^2}i(0^+) = -\frac{1}{RC}di(0^+)/\text{dt}$$

$$\boldsymbol{\frac{d^2}{dt^2}i(0^+) = \frac{V^2}{R^3C^2}A/Sec^2}$$

Transient Response Analysis – Examples

3.For the circuit shown in figure, the switch K is closed at t=0, then find $i(0^+)$, $\frac{d}{dt}i(0^+)$ and $\frac{d^2}{dt^2}i(0^+)$.

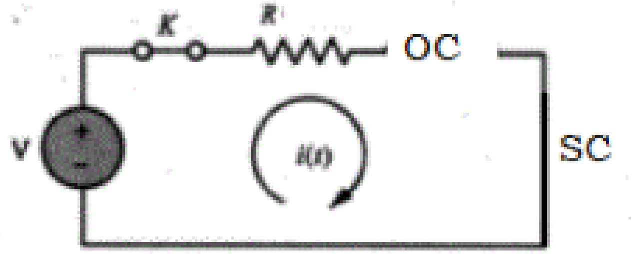


L, C storage elements- Uncharged state

At t=0-

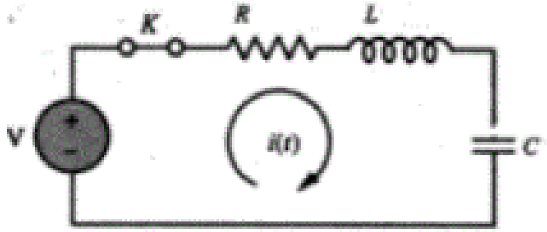
$$i_L(0^-) = i_L(0^+) = 0\ A$$
$$v_C(0^-) = v_C(0^+) = 0\ Volts$$

At t=0+



$$\boldsymbol{i(0^+) = 0\ A}$$

At t>0



$$KVL\ equation$$
$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt ---(1)$$

At t=0+

$$V = Ri(0^+) + L\frac{di(0^+)}{dt} + v_C(0^+)$$
$$\frac{\boldsymbol{di(0^+)}}{\boldsymbol{dt}} = \frac{\mathbf{V}}{\mathbf{L}}\mathbf{A/sec}$$

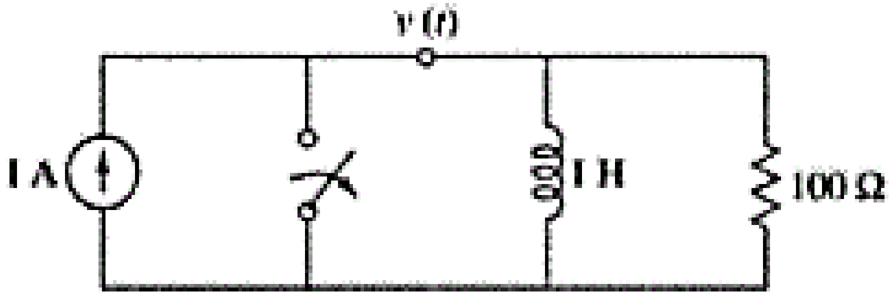
D (2) w.r.t

$$0 = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}\ i(t) ---(2)$$

At t=0+

$$\frac{\boldsymbol{d^2i(0^+)}}{\boldsymbol{dt^2}} = \frac{\mathbf{VR}}{\mathbf{L^2}}\ \mathbf{A/sec^2}$$

Transient Response Analysis – Examples



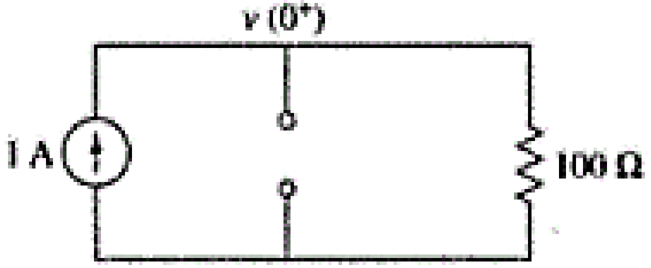
4.For the circuit shown in figure, the switch K is Opened at t=0, then find $v(0^+)$, $\frac{d}{dt}v(0^+)$ and $\frac{d^2}{dt^2}v(0^+)$.

Solution:

L-uncharged state

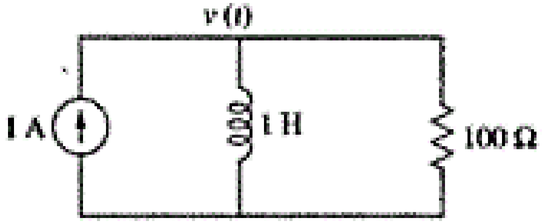
$$i_L(0^-) = i_L(0^+) = 0\ A$$

At t=0+



:s

At t>0



Apply KCL

$$1 = i_1 + i_2$$

$$1 = \frac{1}{L}\int v(t)dt + \frac{v(t)}{100} ---(1)$$

D (1) w.r.t.t

$$0 = \frac{v(t)}{L} + \frac{1}{100}\frac{dv(t)}{dt} ---(2)$$

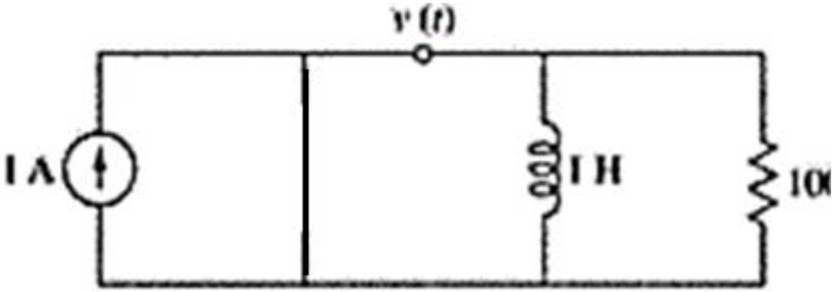
At t=0+

$$\frac{dv(0^+)}{dt} = -100\, v(0^+) \Rightarrow \mathbf{-10^4 V/sec}$$

D. (2) w.r.t.t

$$0 = \frac{dv(t)}{dt} + \frac{1}{100}\frac{d^2v(t)}{dt^2} ---(3)$$

$$\frac{d^2v(t)}{dt^2} = -100\frac{dv(t)}{dt}$$

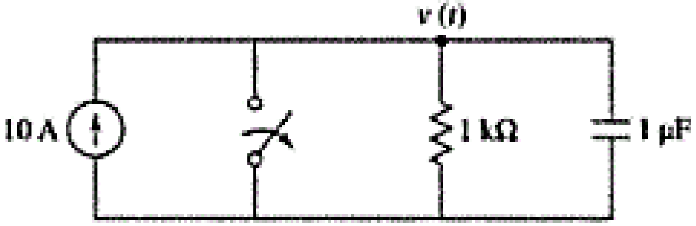


At t=0+

$$\frac{d^2v(0^+)}{dt^2} = -100\frac{dv(0^+)}{dt}$$

$$\boldsymbol{\frac{d^2v(0^+)}{dt^2} = 10^6 v/\mathrm{sec}^2}$$

Transcient Response Analysis – Examples



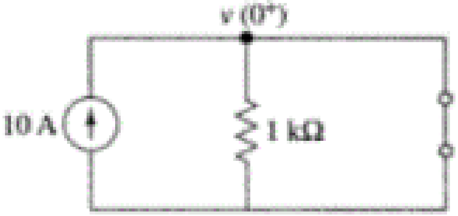
5.For the circuit shown in figure, the switch K is Opened at t=0, then find $v(0^+)$, $\frac{d}{dt}v(0^+)$ and $\frac{d^2}{dt^2}v(0^+)$.

Solution:

C-Uncharged state

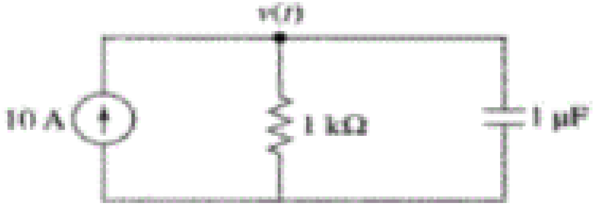
$$v_C(0^-) = v_C(0^+) = 0\,Volts$$

At t=0+



= **0 Volts**

At t>0



Apply KCL

$$10 = \frac{v(t)}{10^3} + 10^{-6}\frac{dv(t)}{dt} \quad ---(1)$$

$$\frac{dv(t)}{dt} = 10^6\left(10 - \frac{v(t)}{10^3}\right)$$

At t=0+

$$\frac{dv(0^+)}{dt} = 10^6\left(10 - \frac{v(0^+)}{10^3}\right)$$

$$\boldsymbol{\frac{dv(0^+)}{dt} = 10^7\ v/sec}$$

D (1) w.r.t.t

$$0 = 10^{-3}\frac{dv(t)}{dt} + 10^{-6}\frac{d^2v(t)}{dt^2} \quad ---(2)$$

At t=0+

$$0 = 10^{-3}\frac{dv(t)}{dt} + 10^{-6}\frac{d^2v(t)}{dt^2}$$

$$\frac{d^2v(0^+)}{dt^2} = -\frac{10^3 dv(0^+)}{dt}$$

$$\boldsymbol{\frac{d^2v(0^+)}{dt^2} = -10^7 v/sec^2}$$

Transient Response Analysis – Examples

Important points and expressions to be remembered

1. Voltage across the short circuit is zero
2. Voltage across the open circuit is maximum
3. Current through the open circuit is zero
4. Current through the short circuit is maximum
5. R

$v(t) = Ri(t)$ and $i(t) = \frac{v(t)}{R}$

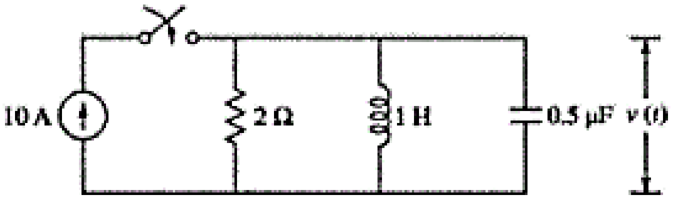
6. C

$$v(t) = \frac{1}{C}\int i(t)dt \;\; and \; i(t) = C\frac{dv(t)}{dt}$$

7. L

$$v(t) = L\frac{di(t)}{dt} \; and \; i(t) = \frac{1}{L}\int v(t)dt.$$

Transient Response Analysis – Examples



6.For the circuit shown in figure, the switch K is closed at t=0, then find $v(0^+)$, $\frac{d}{dt}v(0^+)$ and $\frac{d^2}{dt^2}v(0^+)$.

Solution:

L, C – uncharged state.

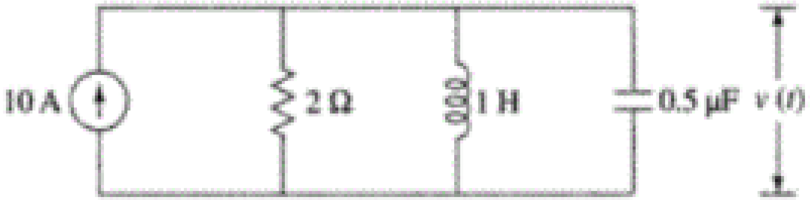
$$i_L(0^-) = i_L(0^+) = 0\ A$$

$$v_C(0^-) = v_C(0^+) = 0\ Volts.$$

At t=0+

$$\boldsymbol{v(0^+) = 0\ Volts}$$

At t>0



Apply KCL

$$10 = \frac{v(t)}{2} + \int v(t)dt + 0.5\ x10^{-6}\frac{dv(t)}{dt} ---(1)$$

At t=0+

$$\boldsymbol{\frac{dv(0^+)}{dt} = 20x10^6\frac{volts}{sec}}.$$

D (1) w.r.t. t

$$0 = \frac{1}{2}\frac{dv(t)}{dt} + v(t) + 0.5x10^{-6}\frac{d^2v(t)}{dt^2} ---(2)$$

At t=0+

$$0.5x10^{-6}\frac{d^2v(t)}{dt^2} = -\frac{1}{2}\frac{dv(t)}{dt} - v(t)$$

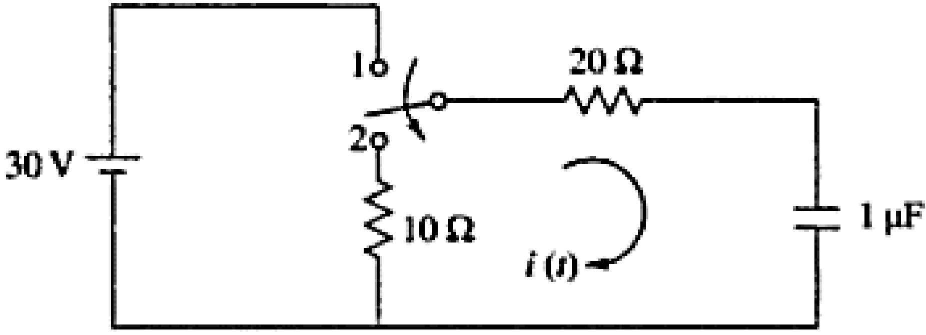
$$0.5x10^{-6}\frac{d^2v(t)}{dt^2} = -\frac{1}{2}20x10^6 - 0$$

$$0.5x10^{-6}\frac{d^2v(t)}{dt^2} = -10^7$$

$$\frac{d^2v(t)}{dt^2} = -\frac{10^7}{0.5x10^{-6}}$$

$$\boldsymbol{\frac{d^2v(0^+)}{dt^2} = -2x10^{13}V/sec^2}$$

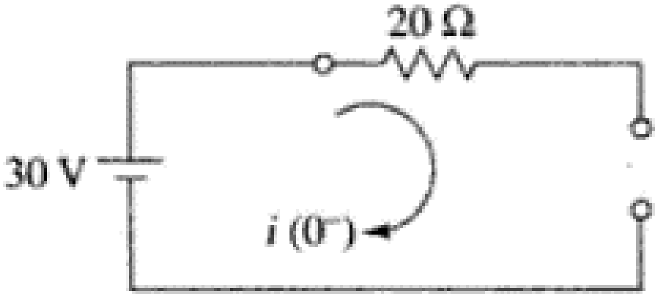
7. In the network shown in figure, the switch is changed from the position 1 to the position 2 at t=0, steady state condition having reached before switching. Find the values of i, di/dt and d^2i/dt^2 at t=0+



Solution:

Capacitor C- Reached steady state condition at t=0-

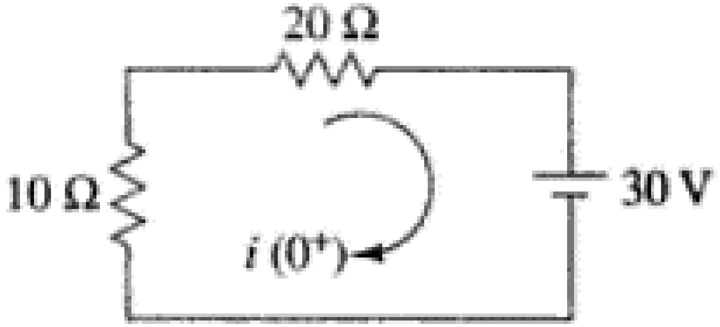
The equivalent circuit at t=0-



$$v_C(0^-) = 30V = v_C(0^+)$$

A

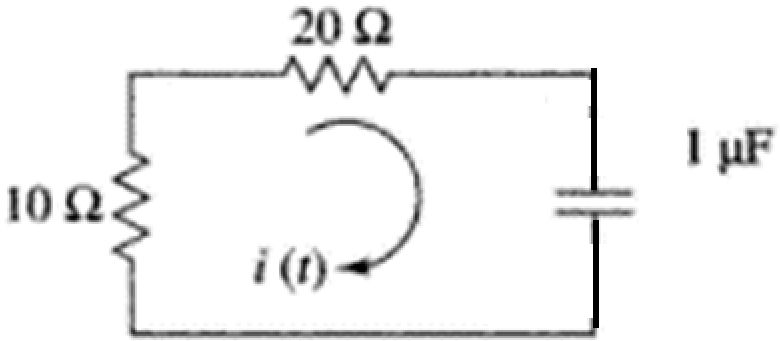
Equivalent circuit



$$30i(0^+) = -30$$

$$\boldsymbol{i(0^+) = -1A.}$$

At t>0



Apply KVL

$$30\, i(t) + 10^6 \int i(t) + 30 = 0 ---(1)$$

Transient Response Analysis – Examples

Apply KVL

$$30\,i(t) + 10^6 \int i(t) + 30 = 0 ---(1)$$

D (1) w.r.t.t.

$$30\frac{di(t)}{dt} + 10^6 i(t) = 0 ---(2)$$

At t=0+

$$\frac{d}{dt}i(0^+) = -\frac{10^6 i(0^+)}{30}$$

$$\mathbf{\frac{d}{dt}i(0^+) = \frac{10^6}{30}\,A/sec}$$

$$D\ (2)w.r.t.t$$

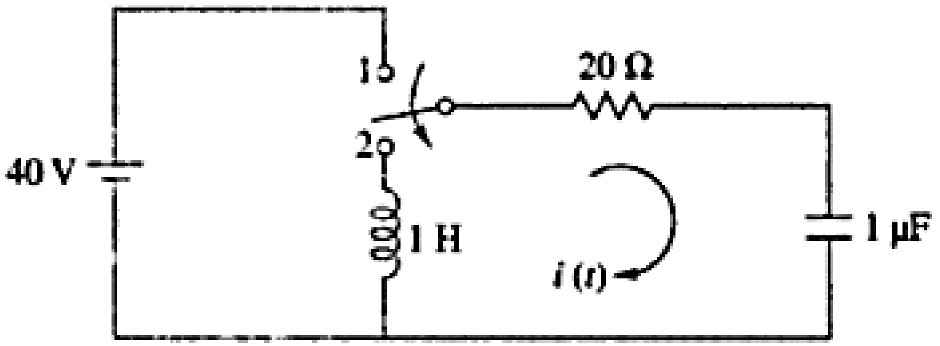
$$\frac{30d^2i(t)}{dt^2} + \frac{10^6 di(t)}{dt} = 0 ---(3)$$

$$\frac{d^2i(t)}{dt^2} = -\frac{10^6}{30}\frac{di(t)}{dt}$$

At t=0+

$$\boldsymbol{\frac{d^2i(0^+)}{dt^2} = -\frac{10^{12}}{900} \Rightarrow \frac{10^{10}}{9} A/sec^2}$$

8. In the network shown in figure, the switch is changed from the position 1 to the position 2 at t=0, steady state condition having reached before switching. Find the values of i, di/dt and d^2i/dt^2 at t=0+



Solution:

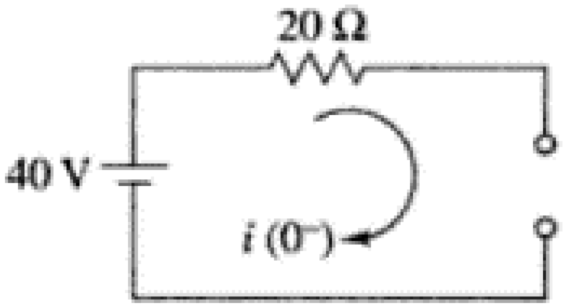
Inductor L- Uncharged state

Capacitor C-Steady state condition at t=0-

$$\boldsymbol{i_L(0^-) = i_L(0^+) = 0\ A}$$

At t=0-

Equivalent circuit

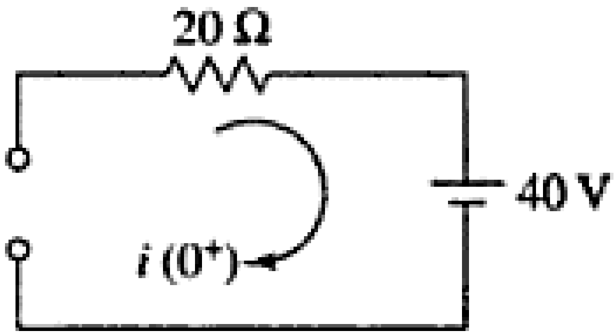


$v_{20} + 40 \Rightarrow 20(-i) + 40$

$\boldsymbol{v_C(0^+)}$

At t=0+

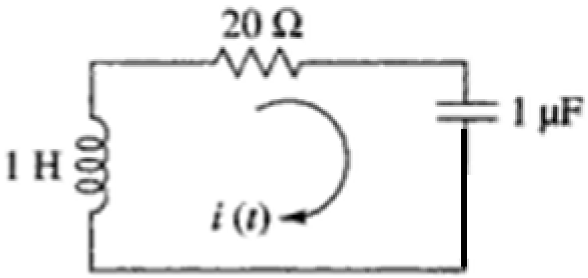
Equivalent circuit



$$\boldsymbol{i(0^+) = 0\ A}$$

Transient Response Analysis – Examples

At t>0
Equivalent circuit



Apply KVL

$$\frac{di(t)}{dt} + 20i(t) + 10^6 \int i(t)dt = 0 ---(1)$$

At t=0+

$$\frac{di(0^+)}{dt} = -20i(0^+) - 40$$

$$\boldsymbol{\frac{di(0^+)}{dt} = -40\ A/sec}$$

D (1) w.r.t.t

$$\frac{d^2i(t)}{dt^2} + 20\frac{di(t)}{dt} + 10^6 i(t) = 0 ---(2)$$

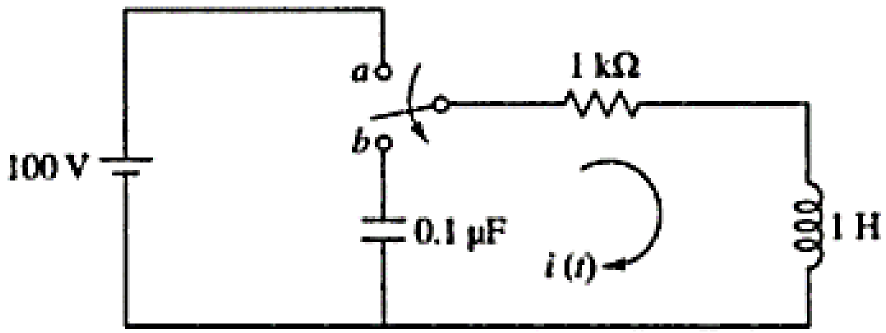
At t=0+

$$\frac{d^2i(0^+)}{dt^2} = -20\frac{di(0^+)}{dt} - 10^6 i(0^+)$$

$$\boldsymbol{\frac{di(0^+)}{dt} = 800\ A/sec^2}$$

Transient Response Analysis – Examples

9. In the network shown in figure, the switch is changed from the position a to the position b at t=0, steady state condition having reached before switching. Find the values of i, di/dt and d^2i/dt^2 at t=0+



Solution:

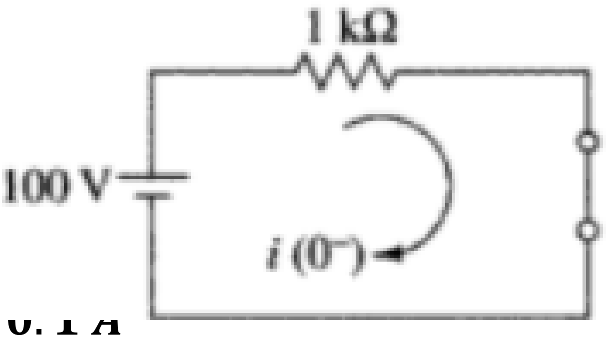
Capacitor C= Uncharged state

$$v_C(0^-) = v_C(0^+) = 0\ V$$

Inductor L=Steady state condition .

At t=0-

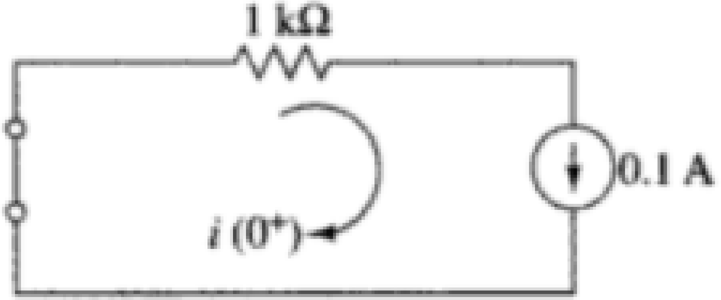
Equivalent circuit.



$i_L(0^-) = i_L(0^+) = 0.1\ A$

At t=0+

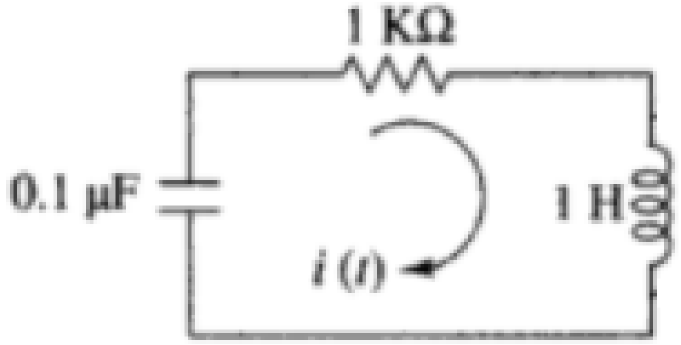
Equivalent circuit.



$i(0^+) = 0.1A$

Transient Response Analysis – Examples

At t>0
Equivalent circuit



Apply KVL

$$10^7 \int i(t)dt + 10^3 i(t) + \frac{di(t)}{dt} = 0 --- (1)$$

At t=0+

$$\frac{di(0^+)}{dt} = -10^3 i(0^+) \Rightarrow \mathbf{-100A/\ sec}$$

D (1) w.r.t.t

$$10^7 i(t) + 10^3 \frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} = 0 --- (2)$$

At t=0+

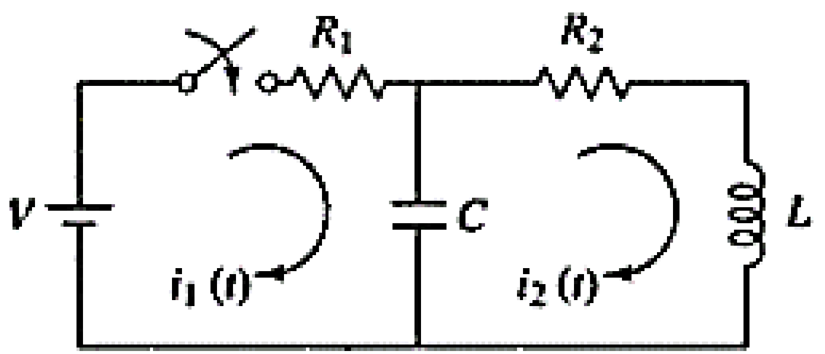
$$10^7 i(0^+) + 10^3 \frac{di(0^+)}{dt} + \frac{d^2 i(0^+)}{dt^2} = 0 --- (2)$$

$$\frac{d^2 i(0^+)}{dt^2} = -10^7 i(0^+) - 10^3 \frac{di(0^+)}{dt}$$

$$\frac{d^2 i(0^+)}{dt^2} = -10^7 x0.1 - 10^3.(-100) \Rightarrow -10x10^5 + 10^5$$

$$\boldsymbol{\frac{d^2 i(0^+)}{dt^2} = -9x10^5 A/sec^2}$$

Transient Response Analysis – Examples



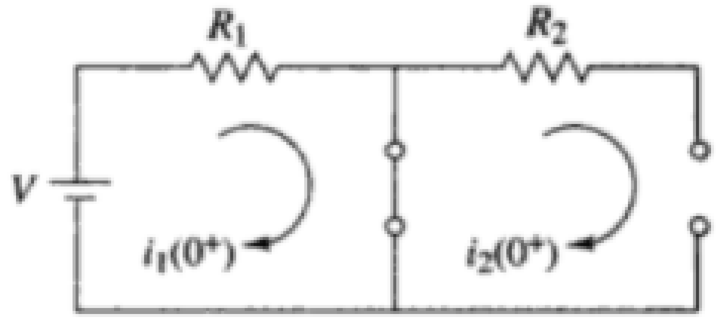
10. In the network shown in figure, the switch is closed at t=0. Find the values of $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}, \frac{d^2i_1}{dt^2}$ and $\frac{d^2i_2}{dt^2}$ at $t = 0^+$

Solution:
Capacitor C=Uncharged
Inductor L=Uncharged

$$\boldsymbol{i_L(0^-) = i_L(0^+) = 0A}$$

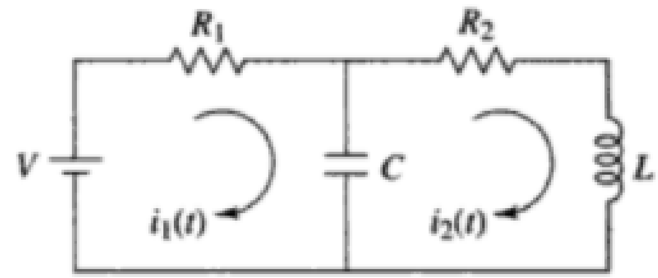
$$\boldsymbol{v_C(0^-) = v_C(0^+) = 0\ V}$$

At t=0+



$$\boldsymbol{i_1(0^+) = \frac{}{R_1} A; i_2(0^+) = 0A}$$

At t>0



Apply KVL

$$V = R_1 i_1(t) + \frac{1}{C}\int \big(i_1(t) - i_2(t)\big)dt \text{ ---(1)}$$

$$0 = \frac{1}{C}\int \big(i_2(t) - i_1(t)\big)dt + R_2 i_2(t) + L\frac{di_2(t)}{dt} \text{ ---(2)}$$

Transient Response Analysis – Examples

Apply KVL

$$V = R_1 i_1(t) + \frac{1}{C}\int \big(i_1(t) - i_2(t)\big)dt \;---(1)$$

$$0 = \frac{1}{C}\int \big(i_2(t) - i_1(t)\big)dt + R_2 i_2(t) + L\frac{di_2(t)}{dt} \;---(2)$$

D (1) w.r.t t

$$0 = R_1\frac{di_1(t)}{dt} + \frac{1}{C}\, i_1(t) - \frac{1}{C} i_2(t) \;---(3)$$

At t=0+

$$\frac{di_1(0^+)}{dt} = -\frac{1}{R_1 C}\, i_1(0^+) + \frac{1}{R_1 C} i_2(0^+)$$

$$\frac{\boldsymbol{di_1(0^+)}}{\boldsymbol{dt}} = -\frac{\mathbf{V}}{\mathbf{CR_1^2}}\,\mathbf{A/sec}$$

From (2)

$$\frac{di_2(t)}{dt} = -\frac{1}{LC}\int \big(i_2(t) - i_1(t)\big)dt - \frac{R_2}{L} i_2(t) \;---(4)$$

$$at\ t = 0^+$$

$$\frac{\boldsymbol{di_2(0^+)}}{\boldsymbol{dt}} = \mathbf{0}\,\frac{\boldsymbol{A}}{\boldsymbol{sec}}$$

D.(3) w.r.t.t.

$$0 = R_1\frac{d^2\, i_1(t)}{dt^2} + \frac{1}{C}\frac{di_1(t)}{dt} - \frac{1}{C}\frac{di_2(t)}{dt} \;---(5)$$

At t=0+

$$\frac{d^2\, i_1(0^+)}{dt^2} = -\frac{1}{CR_1}\frac{di_1(0^+)}{dt} + \frac{1}{CR_2}\frac{di_2(0^+)}{dt}$$

$$\frac{\boldsymbol{d^2\, i_1(0^+)}}{\boldsymbol{dt^2}} = \frac{\boldsymbol{V}}{\boldsymbol{C^2R_1^3}}\,\boldsymbol{A/sec^2}$$

D (2) w.r.t.t

$$0 = \frac{1}{C}\,[i_2(t)] - \frac{1}{C} i_1(t) + R_2\frac{di_2(t)}{dt} + L\frac{d^2 i_2(t)}{dt^2}$$

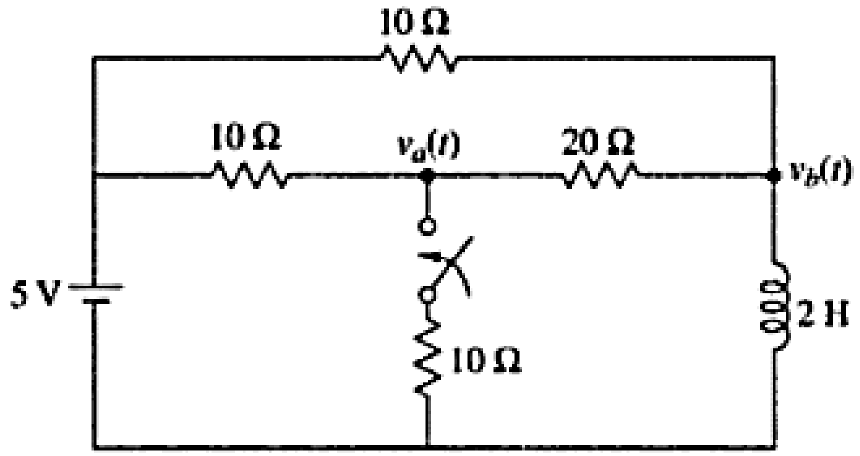
$$\frac{d^2 i_2(t)}{dt^2} = -\frac{1}{CL}\,[i_2(t)] + \frac{1}{CL} i_1(t) - \frac{R_2}{L}\frac{di_2(t)}{dt}$$

$$at\ t = 0^+$$

$$\frac{\boldsymbol{d^2 i_2(0^+)}}{\boldsymbol{dt^2}} = \frac{\boldsymbol{V}}{\boldsymbol{CLR_1}}\,\boldsymbol{A/sec}$$

Transient Response Analysis – Practice Problems

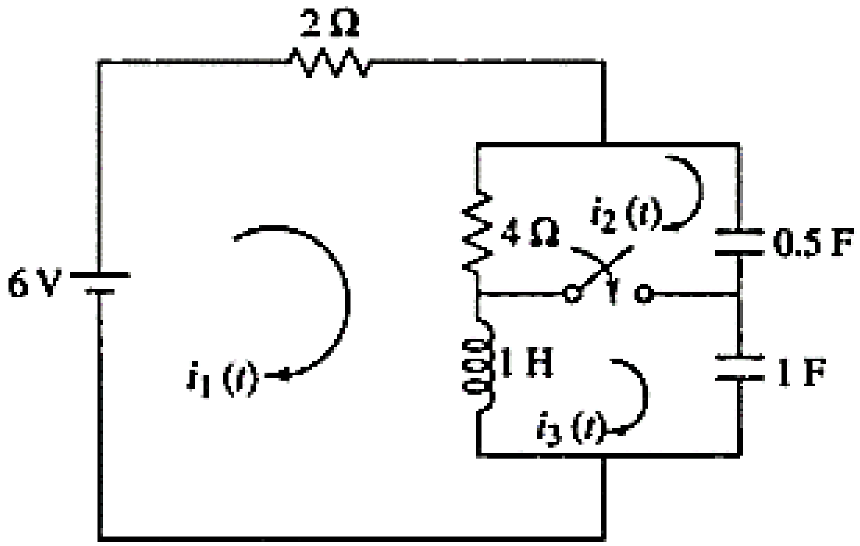
11. In the network shown in figure, the switch is closed at t=0. Find the values of $v_a(0^-), v_b(0^-), v_a(0^+) and\ v_b(0^+)$.



Transient Response Analysis – Practice Problems

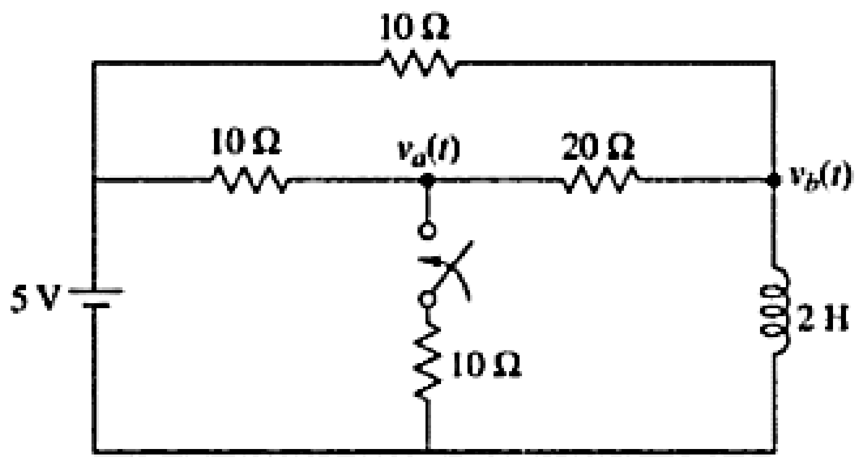
Transient Response Analysis – Practice Problems

12. In the network shown in figure, a steady state has reached with switch open. At t=0 switch is closed. Find the three loop currents at $t=0^+$.



Transient Response Analysis – Practice Problems

Transient Response Analysis – Examples

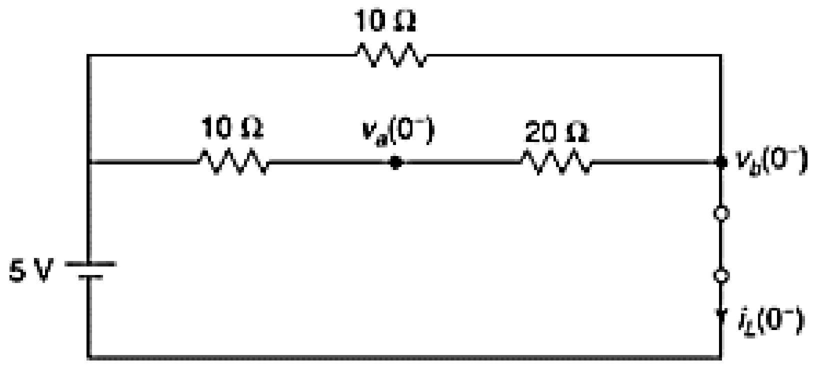


11. In the network shown in figure, the switch is closed at t=0. Find the values of $v_a(0^-), v_b(0^-), v_a(0^+) and\ v_b(0^+)$.

Solution:

L=2H is the energy storage element- attains steady state at t=0$^-$

at t=0-, equivalent circuit.



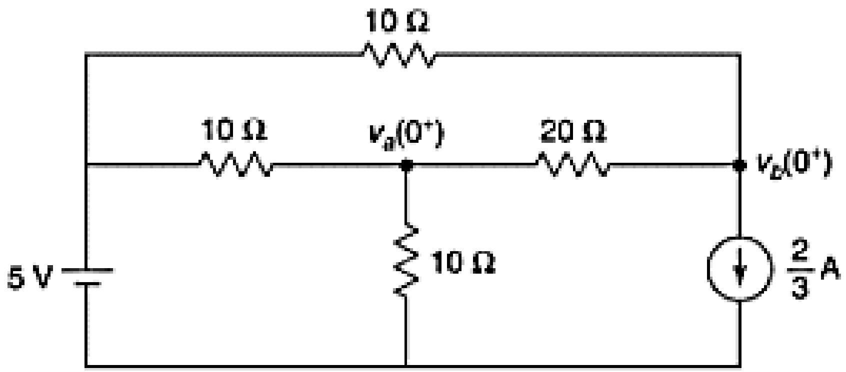
$i_L(0^-) = 5/(30||10) \Rightarrow \mathbf{0.667A}$

$\boldsymbol{V_b(0^-)}$ **= 0 Volts.**

$\boldsymbol{V_a(0^-) = V_{20} \Rightarrow 20 \ * 0.667 * \left(\frac{10}{10+30}\right)}$ **⇒3.33V**

At t=0+

Equivalent circuit



Transient Response Analysis – Examples

Nodal analysis

$$V_1 = 5V$$

$$\frac{5 - V_a(0^+)}{10} = \frac{V_a(0^+)}{10} + \frac{(V_a(0^+) - V_b(0^+))}{20}$$

$$-0.25V_a(0^+) + 0.05V_b(0^+) = -0.5 --- (1)$$

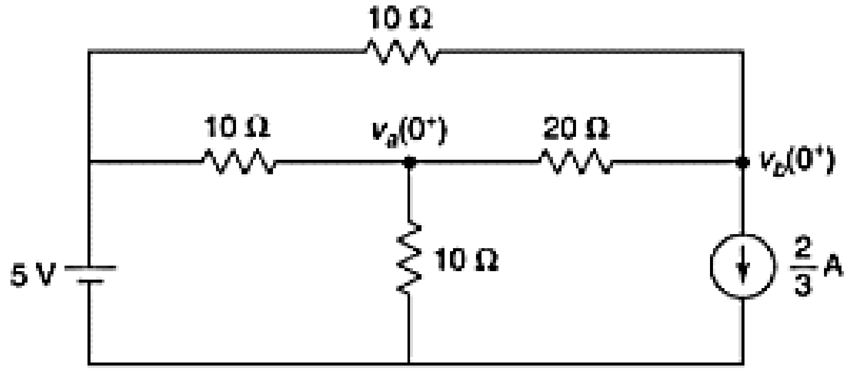
$$\frac{5 - V_b(0^+)}{10} + \frac{(V_a(0^+) - V_b(0^+))}{20} = \frac{2}{3}$$

$$-0.15V_b(0^+) + 0.05V_a(0^+) = 0.1667 --- (2)$$

Solve equations (1) and (2)

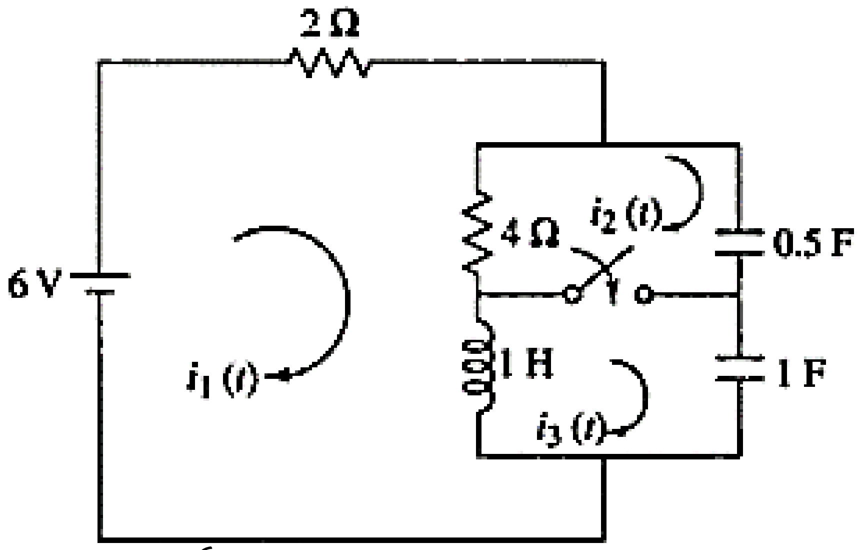
$$\boldsymbol{V_a(0^+) = 1.9\ V}$$

$$\boldsymbol{V_b(0^+) = -0.477V}$$

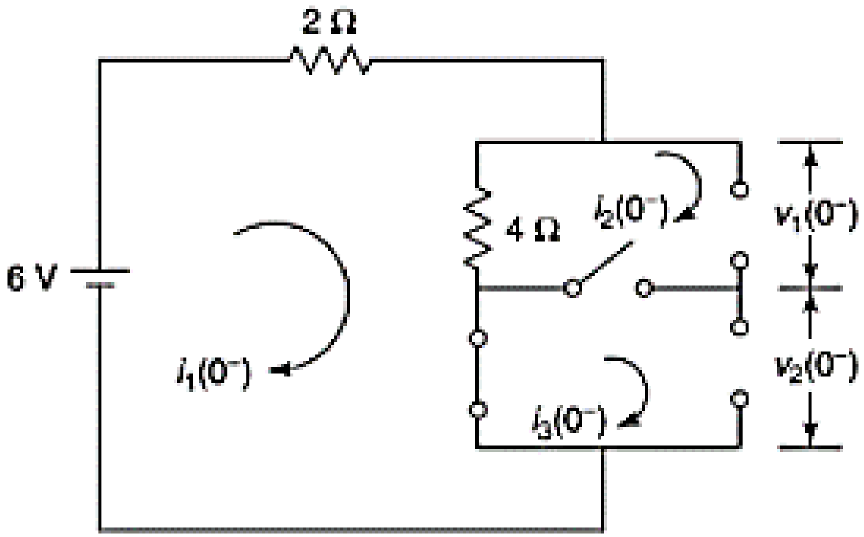


Transient Response Analysis – Examples

12. In the network shown in figure, a steady state has reached with switch open. At t=0 switch is closed. Find the three loop currents at $t=0^+$.



Solution:
L=1H ; attains steady state at t=0-
C1=05F; attains steady state at t=0-
C2=1F; attains steady state at t=0-
At t=0-
Equivalent circuit.



$$I_L(0^-) = \frac{6}{2+4} = \mathbf{1A}$$

$$v_1(0^-) + v_2(0^-) = v_4 \Rightarrow 4V ---(1)$$

The charges on capacitors are equal if the capacitors are Connected in series.

$$Q_1 = Q_2$$
$$C_1 v_1 = C_2 v_2$$
$$0.5 v_1(0^-) = v_2(0^-)$$
$$0.5 v_1(0^-) - v_2(0^-) = 0 ---(2)$$

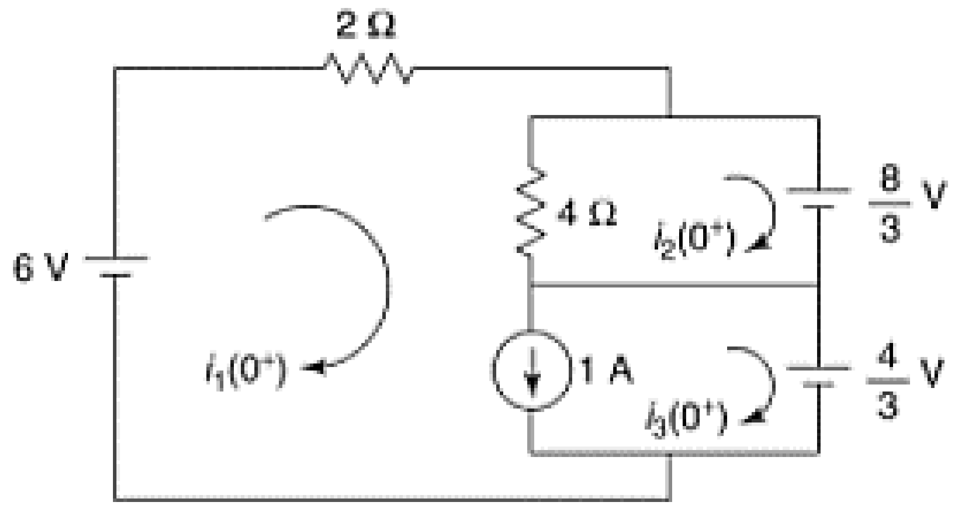
Solve (1) and (2)

$$\boldsymbol{v_1(0^-) = 2.66V\ ;\ and\ v_2(0^-) = 1.33V}$$

Transient Response Analysis – Examples

At t=0+

Equivalent circuit.



$$i_1(0^+) - i_3(0^+) = 1 - - -(1)$$

Apply KVL to super mesh

$$2i_1(0^+) + 4\left(i_1(0^+) - i_2(0^+)\right) + \frac{4}{3} - 6 = 0$$

$$6i_1(0^+) - 4i_2(0^+) = 4.67 - - -(2)$$

Apply KVL to mesh 2

$$4\left(i_2(0^+) - i_1(0^+)\right) + \frac{8}{3} = 0$$

$$-4i_1(0^+) + 4i_2(0^+) = 2.667 - -(3)$$

Solve (1),(2) and (3)

$$\boldsymbol{i_1(0^+) = 1A}$$

$$\boldsymbol{i_2(0^+) = 0.33A}$$

$$\boldsymbol{i_3(0^+) = 0A}$$

Transient Response Analysis - Introduction

Solution of homogeneous differential equation
Consider a differential homogeneous equation of first order.

$$a\frac{di(t)}{dt} + b\,i(t) = 0 ---(1)$$

Rearrange the equation by separating the variables.

$$a\frac{di(t)}{dt} = -bi(t)$$

$$\frac{di(t)}{dt} = \left(-\frac{b}{a}\right) i(t) ---(2)$$

Multiply dt on both sides

$$di(t) = -\frac{b}{a} i(t)dt$$

$$\frac{di(t)}{i(t)} = -\frac{b}{a}dt ---(3)$$

Integrate on both sides
We get,

$$\int \frac{di(t)}{i(t)} = \int -\frac{b}{a}dt$$

$$\int \frac{di(t)}{i(t)} = \int -\frac{b}{a}dt$$

$$\ln i(t) = -\frac{b}{a}t + K_I$$

$$K_I \; is \; defined \; at \; t = -\infty \; to \; 0^- \; or \; at \; t = 0^+$$

$$\ln i(t) = \ln e^{-\frac{b}{a}t} + \ln K$$

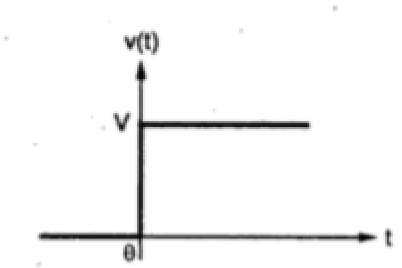
$$\ln i(t) = \ln(Ke^{-\frac{b}{a}t})$$

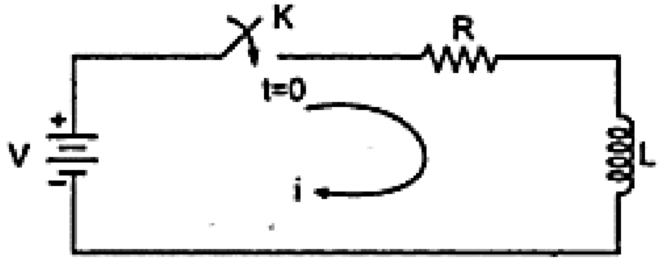
$$\boldsymbol{i(t) = Ke^{-\frac{b}{a}t} ---(4)}$$

General solution- K is unknown
Particular solution – K is known

Transient Response Analysis - Introduction

Step response of RL series circuit





?

At t=0, V is applied to the circuit and at $t=0^-$, inductor is at rest(uncharged condition)

Hence, $i_L(0^-)=0$ Amperes, $i_L(0^+)=0$ Amperes.

At t>0,

KVL equation

$$V = Ri(t) + L\frac{di(t)}{dt} ---(1)$$

$$L\frac{di(t)}{dt} = V - R\,i(t)$$

$$divide\ R\ on\ both\ sides$$

$$\frac{L}{R}\frac{di(t)}{dt} = \frac{V}{R} - i(t)$$

Multiply dt on both sides, we get

Multiply dt on both sides, we get

$$\frac{di(t)}{\frac{V}{R} - i(t)} = \frac{R}{L}dt ---(2)$$

Apply integration on both sides

$$\int \frac{di(t)}{\frac{V}{R} - i(t)} = \int \frac{R}{L}dt$$

$$-\ln\left(\frac{V}{R} - i(t)\right) = \frac{R}{L}t + K$$

Transient Response Analysis - Introduction

$$-\ln\left(\frac{V}{R}-i(t)\right)=\frac{R}{L}t+K\ ---(3)$$

$$at\ t=0^{+}, i(t)=0$$

$$K=-\ln\left(\frac{V}{R}\right)\ ---(4)$$

At t>0

$$-\ln\left(\frac{V}{R}-i(t)\right)=\frac{R}{L}t-\ln\left(\frac{V}{R}\right)$$

$$-\ln\left(\frac{V}{R}-i(t)\right)=\ln e^{\frac{R}{L}t}\ -\ln\left(\frac{V}{R}\right)$$

$$-\ln\left(\frac{V}{R}-i(t)\right)=-(-\ln e^{\frac{R}{L}t}\ +\ln\left(\frac{V}{R}\right))$$

$$\frac{V}{R}-i(t)=\frac{\frac{V}{R}}{e^{\frac{R}{L}t}}$$

$$i(t)=\frac{V}{R}-\frac{\frac{V}{R}}{e^{\frac{R}{L}t}}$$

$$\boldsymbol{i(t)=\frac{V}{R}-\frac{V}{R}e^{-\left(\frac{R}{L}t\right)}\ ---(5)}$$

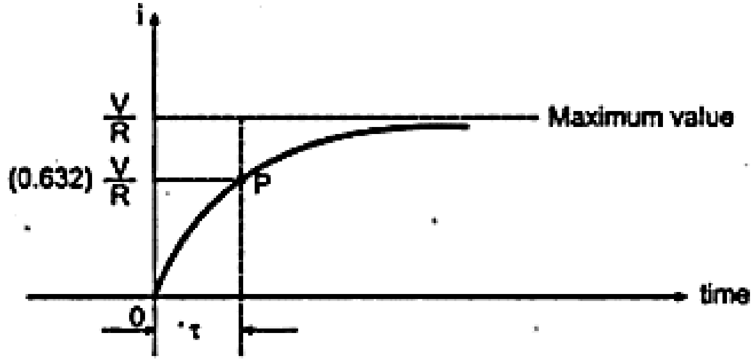
$$\boldsymbol{i(t)=\ i_{ss}+i_{tr}(t)}$$

$$V_L=L\frac{di(t)}{dt}$$

$$V_L=L\frac{d}{dt}\left[\frac{V}{R}-\frac{V}{R}e^{-\left(\frac{R}{L}t\right)}\right]$$

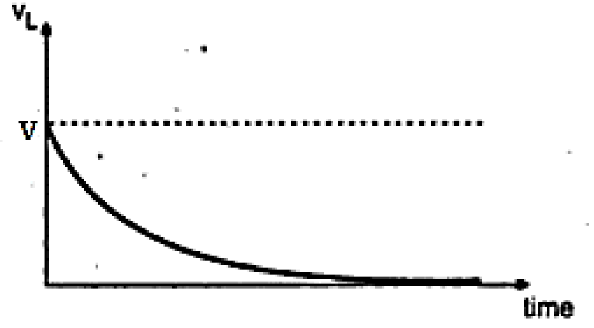
$$V_L=L.\left(-\frac{V}{R}\right).e^{-\frac{R}{L}t}.\left(-\frac{R}{L}\right)$$

$$\boldsymbol{V_L=Ve^{-\frac{R}{L}t}}$$



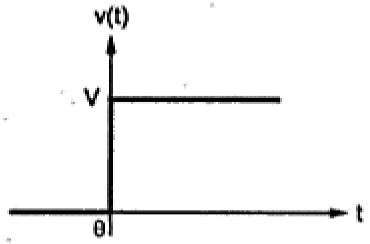
NOTE: $V_L=\mathrm{V}e^{\left(-\frac{R}{L}\right)t}$

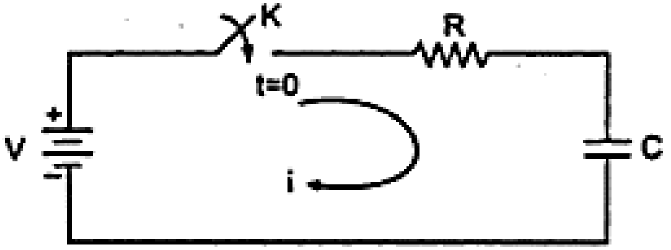
If inductor carries initial current



Transient Response Analysis - Introduction

Step response of RC series circuit





$$V_C(0^-) = V_C(0^+) = 0Volts.$$

At t>0

$$V = V_R + V_C$$
$$V = iR + V_C$$
$$i = i_R = I_C$$
$$w.k.t., I_C = C\frac{dV_C}{dt}$$
$$V = RC\frac{dV_C}{dt} + V_C --- (1)$$

Rearrange the equation by separating the variables

$$V - V_C = RC\frac{dV_C}{dt}$$

$$V - V_C = RC\frac{dV_C}{dt}$$
$$\frac{dV_C}{V - V_C} = \frac{dt}{RC} --- (2)$$

Integration on both sides

$$\int \frac{dV_C}{V - V_C} = \int \frac{dt}{RC} + K$$
$$-\ln(V - V_C) = \frac{1}{RC}t + K --- (3)$$

Where K is the integral constant defined at t=0+

$$-\ln(V - V_C) = \ln e^{\frac{t}{RC}} + K$$

Transient Response Analysis - Introduction

$$-\ln(V - V_C) = \ln e^{\frac{t}{RC}} + K$$

At t=0+

w.k.t. VC=0

$$K = -\ln V$$

$$-\ln(V - V_C) = \ln e^{\frac{t}{RC}} - lnV$$

$$-\ln(V - V_C) + \ln V = \ln e^{\frac{t}{RC}}$$

$$\ln\left(\frac{V}{V - V_C}\right) = \ln e^{\frac{t}{RC}}$$

$$\frac{V}{V - V_C} = e^{\frac{t}{RC}}$$

$$V = Ve^{\frac{t}{RC}} - V_C e^{\frac{t}{RC}}$$

$$V_C e^{\frac{t}{RC}} = Ve^{\frac{t}{RC}} - V$$

$$V_C = V - Ve^{-\frac{t}{RC}} ----(4)$$

$$V = iR + V_C$$

$$iR = V - V_C$$

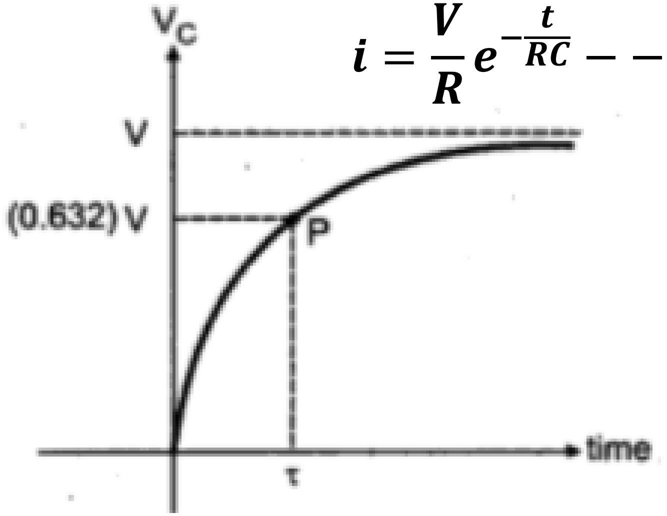
w.k.t., $V_C = V - Ve^{-\frac{t}{RC}}$ -

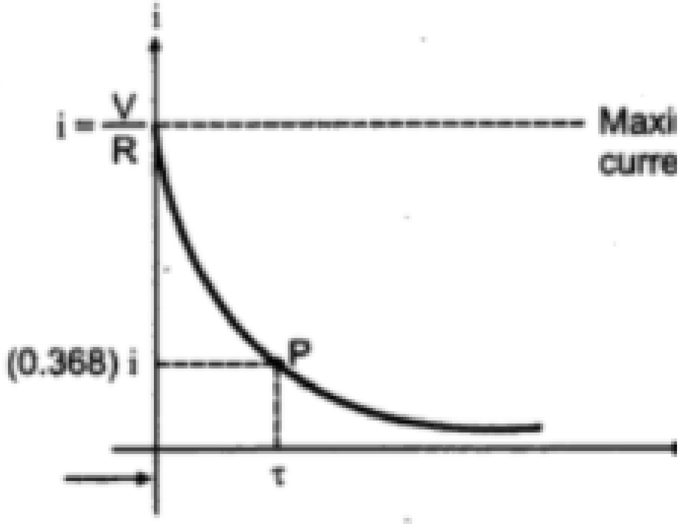
$$iR = V - \left(V - Ve^{-\frac{t}{RC}}\right)$$

$$iR = V - V + Ve^{-\frac{t}{RC}}$$

$$iR = Ve^{-\frac{t}{RC}}$$

$$\boldsymbol{i = \frac{V}{R} e^{-\frac{t}{RC}} --(5)}$$





Transient Response Analysis - Introduction

Summary

Homogeneous differential equation

Form-1

$$a\frac{di}{dt} + b\, i(t) = 0$$

Solution:

$$i(t) = Ke^{-\frac{b}{a}t}$$

Where, K is the initial condition

Or

$$a\frac{dv(t)}{dt} + b\, v(t) = 0$$

Solution:

$$v(t) = Ke^{-\left(\frac{b}{a}\right)t}$$

Where, K is the initial condition

Transient Response Analysis - Introduction

Summary

Homogeneous differential equation

Form-2

$$a\frac{di}{dt} + b\,i(t) = c$$

Solution:

$$i(t) = \frac{c}{b}\left(1 - e^{-\frac{b}{a}t}\right)$$

Zero initial conditions

$$i(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t}$$

Or

$$a\frac{dv(t)}{dt} + b\,v(t) = c$$

Solution:

$$v(t) = \frac{c}{b}\left(1 - e^{-\left(\frac{b}{a}\right)t}\right)$$

Zero initial conditions

Example: $L\frac{di(t)}{dt} + R\,i(t) = V$

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

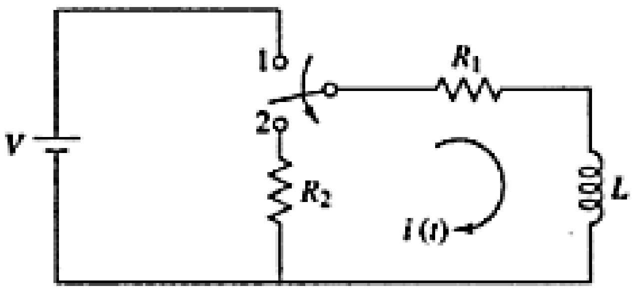
$$i(t) = \frac{V}{R} - Ke^{-\left(\frac{R}{L}\right)t}$$

Example: $\mathrm{RC}\frac{dv_C(t)}{dt} + v_C(t) = V$

$$v_C(t) = V - Ve^{-\left(\frac{1}{RC}\right)t}$$

$$v_C(t) = V - Ke^{-\left(\frac{1}{RC}\right)t}$$

Transient Response Analysis - Introduction



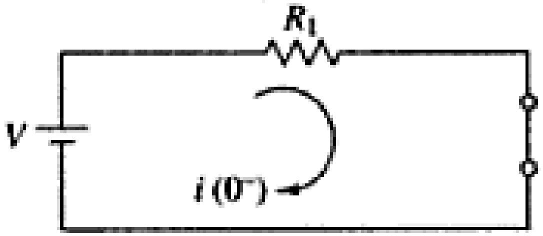
P. In the network shown in figure, the switch is initially at the position 1 and the steady state having reached, the switch is changed to the position 2 at t=0. find current i(t).

Solution:

L- energy storage element-charged state at t=0-.

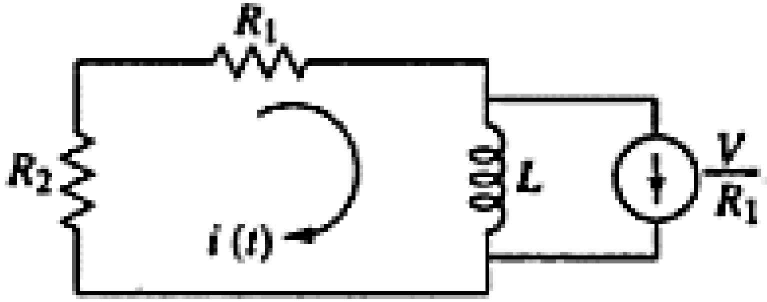
At t=0-

Equivalent circuit



$$i(0^-) = \frac{V}{R_1} A.$$

At t>0, equivalent circuit.



Apply KVL

$$R_1 i(t) + R_2 i(t) + \frac{L di(t)}{dt} = 0$$

$$L\frac{di(t)}{dt} + (R_1{+}R_2)i(t) = 0 ---(1)$$

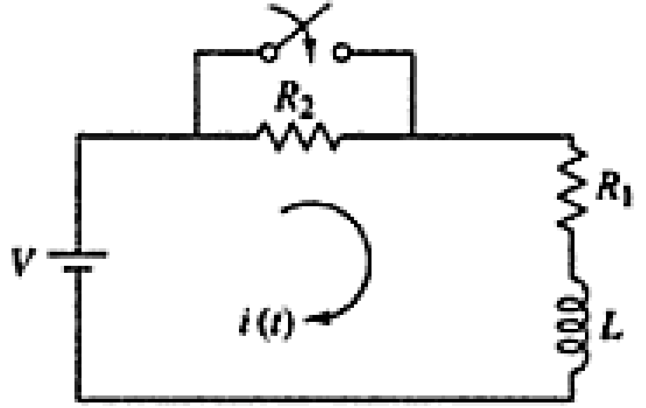
$$i(t) = Ke^{-\left(\frac{R_1+R_2}{L}\right)t} ---(2)$$

Where, K is the initial value of current through he inductor i.e., at 0-.

Therefore K = $\frac{V}{R_1}$ ----(3)

$$\mathbf{i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1+R_2}{L}\right)t} ---(4)}$$

Transient Response Analysis - Introduction



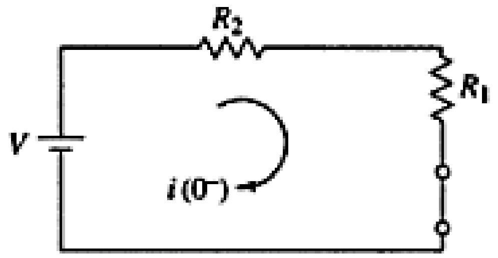
P. In the network shown in figure, the switch is closed at t=0, a steady state having previously been attained. Find current i(t).

Solution:

L- charged state at t=0-

At t=0-

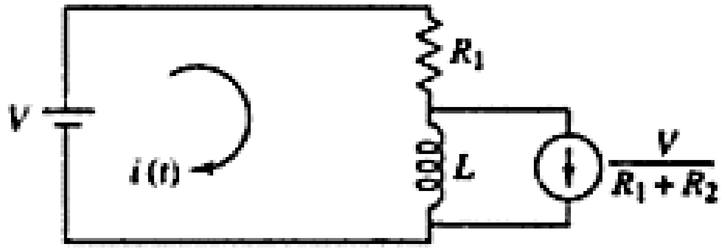
Equivalent circuit



$$i(0^-) = \frac{V}{R_1+R_2}A$$

At t>0

Equivalent circuit



Apply KVL

$$V = R_1 i(t) + L\frac{di(t)}{dt}$$

$$L\frac{di(t)}{dt} + R_1\, i(t) = V \;---(1)$$

w.k.t.,

$$i(t) = \frac{V}{R_1} - Ke^{-\left(\frac{R_1}{L}\right)t} \;---(2)$$

Where, K is the initial value of current through the inductor.

$$at\ t = 0+,$$

$$\frac{V}{R_1 + R_2} = \frac{V}{R_1} - K$$

$$\frac{V}{R_1 + R_2} = \frac{V}{R_1} - K$$

$$K = \frac{V}{R_1 + R_2} - \frac{V}{R_1}$$

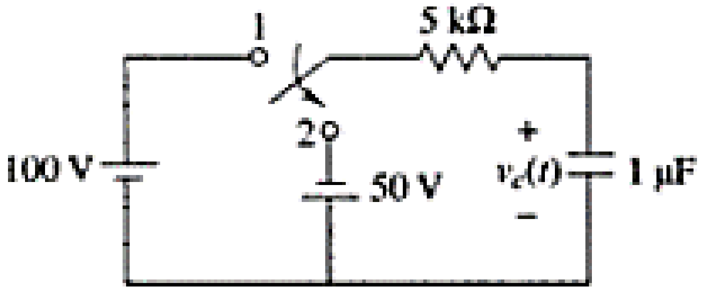
Therefore,

$$i(t) = \frac{V}{R_1} - \left(\frac{V}{R_1 + R_2} - \frac{V}{R_1}\right) e^{-\left(\frac{R_1}{L}\right)t}$$

$$\boldsymbol{i(t) = \frac{V}{R_1}(2 - \frac{R_1}{R_1 + R_2})e^{-\left(\frac{R_1}{L}\right)t}}$$

Transient Response Analysis - Introduction

P. In the network shown in figure, the switch is moved from the position 1 to 2 at t=0. Find $V_C(t)$ at t>0.

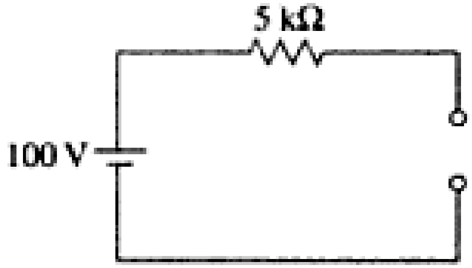


Solution:

C-charged state at t=0-

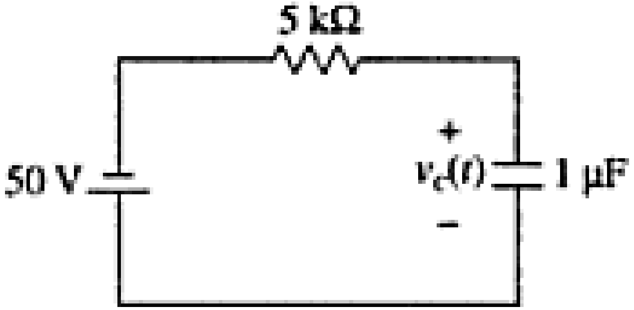
At t=0-

Equivalent circuit



$) = 100V$

At t>0, equivalent circuit.



$$-50 = V_R + V_C$$

$$-50 = 1x10^{-6}\frac{dv_c}{dt}x\ 5000 + V_c$$

$$-50 = \frac{5}{1000}\frac{dv_C}{dt} + vc$$

$$\frac{dv_C(t)}{dt} + 200v_C = -10^4 ---(1)$$

$$v(t) = \frac{c}{b}\left(1 - e^{-\left(\frac{b}{a}\right)t}\right)$$

$$v_C(t) = -\frac{10^4}{200} - Ke^{-(200)t} ---(2)$$

Transient Response Analysis

Summary

Differential equation-1

$$a\frac{dy(t)}{dt} + by(t) = 0$$

Solution:

$$\frac{dy(t)}{dt} = -\frac{b}{a}y(t)$$

$$\frac{dy(t)}{y(t)} = -\frac{b}{a}dt$$

$$\int\frac{dy(t)}{y(t)} = \int -\frac{b}{a}dt$$

$$\ln y(t) = -\frac{b}{a}t + K'$$

$$\ln y(t) = \ln e^{-\left(\frac{b}{a}\right)t} + \ln K --- (1)$$

$$y(t) = Ke^{-\left(\frac{b}{a}\right)t} --- (2)$$

Where, K is integral constant at $t=0^+$.

If $y(t)=0$, at $t=0^+$. from (1).

$$\ln y(t) = \ln e^{-\left(\frac{b}{a}\right)t} + \ln K$$

$$K = 0$$

$$\therefore \mathbf{y(t) = 0}$$

If $y(t)=y(0^+)=x$, at $t=0^+$. from (1).

$$\ln y(t) = \ln e^{-\left(\frac{b}{a}\right)t} + \ln K$$

$$K = x$$

$$\therefore \mathbf{y(t) = xe^{-\left(\frac{b}{a}\right)t}}$$

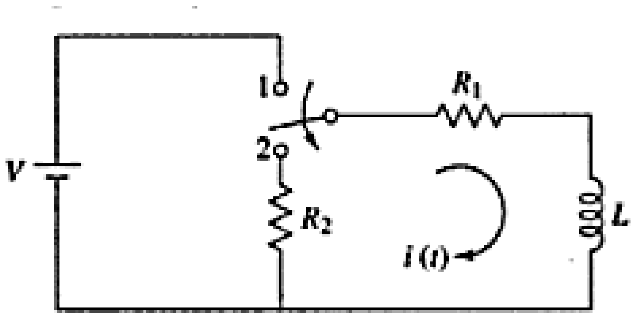
NOTE:

- y(t) may be current i(t) or voltage v(t)
- i(t) may be branch currents/loop currents/$i_L(t)$
- v(t) may be branch voltages/Node voltages/$v_C(t)$.

Transient Response Analysis

Example:

- Consider a circuit.



- At t>0
- KVL equation

$$R_1 i(t) + R_2 i(t) + \frac{Ldi(t)}{dt} = 0$$

$$L\frac{di(t)}{dt} + (R_1 + R_2)i(t) = 0$$

- This is in the form of differential equation-1

$$a\frac{di(t)}{dt} + bi(t) = 0$$

- w.k.t., the solution for differential equation.

$$i(t) = Ke^{-\left(\frac{b}{a}\right)t}, \text{Where, K is integral constant at } t = 0^+.$$

If $i(t)=0$, at $t=0^+$. from (1).

$$\ln i(t) = \ln e^{-\left(\frac{b}{a}\right)t} + \ln K$$

$$K = 0$$

$$\therefore \mathbf{i(t) = 0}$$

If $i(t)=\frac{V}{R_1}$, at $t=0^+$. from (1).

$$\ln i(t) = \ln e^{-\left(\frac{b}{a}\right)t} + \ln K$$

$$K = \frac{V}{R_1}$$

$$\therefore \mathbf{i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1+R_2}{L}\right)t}}$$

Transient Response Analysis - Introduction

Differential equation-2

$$a\frac{dy(t)}{dt} + b\,y(t) = c$$

Solution:

$$a\frac{dy(t)}{dt} = c - by(t)$$

$$\frac{dy(t)}{c - by(t)} = \frac{1}{a}dt$$

$$\int \frac{dy(t)}{c - by(t)} = \int \frac{1}{a}dt$$

$$-\left(\frac{1}{b}\right)\ln\big(c - by(t)\big) + K' = \frac{b}{a}t$$

$$-\ln\big(c - by(t)\big) + b\ln K = \ln e^{\left(\frac{b}{a}\right)t}$$

$$b\ln K - \ln e^{\left(\frac{b}{a}\right)t} = \ln(c - by(t)$$

$$\frac{bK}{e^{\left(\frac{b}{a}\right)t}} = c - by(t)$$

Also, $\boldsymbol{K = \frac{c}{b}e^{\left(\frac{b}{a}\right)t} - y(t).e^{\left(\frac{b}{a}\right)t} --- (2)}$

$$bKe^{-\left(\frac{b}{a}\right)t} = c - by(t)$$

$$by(t) = c - bKe^{-\left(\frac{b}{a}\right)t}$$

$$\mathbf{y(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t} --- (3)}$$

If y(t)=0, at t=0^+. from (2).

$$\mathrm{K} = \frac{\mathrm{c}}{\mathrm{b}}\mathrm{e}^{\left(\frac{\mathrm{b}}{\mathrm{a}}\right)\mathrm{t}} - \mathrm{y(t).e}^{\left(\frac{\mathrm{b}}{\mathrm{a}}\right)\mathrm{t}}$$

$$\mathrm{K} = \frac{c}{b}$$

$$\therefore \mathbf{y(t) = \frac{c}{b}\left(1 - e^{-\left(\frac{b}{a}\right)t}\right)}$$

If y(t)=x, at t=0^+. from (1).

$$\mathrm{K} = \frac{\mathrm{c}}{\mathrm{b}}\mathrm{e}^{\left(\frac{\mathrm{b}}{\mathrm{a}}\right)\mathrm{t}} - \mathrm{y(t).e}^{\left(\frac{\mathrm{b}}{\mathrm{a}}\right)\mathrm{t}}$$

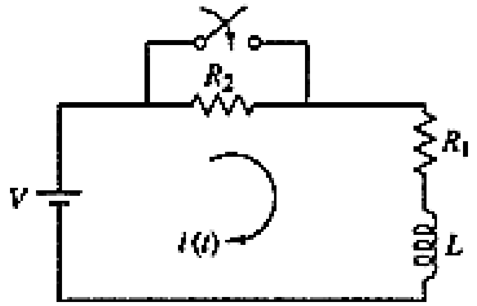
$$\mathrm{K} = \frac{c}{b} - x$$

$$\therefore \mathbf{y(t) = \frac{c}{b} - \left(\frac{c}{b} - x\right)e^{-\left(\frac{b}{a}\right)t}}$$

Transient Response Analysis - Introduction

Example:

- Consider a circuit.



- At t>0
- KVL equation

$$R_1 i(t) + L\frac{di(t)}{dt} = V$$

$$L\frac{di(t)}{dt} + R_1 i(t) = V$$

- This is in the form of differential equation-2

$$a\frac{di(t)}{dt} + bi(t) = c$$

- w.k.t., the solution for the differential equation.

$i(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t}$, Where, K is integral constant at $t = 0^+$.

If i(t)=0, at $t=0^+$.

$$K = \frac{c}{b}e^{\left(\frac{b}{a}\right)t} - i(t).e^{\left(\frac{b}{a}\right)t}$$

$$K = \frac{V}{R_1}$$

$$\therefore \mathbf{i(t)} = \frac{\mathbf{V}}{\mathbf{R_1}}\left(\mathbf{1} - \mathbf{e}^{-\left(\frac{R_1}{L}\right)t}\right)$$

If $i(t)=\frac{V}{R_1+R_2}$, at $t=0^+$.

$$K = \frac{c}{b}e^{\left(\frac{b}{a}\right)t} - i(t).e^{\left(\frac{b}{a}\right)t}$$

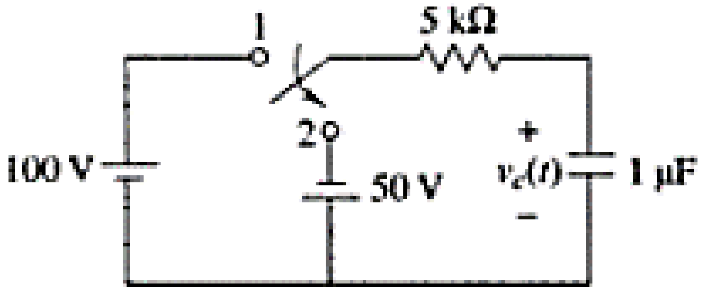
$$K = \frac{V}{R_1} - \frac{V}{R_1 + R_2}$$

$$\therefore \mathbf{i(t)} = \frac{\mathbf{V}}{\mathbf{R_1}} - \left(\frac{V}{R_1} - \frac{V}{R_1 + R_2}\right)\mathbf{e}^{-\left(\frac{R_1}{L}\right)t}$$

$$\mathbf{i(t)} = \frac{\mathbf{V}}{\mathbf{R_1}} - \frac{\mathbf{V}}{\mathbf{R_1}}\mathbf{e}^{-\left(\frac{R_1}{L}\right)t} + \frac{\mathbf{V}}{\mathbf{R_1} + \mathbf{R_2}}\mathbf{e}^{-\left(\frac{R_1}{L}\right)t}$$

Transient Response Analysis - Introduction

P. In the network shown in figure, the switch is moved from the position 1 to 2 at t=0. Find $V_C(t)$ at t>0.

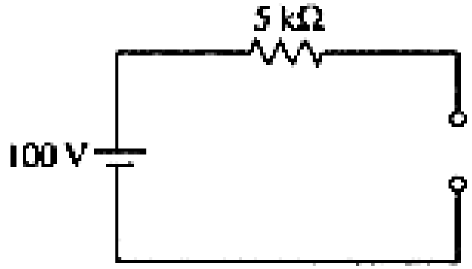


Solution:

C-charged state at t=0-

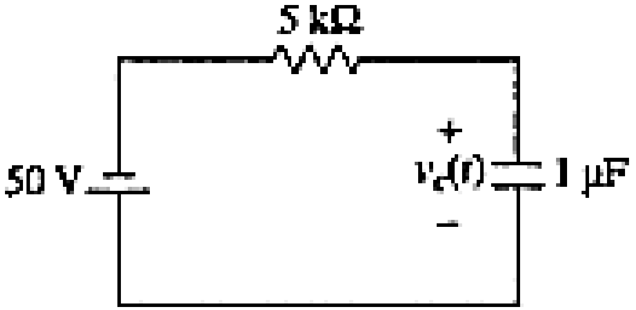
At t=0-

Equivalent circuit



$v_C(o$

At t>0, equivalent circuit.



$$-50 = 5000 * 10^{-6}\frac{dv_C(t)}{dt} + v_C(t)$$

$$5 * 10^{-3}\frac{dv_C(t)}{dt} + v_C(t) = -50 ---(1)$$

Equation (1) is in the form of differential equation-2

$$a\frac{dv(t)}{dt} + b\,v(t) = c$$

Solution for the above equation is

$$v(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t}$$

Now, $a = 5 * 10^{-3}, b = 1\ and\ c = -50$

Also

$$also\ K = \frac{c}{b} - x$$

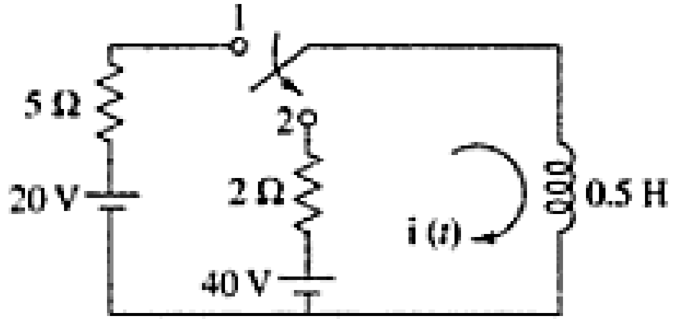
Where, $x = v_C(0^+) = 100$

$$\therefore v_C(t) = -50 - (-50 - 100)e^{-\left(\frac{1}{5*10^{-3}}\right)t}$$

$$\mathbf{v_c(t) = -50 + 150e^{-200t}}$$

Transient Response Analysis - Introduction

P. In the network shown in figure, the switch is moved from the position 1 to 2 at t=0. Find i(t) at t>0.

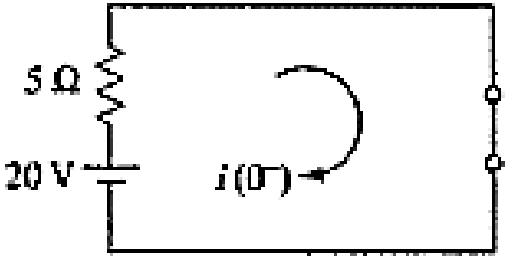


Solution:

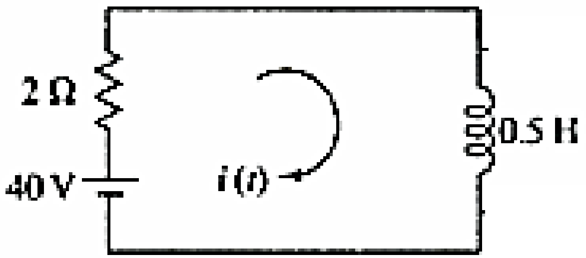
L-charged state at t=0-

At t=0-

Equivalent circuit



At t>0, equivalent circuit.



$$40 = 2i(t) + 0.5\frac{di(t)}{dt}$$

$$0.5\frac{di(t)}{dt} + 2i(t) = 40 ---(1)$$

Equation (1) is in the form of differential equation-2

$$a\frac{di(t)}{dt} + b\,i(t) = c$$

Solution for the above equation is

$$i(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t}$$

Now, $a = 0.5, b = 2$ and $c = 40$

Also

$$also\ K = \frac{c}{b} - x$$

Where, $x = i_L(0^-) = 4$

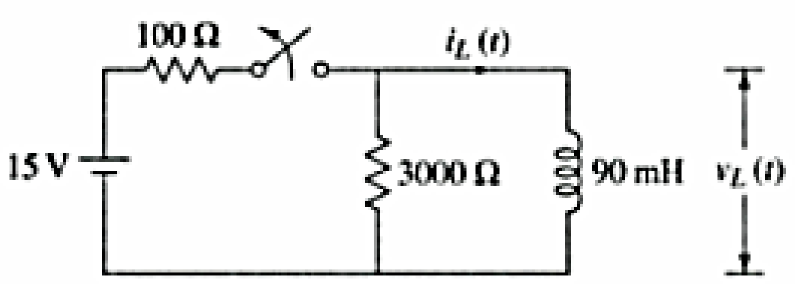
$$i(t) = \frac{40}{2} - \left(\frac{40}{2} - 4\right)e^{-\frac{2}{0.5}t}$$

$$i(t) = 20 - (20 - 4)e^{-4t}$$

$$\mathbf{i(t) = 20 - 16e^{-4t}}$$

Transient Response Analysis - Introduction

P. For the network shown in figure, steady state is reached with the switch closed. The switch is opened at t=0. Obtain expression for $i_L(t) and\ v_L(t)$.

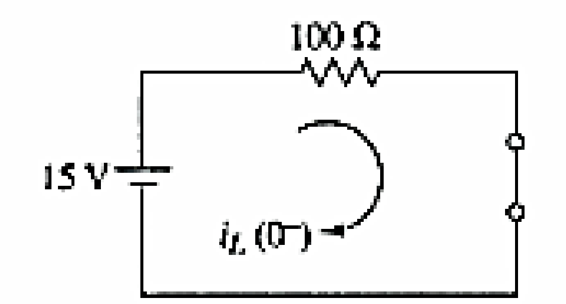


Solution:

L-charged state at t=0-

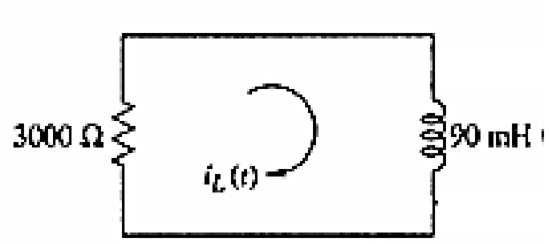
At t=0-

Equivalent circuit



$i_L(0^-) = 0.15A$

At t>0, equivalent circuit.



$$3000 i_L(t) + 90 * 10^{-3} \frac{di_L(t)}{dt} = 0$$

$$0.09 \frac{di_L(t)}{dt} + 3000 i_L(t) = 0 --- (1)$$

Equation (1) is in the form of differential equation-1

$$a \frac{di_L(t)}{dt} + b\, i_L(t) = 0$$

Solution for the above equation is

$$i_L(t) = K e^{-\left(\frac{b}{a}\right)t}$$

Now, $a = 0.09, b = 3000$

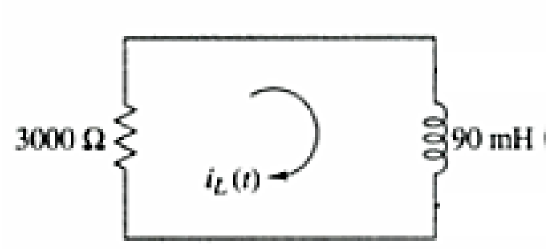
Also

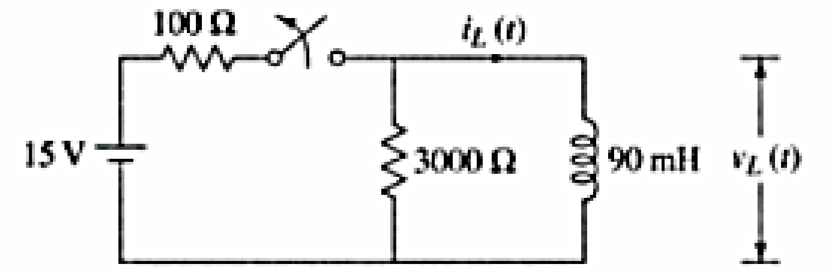
$$also\ K = x$$

Where, $x = i_L(0^-) = 0.15$

$$\boldsymbol{i_L(t) = 0.15 e^{-33.3*10^3 t}}$$

Transient Response Analysis - Introduction





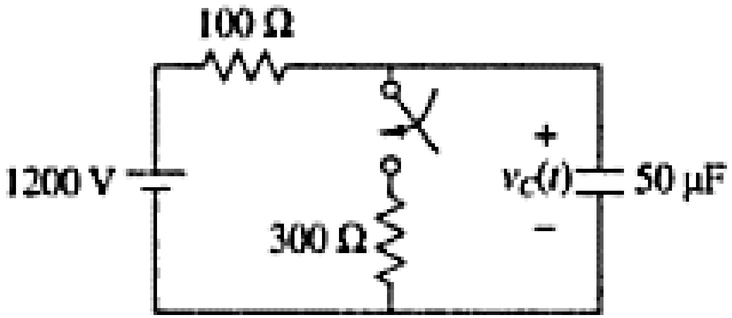
$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$v_L(t) = 0.09\frac{d}{dt}\left(\mathbf{0.15e^{-33.3*10^3 t}}\right)$$

$$v_L = 0.09 * 0.15 * (-33.33 * 10^3)e^{-33.33*10^3 t}$$

$$\boldsymbol{v_L(t) = -450e^{(-33.33*10^3)t}}$$

Transient Response Analysis - Introduction



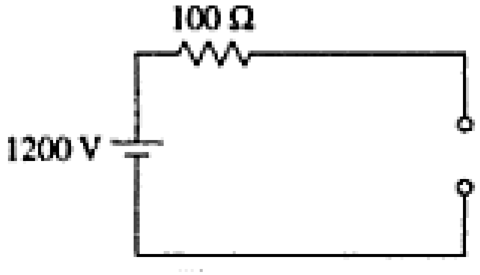
P. For the network shown in figure, the switch is open for a long time and closes at t=0. Determine $v_C(t)$.

Solution:

C-charged state at t=0-

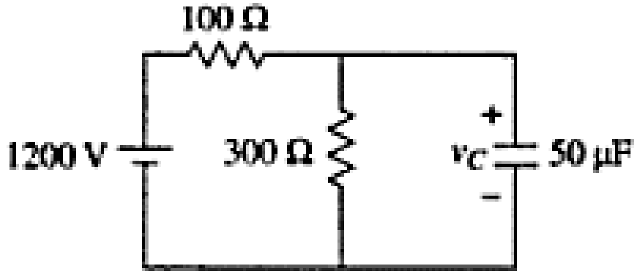
At t=0-

Equivalent circuit



$v_C(o^-)$ =1200V

At t>0, equivalent circuit.



Apply KCL

$$\frac{1200 - v_c(t)}{100} = \frac{v_C(t)}{300} + 50 * 10^{-6} \frac{dv_C(t)}{dt}$$

$$50 * 10^{-6} \frac{dv_C(t)}{dt} + 0.0133 v_C(t) = 12 ---(1)$$

Equation (1) is in the form of differential equation-2

$$a\frac{dy(t)}{dt} + b\, y(t) = c$$

Solution for the above equation is

$$y(t) = \frac{c}{b} - Ke^{-\left(\frac{b}{a}\right)t}$$

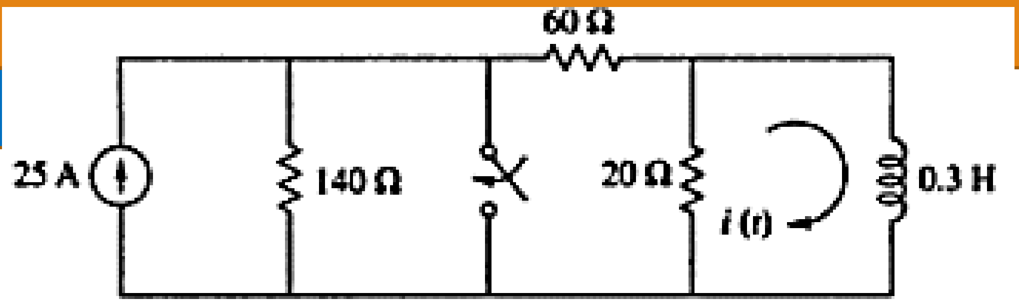
Now, a $= 50 * 10^{-6}, b = 0.0133\ and\ c = 12.$

Also, $K = \frac{c}{b} - x, where\ x = v_C(0^-) = 1200$

$$v_C(t) = 900 - (900 - 1200)e^{-266t}$$

$$\mathbf{v_C(t) = 900 + 300}e^{-266t}$$

Transient Response Analysis - Introduction



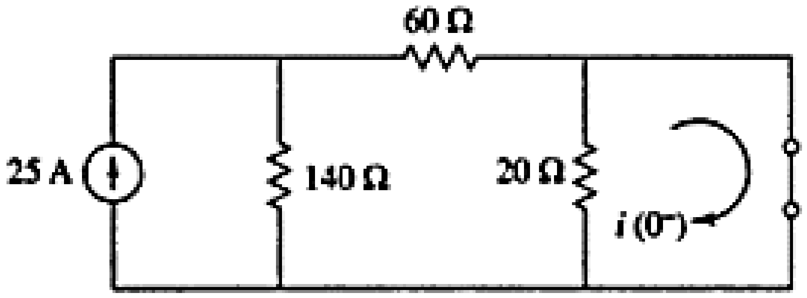
P. Find i(t) for t>0.

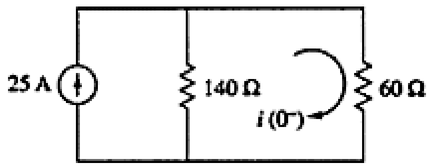
Solution:

L-charged state at t=0-

At t=0-

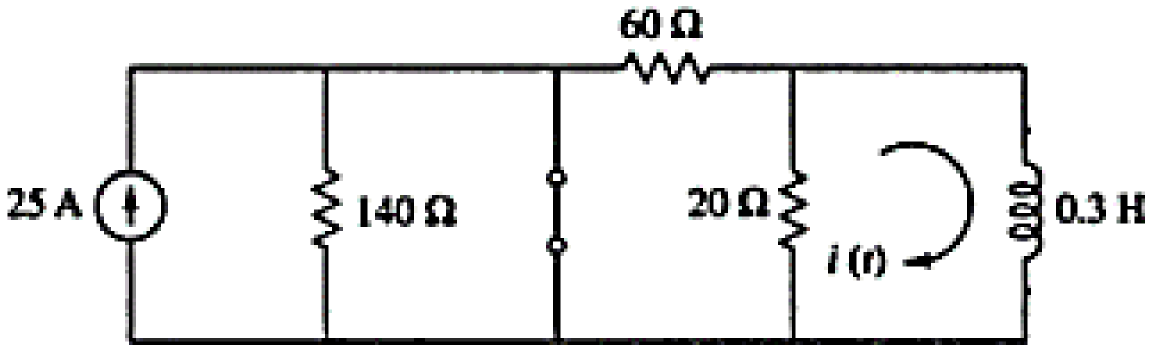
Equivalent circuit

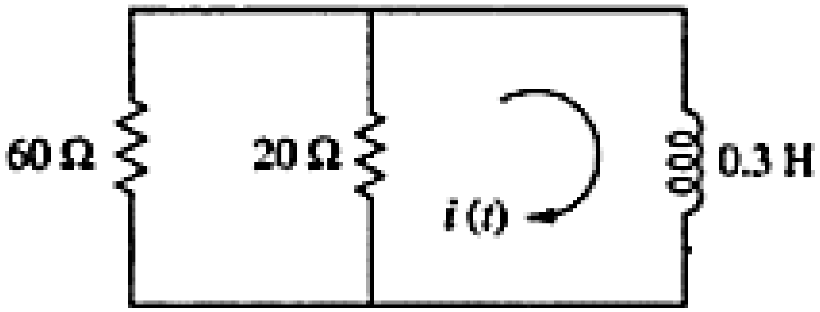


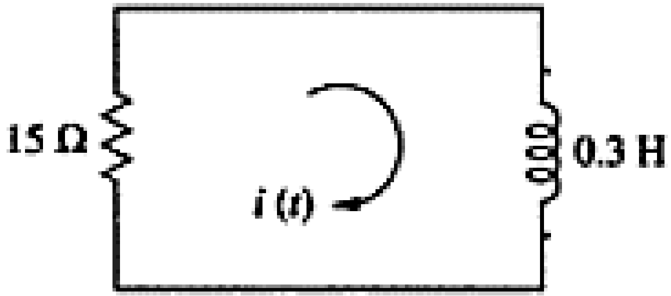


$i_L(0^-)$ =17.5 A.

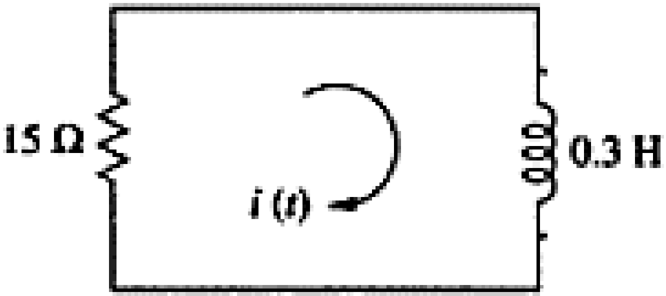
At t>0, equivalent circuit







Transient Response Analysis - Introduction



Apply KVL

$$0.3\frac{di(t)}{dt} + 15i(t) = 0 --- (1)$$

Equation (1) is in the form of differential equation-1

$$a\frac{di(t)}{dt} + b\, i(t) = 0$$

Solution for the above equation is

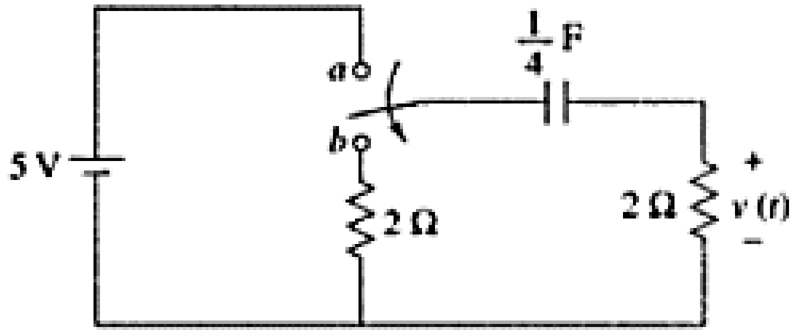
$$i(t) = Ke^{-\left(\frac{b}{a}\right)t}$$

Now,a $= 0.3\ and\ \ b = 15.$

Also, $K = x, where\ x = i_L(0^-) = 17.5$

$$\boldsymbol{i(t) = 17.5e^{-(50)t}}$$

Transient Response Analysis - Introduction



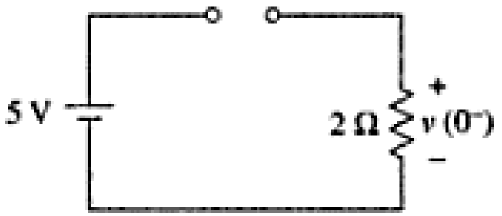
P. For the network shown in figure, the switch is changed the position from a to b at t=0. Find $v(t) for\ t > 0$.

Solution:

C-charged state at t=0-

At t=0-

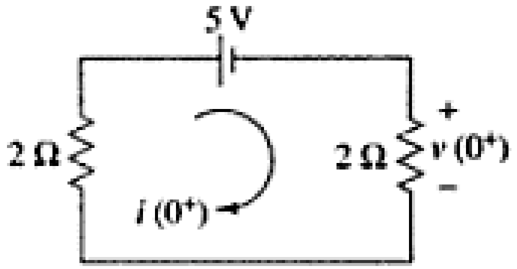
Equivalent circuit



$v_C(o^-) = 5$V

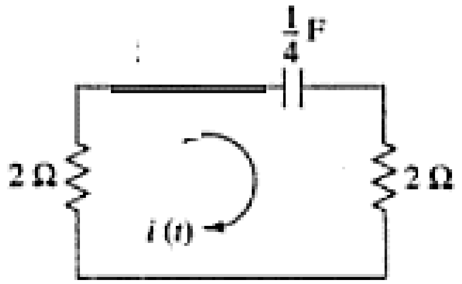
$v(0^-) = 0\ V.$

At t=0+, equivalent circuit.



$$v(0^+) = -2.5V \text{ and } i(0^+) = -1.25A$$

At t>0, equivalent circuit.



Apply KVL

$$4i(t) + \frac{1}{\frac{1}{4}} \int i(t)dt = 0$$

Differentiate the equation w.r.t.t

$$4\frac{di(t)}{dt} + 4i(t) = 0 --- (1)$$

Transient Response Analysis - Introduction

$$4\frac{di(t)}{dt} + 4i(t) = 0 --- (1)$$

Equation (1) is in the form of differential equation-1

$$a\frac{di(t)}{dt} + b\,i(t) = 0$$

Solution for the above equation is

$$i(t) = Ke^{-\left(\frac{b}{a}\right)t}$$

Now, $a = 4, b = 4$

Also, $K = x, where\ x = i(0^+) = -1.25$

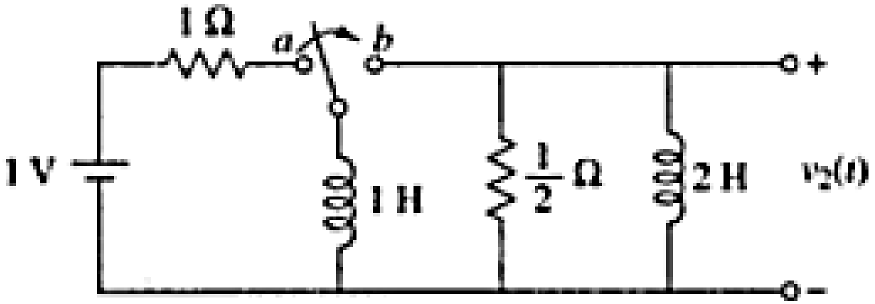
$$\boldsymbol{i(t) = -1.25e^{-t}}$$

From the circuit

$$\boldsymbol{v(t) = 2i(t)}$$

$$\boldsymbol{v(t) = -2.5e^{-t}}$$

Transient Response Analysis - Introduction



P. In the network shown in figure, the switch is in position 'a' for a long time. At t=0, the switch is moved from a to b. Find $v_2(t)$. Assume that the initial current in the 2H inductor is zero.

Solution:

L1=1H-charged state at t=0-

L2=2H-Uncharged state at t=0-

At t=0-

$i_{L_2}(0^-) = 0A.$

$i_{L_1}(0^-) = 1A$

At t=0+

$\boldsymbol{v_2(0^+) = -0.5V}$

At t>0

$$\frac{1}{L_1}\int v_2(t)dt + \frac{v_2(t)}{\frac{1}{2}} + \frac{1}{L_2}\int v_2(t)dt = 0$$

$$\int v_2(t)dt + 2v_2(t) + 0.5\int v_2(t)dt = 0$$

d.w.r.t.t

$$v_2(t) + 2\frac{dv_2(t)}{dt} + 0.5v_2(t) = 0$$

$$2\frac{dv_2(t)}{dt} + 1.5v_2(t) = 0 ---(1)$$

Equation (1) is in the form of

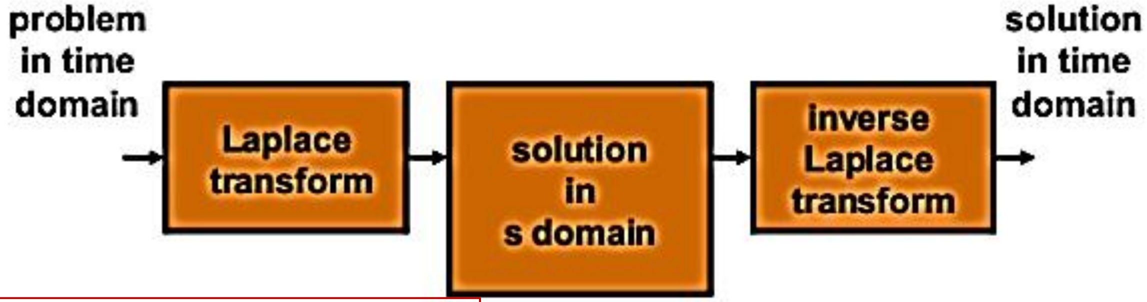
$$a\frac{dy(t)}{dt} + b\,y(t) = 0$$

Hence, the solution is

$$\boldsymbol{v_2(t) = Ke^{-\left(\frac{b}{a}\right)t} \Rightarrow -0.5e^{-(0.75t)}}$$

Laplace Transforms-Introduction

Laplace Transforms:
Laplace transform is a mathematical technique, which converts time domain equations into frequency domain.



- Integro-differential equations modelling is the well known/conventional mathematical model used to describe the electrical systems.
- Analysis of systems using IDE and finding the solution is difficult for higher order systems.
- Difficult to incorporate initial conditions.

- Laplace Transforms converts Integro-differential equations into simple algebraic equations.
- Analysis of systems in frequency domain is easy even for higher systems.
- Initial conditions are automatically incorporated.

Laplace Transforms-Introduction

Laplace Transforms:

Definition:

Any continuous time function f(t) defined for t≥0 and its Laplace transform is given by

$$L\{f(t)\} \Rightarrow F[s] = \int_0^{\infty} f(t).e^{-(st)}\,dt ---(1)$$

Where, **s** is the complex variable, i.e., $\boldsymbol{s = \sigma + j\omega}$

Where $\boldsymbol{\sigma}$ is the real part, which controls the amplitude and

$\boldsymbol{\omega}$ is the imaginary part, which controls the frequency.

Inverse Laplace transform:

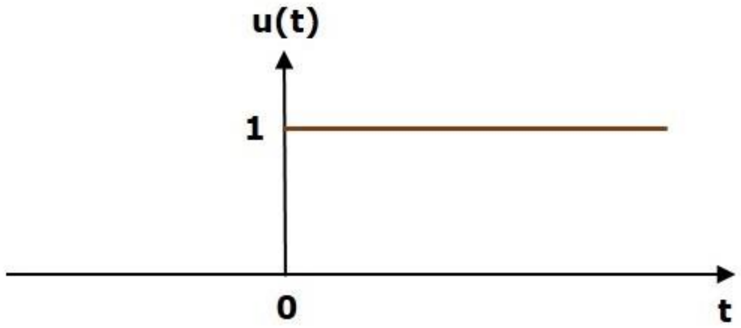
Definition:

$$L^{-1}\{F[s]\} \Rightarrow f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F[s]e^{st}ds ---(2)$$

Laplace Transforms-Important Functions

Laplace Transforms of standard test input signals:

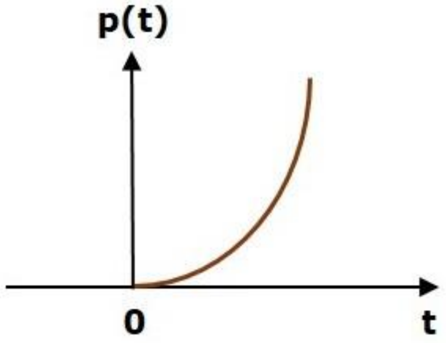
Unit Step Signal



$$r(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \frac{1}{s}$$

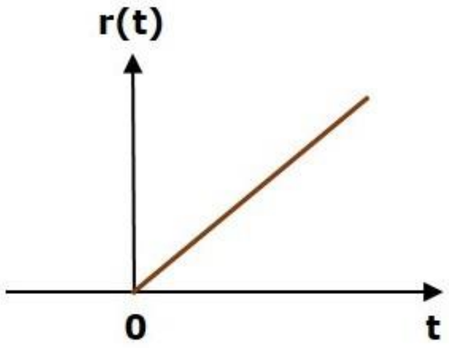
Unit Parabolic Signal



$$r(t) = \begin{cases} t^2 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \frac{2}{s^3}$$

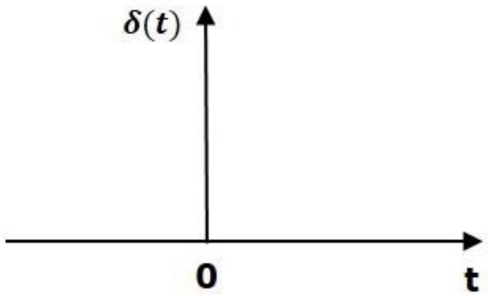
Unit Ramp Signal



$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \frac{1}{s^2}$$

Unit Impulse Signal



$$r(t) = \begin{cases} 1 & t = 0 \\ 0 & else \end{cases}$$

$$R(s) = 1$$

$$L\{f(t)\} \Rightarrow F[s] = \int_0^\infty f(t).\, e^{-(st)}\, dt ---(1)$$

1. Step signal

$$f(t) = u(t) = \begin{Bmatrix} A \middle| t \geq 0 \\ 0 \middle| t < 0 \end{Bmatrix}$$

$$F(s) = \int_0^\infty A.\, e^{-st} dt$$

$$F(s) = A.\frac{e^{-st}}{-s} \mid t = 0\ to\ \infty$$

$$F(s) = A.\left[\frac{e^{-\infty}}{-s} - \frac{e^0}{-s}\right]$$

$$F(s) = \frac{\mathrm{A}}{\mathrm{s}}$$

$$for\ unit\ step\ signal\ A = 1$$

$$F(s) = \frac{1}{s}$$

$$L\{f(t)\} \Rightarrow F[s] = \int_0^\infty f(t).\, e^{-(st)}\, dt ---(1)$$

2. Ramp signal

$$f(t) = r(t) = \begin{Bmatrix} At \middle| t \geq 0 \\ 0 \middle| t < 0 \end{Bmatrix}$$

$$F(s) = \int_0^\infty At.\, e^{-st} dt$$

$$F(s) = \frac{A}{s^2}$$

$$\int_a^b u \cdot dv = u \cdot v\Big|_a^b - \int_a^b v \cdot du$$

$$du = dt \qquad u = t$$

$$v = -\frac{1}{s}e^{-st} \qquad dv = e^{-st}$$

$$Y(s) = \left[-\frac{t}{s}e^{-st}\right]_{0^-}^{\infty} - \left[\int_{0^-}^{\infty} -\frac{1}{s}e^{-st} \cdot dt\right]$$

$$= [0-0] - \left[-\frac{1}{s}\int_{0^-}^{\infty} e^{-st} \cdot dt\right] = \frac{1}{s}\int_{0^-}^{\infty} e^{-st} \cdot dt = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

$$t \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s^2}$$

$$L\{f(t)\} \Rightarrow F[s] = \int_0^{\infty} f(t).e^{-(st)}\,dt ---(1)$$

3. Unit Impulse signal

$$f(t) = \delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t < 0 \text{ or } t > 0 \end{cases}$$

$$F(s) = \int_0^{0} 1.e^{-st}dt$$

$$F(s) = 1.e^{-st}|t = 0.$$

$$F(s) = 1$$

1. Coswt
2. Sinhwt
3. Coshwt
4. e^{at}
5. e^{at}sinwt
6. e^{at}coswt
7. e^{-at}
8. e^{-at}sinwt
9. e^{-at}coswt

$$L\{f(t)\} \Rightarrow F[s] = \int_0^{\infty} f(t).e^{-(st)}\,dt ---(1)$$

4. f(t)=sinwt

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$Y(s) = \int_{0^-}^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{2j} e^{-st}dt = \frac{1}{2j}\int_{0^-}^{\infty} e^{j\omega t}e^{-st}dt - \frac{1}{2j}\int_{0^-}^{\infty} e^{-j\omega t}e^{-st}dt$$

$$Y(s) = \frac{1}{2j}\frac{1}{s - j\omega} - \frac{1}{2j}\frac{1}{s + j\omega}$$

$$Y(s) = \frac{1}{2j}\frac{1}{(s - j\omega)}\frac{(s + j\omega)}{(s + j\omega)} - \frac{1}{2j}\frac{1}{(s + j\omega)}\frac{(s - j\omega)}{(s - j\omega)}$$

$$= \frac{1}{2j}\frac{(s + j\omega) - (s - j\omega)}{\left(s^2 - \cancel{sj\omega} + \cancel{sj\omega} - (j\omega)^2\right)} = \frac{1}{2j}\frac{2j\omega}{s^2 + \omega^2}$$

$$= \frac{\omega}{s^2 + \omega^2}$$

$$\sin(\omega t) \xleftrightarrow{\mathcal{L}} \frac{\omega}{s^2 + \omega^2}$$

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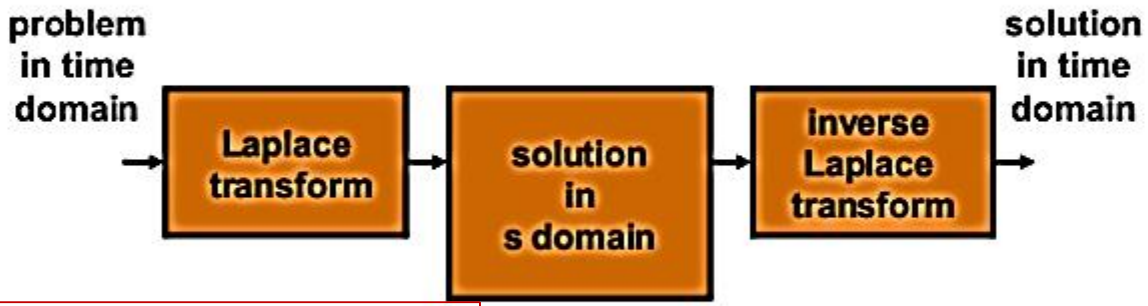
Laplace Transforms-Introduction

Laplace Transforms:
Laplace transform is a mathematical technique, which converts time domain equations into frequency domain.



- Integro-differential equations modelling is the well known/conventional mathematical model used to describe the electrical systems.
- Analysis of systems using IDE and finding the solution is difficult for higher order systems.
- Difficult to incorporate initial conditions.

- Laplace Transforms converts Integro-differential equations into simple algebraic equations.
- Analysis of systems in frequency domain is easy even for higher systems.
- Initial conditions are automatically incorporated.

Laplace Transforms-Introduction

Laplace Transforms:
Definition:
Any continuous time function f(t) defined for t≥0 and its Laplace transform is given by

$$L\{f(t)\} \Rightarrow F[s] = \int_0^{\infty} f(t).e^{-(st)}\,dt \;---(1)$$

Where, **s** is the complex variable, i.e., $\boldsymbol{s = \sigma + j\omega}$
Where $\boldsymbol{\sigma}$ is the real part, which controls the amplitude and
$\boldsymbol{\omega}$ is the imaginary part, which controls the frequency.

Inverse Laplace transform:
Definition:

$$L^{-1}\{F[s]\} \Rightarrow f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F[s]e^{st}ds \;---(2)$$

Laplace Transforms-Examples

1. $\mathbf{f(t) = 1}$

$$L\{1\} => F[s] = \int_0^\infty 1.e^{-st}dt$$

$$L\{1\} = \left.\frac{e^{-st}}{-s}\right|_0^\infty$$

$$L\{1\} = -\frac{1}{s}[e^{-\infty} - e^0]$$

$$\mathbf{L\{1\} = \frac{1}{s}} \quad (\because e^{-\infty} = 0 \text{ and } e^0 = 1)$$

2. $\mathbf{f(t) = A}$

$$L\{A\} => F[s] = \int_0^\infty A.e^{-st}dt$$

$$L\{A\} = A\left.\frac{e^{-st}}{-s}\right|_0^\infty$$

$$L\{A\} = -\frac{A}{s}[e^{-\infty} - e^0]$$

$$\mathbf{L\{A\} = \frac{A}{s}} \quad (\because e^{-\infty} = 0 \text{ and } e^0 = 1)$$

Laplace Transforms-Examples

$$\mathbf{3. f(t) = t^n}$$

$$L\{t^n\} \Rightarrow F[s] = \int_0^\infty t^n . e^{-st} dt$$

We know that, Integral by parts

$$\int_0^\infty u.dv = u.v\Big|_0^\infty - \int_0^\infty v.du$$

Let, $u = t^n$ and $dv = e^{-st}dt$

$du = nt^{n-1}dt$

$$v = -\frac{e^{-st}}{s}$$

$$\therefore L\{t^n\} = -t^n.\frac{e^{-st}}{s}\Bigg|_0^\infty - \int_0^\infty -\frac{e^{-st}}{s}.n.t^{n-1}dt$$

$$L\{t^n\} = 0 + \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}dt \Rightarrow \frac{n}{s}L\{t^{n-1}\}$$

Similarly

$$L\{t^{n-1}\} = \frac{n(n-1)}{s.s}.L\{t^{n-2}\}$$

.

.

So, Generally

$$\mathbf{L\{t^n\} = \frac{n!}{s^{n+1}}}$$

Laplace Transforms-Examples

4. $\mathbf{f(t) = e^{at}}$

$$L\{e^{at}\} => F[s] = \int_0^\infty e^{at}.e^{-st}dt$$

$$L\{e^{at}\} = \int_0^\infty e^{-(s-a)t}dt$$

$$L\{e^{at}\} = \left.\frac{e^{-(s-a)t}}{-(s-a)}\right|_0^\infty$$

$$L\{\mathbf{e^{at}1}\} = \frac{\mathbf{1}}{\mathbf{s-a}}$$

Similarly

$$\mathbf{L\{e^{-at}\}} = \frac{\mathbf{1}}{\mathbf{s+a}}$$

5. $\mathbf{f(t) = sin\omega t}$

$$L\{\sin\omega t\} = \int_0^\infty \sin\omega t.e^{-st}dt$$

We know that

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\boldsymbol{L}\{\sin\omega t\} = \int_0^\infty \frac{e^{j\omega t} - e^{-j\omega t}}{2j}.e^{-st}dt$$

$$L\{\sin\omega t\} = \frac{1}{2j}\int_0^\infty (e^{j\omega t}e^{-st} - e^{-j\omega t}e^{-st})dt$$

$$L\{\sin\omega t\} = \frac{1}{2j}\int_0^\infty e^{-(s-j\omega)t}dt - \frac{1}{2j}\int_0^\infty e^{-(s+j\omega)t}dt$$

$$L\{\sin\omega t\} = \frac{1}{2j}[-\left.\frac{e^{-(s-j\omega)t}}{s-j\omega}\right|_0^\infty - \left.\frac{e^{-(s+j\omega)t}}{s+j\omega}\right|_0^\infty$$

$$\boldsymbol{L\{\sin\omega t\}} = \frac{\boldsymbol{\omega}}{\mathbf{s^2 + \omega^2}}$$

Laplace Transforms-Examples

6. $\mathbf{f(t) = cos\omega t}$

$$L\{\mathbf{cos\omega t}\} = \int_0^\infty \mathbf{cos\omega t}.\,\mathrm{e}^{-\mathrm{st}}dt$$

We know that

$$\mathbf{cos\omega t} = \frac{\mathrm{e}^{j\omega t} + e^{-j\omega t}}{2}$$

$$\boldsymbol{L}\{\mathbf{cos\omega t}\} = \int_0^\infty \frac{\mathrm{e}^{j\omega t} + e^{-j\omega t}}{2}.\,\mathrm{e}^{-\mathrm{st}}dt$$

$$L\{\mathbf{cos\omega t}\} = \frac{1}{2}\int_0^\infty (e^{j\omega t}e^{-st} + e^{-j\omega t}e^{-st})dt$$

$$L\{\ \mathbf{cos\omega t}\} = \frac{1}{2}\int_0^\infty e^{-(s-j\omega)t}dt + \frac{1}{2j}\int_0^\infty e^{-(s+j\omega)t}dt$$

$$L\{\mathbf{cos\omega t}\} = \frac{1}{2}[-\left.\frac{e^{-(s-j\omega)t}}{s-j\omega}\right|_0^\infty + \left.\frac{e^{-(s+j\omega)t}}{s+j\omega}\right|_0^\infty$$

$$\boldsymbol{L}\{\mathbf{cos\omega t}\} = \frac{\mathbf{s}}{\mathbf{s^2+\omega^2}}$$

7. $\mathbf{f(t) = sinh\omega t}$

$$L\{\mathrm{sin\omega t}\} = \int_0^\infty \mathrm{sin\omega t}.\,\mathrm{e}^{-\mathrm{st}}dt$$

We know that

$$\mathrm{sinh\omega t} = \frac{\mathrm{e}^{\omega t} - e^{-\omega t}}{2}$$

$$L\{\mathrm{sinh\omega t}\} = \int_0^\infty \frac{\mathrm{e}^{\omega t} - e^{-\omega t}}{2}.\,\mathrm{e}^{-\mathrm{st}}dt$$

$$L\{\mathrm{sinh\omega t}\} = \frac{1}{2}\int_0^\infty (e^{\omega t}e^{-st} - e^{-\omega t}e^{-st})dt$$

$$L\{\ \mathrm{sinh\omega t}\} = \frac{1}{2}\int_0^\infty e^{-(s-\omega)t}dt - \frac{1}{2}\int_0^\infty e^{-(s+\omega)t}dt$$

$$L\{\mathrm{sinh\omega t}\} = \frac{1}{2}[-\left.\frac{e^{-(s-\omega)t}}{s-\omega}\right|_0^\infty - \left.\frac{e^{-(s+\omega)t}}{s+\omega}\right|_0^\infty$$

$$\boldsymbol{L}\{\mathbf{sinh\omega t}\} = \frac{\boldsymbol{\omega}}{\mathbf{s^2 - \omega^2}}$$

`

Laplace Transforms-Examples

8. $\mathbf{f(t) = cosh\omega t}$

$$L\{\mathbf{cosh\omega t}\} = \int_0^\infty \mathbf{cosh\omega t}.\, \mathrm{e}^{-\mathrm{st}} dt$$

We know that

$$\mathbf{cosh\omega t} = \frac{\mathrm{e}^{\omega t} + e^{-\omega t}}{2}$$

$$\boldsymbol{L}\{\mathbf{cosh\omega t}\} = \int_0^\infty \frac{\mathrm{e}^{\omega t} + e^{-\omega t}}{2}.\, \mathrm{e}^{-\mathrm{st}} dt$$

$$L\{\mathbf{cosh\omega t}\} = \frac{1}{2}\int_0^\infty (e^{\omega t}e^{-st} + e^{-\omega t}e^{-st})dt$$

$$L\{\mathbf{\ cosh\omega t}\} = \frac{1}{2}\int_0^\infty e^{-(s-\omega)t}dt + \frac{1}{2j}\int_0^\infty e^{-(s+\omega)t}dt$$

$$L\{\mathbf{cosh\omega t}\} = \frac{1}{2}[-\frac{e^{-(s-\omega)t}}{s-\omega}\Bigg|_0^\infty + \frac{e^{-(s+\omega)t}}{s+\omega}\Bigg|_0^\infty$$

$$\boldsymbol{L}\{\mathbf{cosh\omega t}\} = \frac{\mathbf{s}}{\mathbf{s^2 - \boldsymbol{\omega}^2}}$$

$$\boldsymbol{L}\{\sqrt{\boldsymbol{t}}\} = ?$$

$$\boldsymbol{L}\{\mathbf{1}/\sqrt{\boldsymbol{t}}\} = ?$$

Exercise:

Find the Laplace transform of the following functions.

1. $cos\ 3t$

2. $sinh\ 4t$

3. e^{-10t}

4. t^3

Laplace Transforms-Properties

1. $\cos 3t$

$L\{\cos 3t\} = \frac{s}{s^2+9}$ (since, $L\{\cos wt = \frac{s}{s^2+w^2}\}$)

2. $\sinh 4t$

$$L\{\sinh 4t\} = \frac{4}{s^2 - 16}$$

3. e^{-10t}

$$L\{e^{-10t}\} = \frac{1}{s + 10}$$

4. t^3

$$L\{t^3\} = \frac{6}{s^4}$$

`

Laplace Transforms-Properties

1. Linearity

$$\boldsymbol{L\{f(t) \pm g(t)\} = F[s] \pm G[s]}$$

Proof:

$$\boldsymbol{L}\{f(t) \pm g(t)\} = \int_0^\infty (f(t) \pm g(t))e^{-st}\, dt.$$

$$L\{f(t) \pm g(t)\} = \int_0^\infty (f(t))\, e^{-st} dt \ \pm \int_0^\infty (g(t))\, e^{-st} dt\ .$$

$$\boldsymbol{L\{f(t) \pm g(t)\} = F[s] \pm G[s]}$$

Example:

$$\boldsymbol{L\{cos4t + t^2\} = L\{cos4t\} + L\{t^2\} => \frac{s}{s^2 + 16} + \frac{2}{s^3}}$$

Laplace Transforms-Properties

2. Function Scaling

$$\boldsymbol{L\{af(t)\} = aL\{f(t)\} => aF[s]}$$

Proof:

$$L\{af(t)\} = \int_0^\infty af(t)e^{-st}dt$$

$$L\{af(t)\} = a\int_0^\infty f(t)e^{-st}dt$$

$$\boldsymbol{L\{af(t)\} = aF[s]}$$

Example:

$$\boldsymbol{L\{5e^{-3t}\} = 5L\{e^{-3t}\} => \frac{5}{s+3}}$$

`

Laplace Transforms-Properties

3. Time shifting

$$\boldsymbol{L\{f(t-T)\} = e^{-sT}F[s]}$$

Proof:

$$L\{f(t-T)\} = \int_0^{\infty} f(t-T)e^{-st}dt$$

Let, $t - T = \tau$

$$t = \tau + T$$

$$dt = d\tau$$

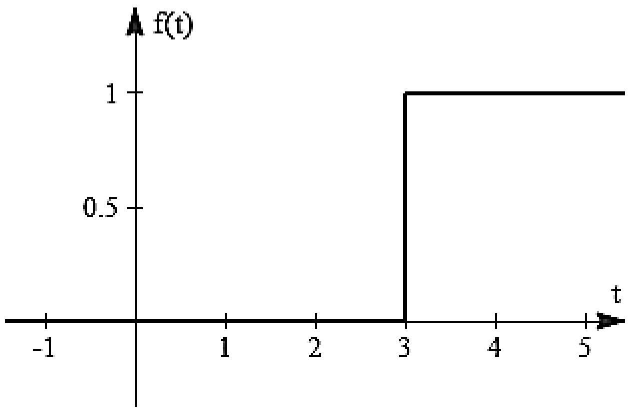
$$\therefore L\{f(t-T)\} = \int_0^{\infty} f(\tau)e^{-s(\tau+T)}d\tau$$

$$L\{f(t-T)\} = \int_0^{\infty} f(\tau)e^{-sT}.e^{-s\tau}d\tau$$

$$L\{f(t-T)\} = e^{-sT}\int_0^{\infty} f(\tau)e^{-s\tau}d\tau$$

$$\therefore \boldsymbol{L\{f(t-T)\} = e^{-sT}F[s]}$$

Example:



$$\boldsymbol{L}\{u(t-3)\} = e^{-3s}L\{u(t)\}$$

$$\boldsymbol{L\{u(t-3)\} = e^{-3s}.\frac{1}{s}}$$

Laplace Transforms-Properties

4. Frequency shifting

$$L\{e^{at}f(t)\} = F(s-a)$$

Proof:

$$L\{e^{at}f(t)\} = \int_0^\infty e^{at}f(t)e^{-st}dt$$

$$L\{e^{at}f(t)\} = \int_0^\infty f(t)e^{-(s-a)t}dt$$

$$L\{e^{at}f(t)\} = F(s-a)$$

Similarly

$$L\{\, e^{-at}f(\mathrm{t})\} = \mathrm{F}(\mathrm{s}+\mathrm{a})$$

Example:

$$L\{e^{-at}sinbt\} = \frac{b}{(s+a)^2+b^2}$$

$$L\{sinbt\} = \frac{b}{s^2+b^2}$$

$$L\{e^{-at}sinbt\} = \frac{b}{(s+a)^2+b^2}$$

$$L\{e^{at}cosbt\} \Rightarrow$$

$$L\{cosbt\} = \frac{s}{s^2+b^2}$$

$$L\{e^{at}cosbt\} = \frac{s-a}{(s-a)^2+b^2}$$

Laplace Transforms-Properties

5. Time scaling

$$\boldsymbol{L\{f(at)\} = \frac{1}{a} F[\frac{s}{a}]}$$

Proof:

$$L\{f(at)\} = \int_0^{\infty} f(at)e^{-st}dt.$$

Let, $at = \tau$

$$t = \frac{\tau}{a}$$

$$dt = \frac{1}{a}d\tau$$

$$\therefore L\{f(at)\} = \int_0^{\infty} \frac{f(\tau)e^{-\left(\frac{s}{a}\right)\tau}d\tau}{a}$$

$$\therefore L\{f(at)\} = \frac{1}{a}\int_0^{\infty} f(\tau)e^{-\left(\frac{s}{a}\right)\tau}d\tau$$

$$\boldsymbol{\therefore L\{f(at)\} = \frac{1}{a} F[\frac{s}{a}]}$$

`

Laplace Transforms-Properties

1. $f(t) = e^{at} coshwt$

$$L\{coshwt\} = \frac{s}{s^2 - w^2}$$

$$L\{e^{at} coshwt\} = \frac{s-a}{(s-a)^2 - w^2}$$

2. $f(t) = 5cos5t + \frac{t^2}{2} + u(t-5) + r(t-2)$

$$F(s) = \frac{5s}{s^2+25} + \frac{1}{s^3} + \frac{e^{-5s}}{s} + \frac{e^{-2s}}{s^2}$$

3. $f(t) = e^{at} u(t-T)$

$$L\{u(t-T)\} = \frac{e^{-Ts}}{s}$$

$$L\{e^{at} u(t-T)\} = \frac{e^{-T(s-a)}}{\mathrm{s-a}}$$

CIE-II Portions

Unit-III: Transient behaviour and initial conditions

Unit-IV-Laplace transforms basic definition and properties

With examples.

Laplace Transforms- Theorems

1. Differentiation theorem (Laplace of derivative of a function).

Statement: Let f(t) is a continuous time function defined for t≥0,

if L{f(t)}=F(s), then $L\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0)$.

In general, $L\left\{\frac{d^n}{dt^n}f(t)\right\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) \ldots..$

Proof:

$$\boldsymbol{L}\left\{\frac{df(t)}{dt}\right\} = \int_0^\infty \frac{df(t)}{dt}.e^{-st}dt.$$

Integration by parts,

$$\int u.dv = u.v - \int v.du$$

Let, $u = e^{-st}$ and $dv = \frac{d}{dt}f(t)$

$du = -se^{-st}dt$ and $v = f(t)$

Therefore,

$$\boldsymbol{L}\left\{\frac{df(t)}{dt}\right\} = e^{-st}f(t)\,|_0^\infty - \int_0^\infty f(t).-se^{-st}dt$$

$$\boldsymbol{L}\left\{\frac{df(t)}{dt}\right\} = s\int_0^\infty f(t).e^{-st}dt - f(0)$$

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

In general,

$$L\left\{\frac{d^n}{dt^n}f(t)\right\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) \ldots..$$

Laplace Transforms- Theorems

2. Integration theorem (Laplace of Integral of a function).

Statement: Let f(t) is a continuous time function defined for t≥0,

if L{f(t)}=F(s), then $L\{\int_0^t f(t)dt\} = \frac{F(s)}{s}$.

In general, $L\{\int_0^{t_1}\int_0^{t_2} \ldots . \int_0^{t_n} f(t)dt_1 . dt_2 \ldots . dt_n\} = \frac{F(s)}{s^n}$

Proof:

$$\boldsymbol{L}\left\{\int_0^t f(\tau)d\tau\right\} = \int_0^{\infty}\int_0^t f(t)dt\,.\,e^{-st}dt.$$

Integration by parts,

$$\int u.dv = u.v - \int v.du$$

Let, $u = \int_0^t f(t)dt$ and $dv = e^{-st}dt$

$du = f(\tau)d\tau$ and $v = \frac{e^{-st}}{-s}$

Therefore,

$$\boldsymbol{L}\left\{\int_0^t f(\tau)d\tau\right\} = \int_0^t f(\tau)d\tau\,.\frac{e^{-st}}{-s}\bigg|_0^{\infty} - \int_0^{\infty} f(t).\frac{e^{-st}}{-s}dt$$

$$\boldsymbol{L}\left\{\int_0^t f(\tau)d\tau\right\} = -\int_0^{\infty} f(t).\frac{e^{-st}}{-s}dt$$

$$\boldsymbol{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}\int_0^{\infty} f(t).e^{-st}dt$$

$$\boldsymbol{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$$

In general,

$$L\{\int_0^{t_1}\int_0^{t_2} \ldots . \int_0^{t_n} f(t)dt_1 . dt_2 \ldots . dt_n\} = \frac{F(s)}{s^n}$$

Laplace Transforms- Theorems

3. Differentiation by s (Multiplication by t)

Statement: If F(s) is the Laplace transform of f(t) then the differentiation by s in the frequency domain corresponds to the multiplication by t in time domain.

i.e., $L\{tf(t)\} = -\frac{dF(s)}{ds}$, In general, $L\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n}$

Proof:

we know that,

$$F(s) = \int_0^\infty f(t).e^{-st}dt --- (1)$$

Differentiating both sides with respect to s,

$$\frac{dF(s)}{ds} = \int_0^\infty f(t).-t.e^{-st}dt$$

$$-\frac{dF(s)}{ds} = \int_0^\infty f(t).t.e^{-st}dt$$

$$-\frac{dF(s)}{ds} = L\{t.f(t)\}$$

Laplace Transforms- Theorems

4. Integration by s (divided by t)

Statement: If F(s) is the Laplace transform of f(t) then the Integration by s in the frequency domain corresponds to the division of t in time domain.

i.e., $L\left\{\frac{f(t)}{t}\right\} = \int_0^{\infty} F(s)ds$. In general, $L\left\{\frac{f(t)}{t^n}\right\} = \int_0^{s_1}\int_0^{s_2} \dots \int_0^{s_n} F(s)ds_1.ds_2 \dots .ds_n$

Proof:

we know that,

$$F(s) = \int_0^{\infty} f(t).e^{-st}dt \text{ ---(1)}$$

Integrating on both sides with respect to s,

$$\int_0^{\infty} F(s)ds = \int_0^{\infty}\int_0^{\infty} f(t).e^{-st}dt\,ds$$

$$\int_0^{\infty} F(s)ds = \int_0^{\infty}\int_0^{\infty} f(t).e^{-st}dtds$$

$$\int_0^{\infty} F(s)ds = \int_0^{\infty} f(t)dt \int_0^{\infty} e^{-st}\ ds$$

$$\int_0^{\infty} F(s)ds = \int_0^{\infty} f(t)dt\ \left[\frac{e^{-st}}{-t}\right]\Bigg|_0^{\infty} ds$$

$$\int_0^{\infty} F(s)ds = \int_0^{\infty} f(t)\frac{e^{-st}}{t}dt$$

$$\int_0^{\infty} F(s)ds = L\left\{\frac{f(t)}{t}\right\}$$

Laplace Transforms- Theorems

5. Initial Value Theorem

Statement: If F(s) is the Laplace transform of f(t) then

$$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s).$$

Proof:

we know that,

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

$$\int_0^\infty \frac{df(t)}{dt}.e^{-st}dt = sF(s) - f(0) ---(1)$$

Take limit as $s \to \infty$ on both sides

$$\lim_{s\to\infty} \int_0^\infty \frac{df(t)}{dt}.e^{-st}dt = \lim_{s\to\infty}[sF(s) - f(0)]$$

$$0 = \lim_{s\to\infty} sF(s) - f(0)$$

$$f(0) = \lim_{s\to\infty} sF(s)$$

$$\boldsymbol{f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)}.$$

6. Final Value Theorem

Statement: If F(s) is the Laplace transform of f(t) then

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s).$$

Proof:

we know that,

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

$$\int_0^\infty \frac{df(t)}{dt}.e^{-st}dt = sF(s) - f(0) ---(1)$$

Take limit as $s \to 0$ on both sides

$$\lim_{s\to 0} \int_0^\infty \frac{df(t)}{dt}.e^{-st}dt = \lim_{s\to 0}[sF(s) - f(0)]$$

$$f(t)\Big|_0^\infty = \lim_{s\to 0} sF(s) - f(0)$$

$$f(\infty) - f(0) = \lim_{s\to 0} sF(s) - f(0)$$

$$\boldsymbol{f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}.$$

Laplace Transforms- Theorems

1. **Find the initial and final value for the system equation $X(s) = \frac{1}{s(s+2)}$**

Solution:

w.k.t. IVT

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

$$f(0) = \lim_{s\to\infty} s.\frac{1}{s(s+2)} \Rightarrow \mathbf{0}$$

w.k.t. FVT

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$f(\infty) = \lim_{s\to 0} s.\frac{1}{s(s+2)} \Rightarrow \mathbf{\frac{1}{2}}$$

2. Verify the final value theorem for the function $f(t) = 2 + e^{-3t}cos2t$

Solution:

LHS is in time domain

$$f(\infty) = lim_{t\to\infty}\left[2 + e^{-3t}cos2t\right]$$

$$f(\infty) = 2$$

RHS is in frequency domain

$$F(s) = \frac{2}{s} + \frac{s+3}{(s+3)^2+4}$$

$$F(s) = \frac{2}{s} + \frac{s+3}{s^2+6s+13}$$

$$f(\infty) = lim_{s\to 0}s.\left(\frac{2}{s} + \frac{s+3}{s^2+6s+13}\right)$$

$$f(\infty) = lim_{s\to 0}\left(2 + \frac{s(s+3)}{s^2+6s+13}\right)$$

$$f(\infty) = 2$$

Laplace Transforms- Theorems

3. Find the Laplace transform of

$$f(t) = \frac{cos4t - cos5t}{t}$$

Solution:

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left[\frac{\cos 4t - \cos 5t}{t}\right]$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left[\frac{\cos 4t}{t}\right] - \mathcal{L}\left[\frac{\cos 5t}{t}\right]$$

Since

$$\mathcal{L}(\cos bt) = \frac{s}{s^2 + b^2}$$

Then,

$$\mathcal{L}\{f(t)\} = \int_s^\infty \frac{u\,du}{u^2+4^2} - \int_s^\infty \frac{u\,du}{u^2+5^2}$$

$$\mathcal{L}\{f(t)\} = \int_s^\infty \left(\frac{u}{u^2+16} - \frac{u}{u^2+25}\right) du$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\int_s^\infty \left(\frac{2u}{u^2+16} - \frac{2u}{u^2+25}\right) du$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\Big[\ln(u^2+16) - \ln(u^2+25)\Big]_s^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln\frac{u^2+16}{u^2+25}\right]_s^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln\frac{\frac{u^2}{u^2}+\frac{16}{u^2}}{\frac{u^2}{u^2}+\frac{25}{u^2}}\right]_s^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln\frac{1+\frac{16}{u^2}}{1+\frac{25}{u^2}}\right]_s^\infty$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln\frac{1+\frac{16}{\infty^2}}{1+\frac{25}{\infty^2}} - \ln\frac{1+\frac{16}{s^2}}{1+\frac{25}{s^2}}\right]$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln\frac{1+0}{1+0} - \ln\frac{\frac{s^2+16}{s^2}}{\frac{s^2+25}{s^2}}\right]$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left[\ln 1 - \ln\frac{s^2+16}{s^2+25}\right]$$

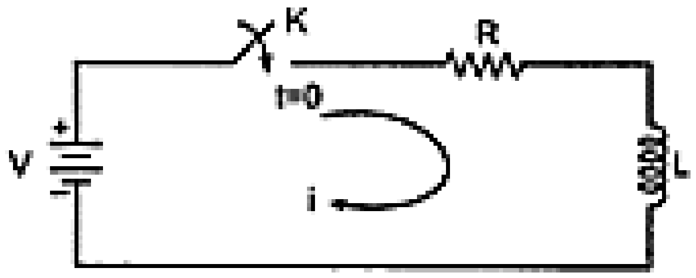
$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left\{0 - \left[\ln(s^2+16) - \ln(s^2+25)\right]\right\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\left\{\ln(s^2+25) - \ln(s^2+16)\right\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2}\ln\frac{s^2+25}{s^2+16} \qquad \textit{answer}$$

Laplace Transforms- Applications

4. In the circuit shown in figure, the switch is closed at t=0. find the current i(t) at t>0 using Laplace transforms.



Solution:

At t=0-, Inductor is uncharged and hence,

$i_L(0-)=0A=i_L(0+)=i(0+)$

At t>0

Apply KVL

$$V = Ri(t) + L\frac{di(t)}{dt} --- (1)$$

Apply Laplace transform on both sides

$$V = RI(s) + L[sI(s) - i(0)] --- (2)$$

$$V = RI(s) + LsI(s)$$

$$V = I(s)(R + Ls)$$

$$I(s) = \frac{V}{R + Ls}$$

$$I(s) = \frac{V}{L(\frac{R}{L} + s)} \Rightarrow \frac{V}{L}\frac{1}{(s + \frac{R}{L})}$$

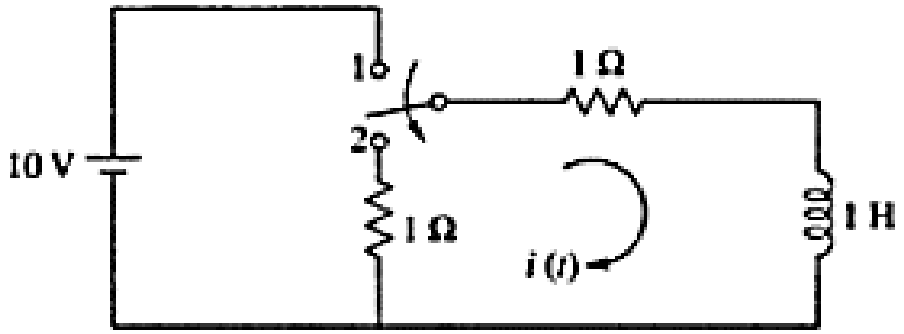
Apply Inverse LT.

$$L^{-1}\{I(s)\} = \frac{V}{L}L^{-1}\left\{\frac{1}{s + \frac{R}{L}}\right\}$$

$$i(t) = \frac{V}{L}.e^{-\left(\frac{R}{L}\right)t}$$

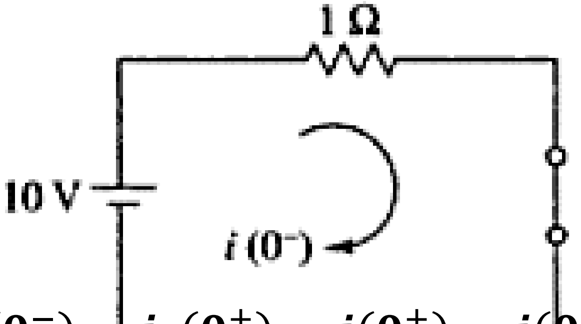
Laplace Transforms- Applications

5. In the circuit shown in figure, the switch changed the position from 1 to 2 at t=0. find the current i(t) at t>0 using Laplace transforms.



Solution:

L is the energy storage element-charged state(reached steady state at t=0-)



$$i_L(0^-) = i_L(0^+) = i(0^+) = i(0^-) = 10A.$$

At t>0

Apply KVL to the circuit

$$1i(t) + 1i(t) + 1\frac{di(t)}{dt} = 0 --- (1)$$

Apply L.T.

$$I(s) + I(s) + sI(s) - i(0) = 0$$

$$I(s) + I(s) + sI(s) - i(0) = 0$$

$$2I(s) + sI(s) = 10 --- (2)$$

$$I(s) = \frac{10}{2+s} --- (3)$$

ILT.

$$i(t) = 10.e^{-2t}$$

Laplace Transforms- Applications

6. In the circuit shown in figure, the switch is closed at t=0. find the Voltage v_Ct) at t>0 using Laplace transforms.

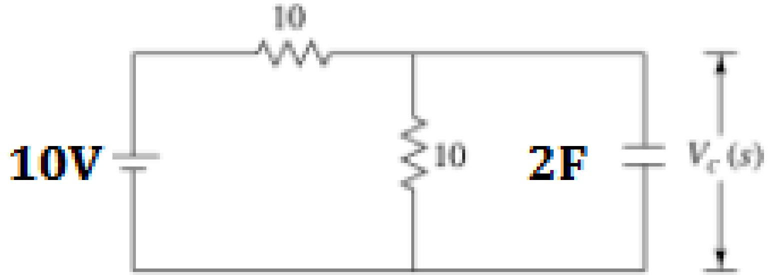
Solution:

C –energy storage element-uncharged state at t=0-.

$$v_C(0^-) = v_C(0^+) = 0V$$

At t>0

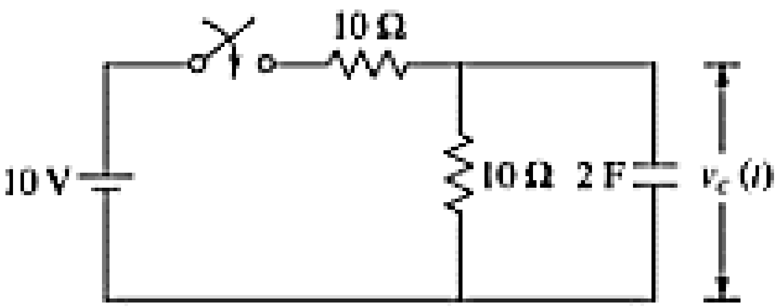
KCL at node $v_C(t)$



$$\frac{10 - v_C(t)}{10} = \frac{v_C(t)}{10} + 2\frac{dv_C(t)}{dt} = 0 ---(1)$$

$$1 - \frac{v_C(t)}{10} - \frac{v_C(t)}{10} = 2\frac{dv_C(t)}{dt}$$

$$2\frac{dv_C(t)}{dt} + \frac{v_C(t)}{5} = 1 ---(2)$$



$$2\frac{dv_C(t)}{dt} + \frac{v_C(t)}{5} = 1 ---(2)$$

LT.

$$2[sv_C(s) - v_C(0)] + \frac{1}{5}v_C(s) = \frac{1}{s}$$

$$2sv_C(s) + 0.2v_C(s) = \frac{1}{s}$$

$$v_C(s)[2s + 0.2] = \frac{1}{s}$$

$$v_C(s) = \frac{1}{s(2s + 0.2)} ---(3)$$

$$v_C(s) = \frac{1}{s(2s+0.2)} --- (3)$$

$$v_C(s) = \frac{A}{s} + \frac{B}{2s+0.2} --- (4)$$

$$A(2s+0.2) + Bs = 1$$

If s=0, A=5, and if s=-0.1, B=-10

$$v_C(s) = \frac{5}{s} - \frac{10}{2s+0.2}$$

$$v_C(s) = \frac{2}{s} - \frac{5}{s+0.1} --- (5)$$

ILT

$$\boldsymbol{v_C(t) = 5 - 5e^{-0.1t}}$$

Laplace Transforms- Applications

7. In the circuit shown in figure, the switch changed the position from a to b at t=0. find the Voltage v(t) at t>0 using Laplace transforms.

Solution:

2H and 1H are energy storage elements

2H inductor reached steady state at t=0- and 1H inductor is in uncharged state.

$$i_{L1}(0^-) = i_{L1}(0^+) = 1A.$$
$$i_{L2}(0^-) = i_{L2}(0^+) = 0A.$$

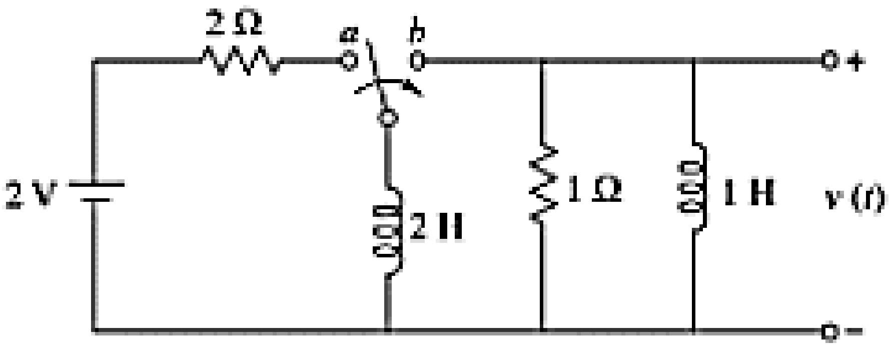
At t>0

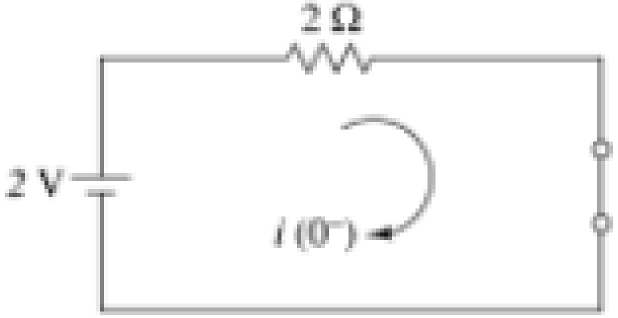
KCL

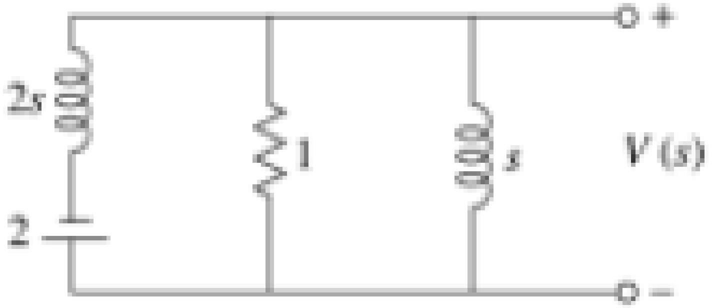
$$\frac{v(s)-2}{2s} + \frac{v(s)}{1} + \frac{v(s)}{s} = 0 --- (1)$$

$$\frac{v(s)}{2s} - \frac{2}{2s} + v(s) + \frac{v(s)}{s} = 0$$

$$v(s)\left[\frac{1}{2s} + 1 + \frac{1}{s}\right] = \frac{1}{s}$$







Laplace Transforms- Applications

$$\boldsymbol{v(s)\left[\frac{1}{2s}+1+\frac{1}{s}\right]=\frac{1}{s}}$$

$$v(s)\left[\frac{1+2s+2}{2s}\right]=\frac{1}{s}$$

$$\frac{v(s)(2s+3)}{2s}=\frac{1}{s}$$

$$v(s)=\frac{2}{2s+3}$$

$$v(s)=\frac{1}{s+1.5}---(2)$$

ILT

$$\boldsymbol{v(t)=e^{-1.5t}}$$

Laplace Transforms- **Analysis of electrical circuits**

Procedure to Analyse electrical circuits using Laplace Transforms:

Step-1: Identify the energy storage elements.
Step-2: Find the state of energy storage elements at t=0-, by either general prediction or calculating by drawing equivalent circuit.

Example: $\boldsymbol{i_L(0^-), i_L(0^+), v_C(0^-)}$ **and** $\boldsymbol{v_C(0^+)}$

Step-3: Draw the equivalent circuit at t=0+. And find the initial values of branch currents/loop currents/branch voltages/node voltages depending on the load quantity.

Example: $i(0^+), v(o^+)$ etc.

Step-4: Draw the equivalent circuit at t>0.
Step-5: Describe the behaviour of the given electrical circuit using differential equations (Apply KVL or KCL).
Step-6: Covert Differential equations to algebraic equations by applying Laplace Transform.
Step-7: Simplify the equation for required variable in frequency domain.
Step-8: Apply partial fractions.
Step-9: Take inverse Laplace transform to get the solution in time domain.

Laplace transform.

$$i(t) \rightarrow I(s)$$
$$v(t) \rightarrow V(s)$$

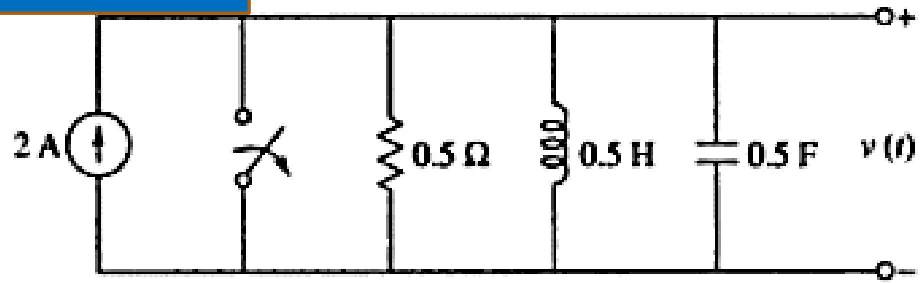
$$Ri(t) \rightarrow RI(s)$$
$$\frac{v(t)}{R} \rightarrow \frac{V(s)}{R}$$

$$L\frac{di(t)}{dt} \rightarrow L[sI(s) - i(0)] = LsI(s) - Li(0)$$
$$C\frac{dv(t)}{dt} \rightarrow C[sV(s) - v(0)] = CsV(s) - Cv(0)$$
$$\frac{1}{L}\int v(t)dt \rightarrow \int_{-\infty}^{t} v(t) \Rightarrow \frac{1}{L}\int_{-\infty}^{0} v(t)dt + \frac{1}{L}\int_{0}^{t} v(t)dt \Rightarrow i_L(0) + \frac{1}{L}\int_{0}^{t} v(t)dt \Rightarrow \frac{1}{Ls}V(s) + \frac{i_L(0)}{s}$$
$$\frac{1}{C}\int i(t)dt \rightarrow \frac{1}{Cs}I(s) + \frac{v_C(0)}{s}$$

Laplace Transforms- **Analysis of electrical circuits**

Examples

1. Find the voltage v(t) at t>0 for the electrical circuit shown in figure.

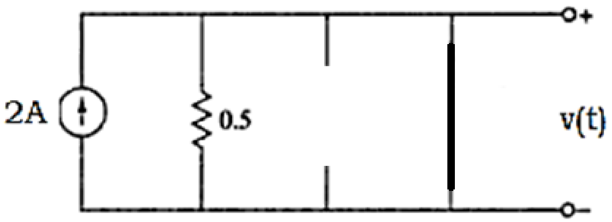


Solution:

0.5H and 0.5F are energy storage elements, and they are in uncharged state at t=0-. Hence $\boldsymbol{i_L(0^-) = i_L(0^+) = 0A}$.

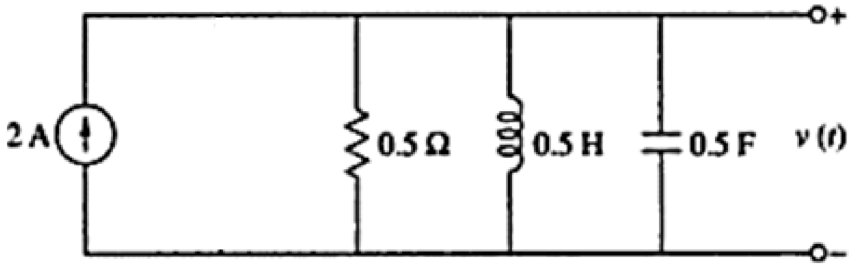
$$\boldsymbol{v_C(0^-) = v_C(0^+) = 0V}.$$

At t=0+



$$\boldsymbol{v(0^+) = 0\ volts}.$$

At t>0



Apply KCL at node v(t)

$$2 = \frac{v(t)}{0.5} + \frac{1}{L}\int v(t)dt + C\frac{dv(t)}{dt} ---(1)$$

Apply L.T

$$\frac{2}{s} = \frac{V(s)}{0.5} + 2\frac{V(s)}{s} + i\ (0) + 0 + 0.5\,[sV(s) - v(0)]$$

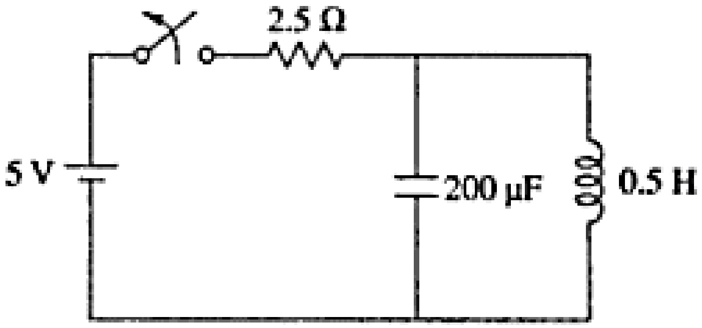
$$\frac{2}{s} = 2V(s) + \frac{2}{s}V(s) + 0.5sV(s)$$

$$\frac{2}{s} = V(s)\left[2 + \frac{2}{s} + 0.5s\right]$$

$$V(s) = \frac{\frac{2}{s}}{\frac{2s + 2 + 0.5s^2}{s}} \Rightarrow \frac{4}{s^2 + 4s + 4} \Rightarrow \frac{4}{(s+2)^2}$$

Apply ILT, $\boldsymbol{v(t) = 4.t.e^{-2t}}$

Laplace Transforms- **Analysis of electrical circuits**



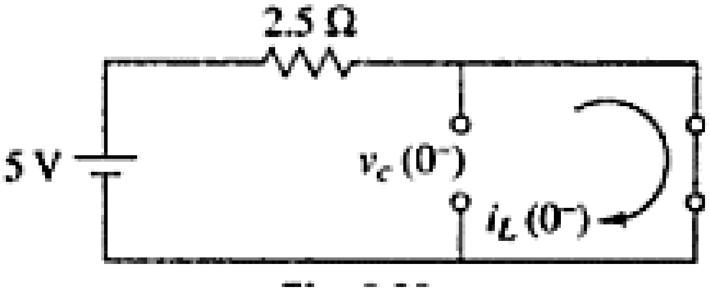
Examples

2. Find the current through the inductor at t>0 for the electrical circuit shown in figure.

Solution:

200uF and 0.5H are energy storage elements, and attained steady state at t=0-

At t=0-, the equivalent circuit.

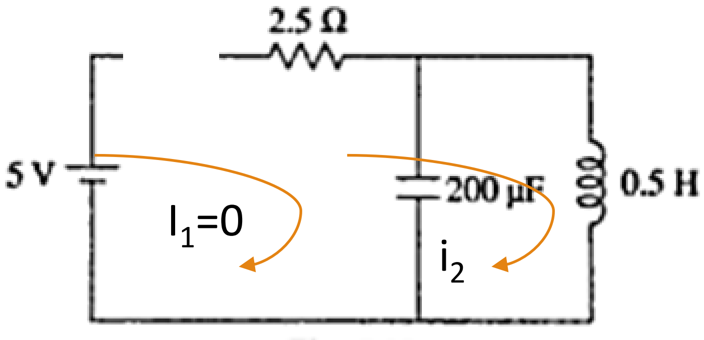


$$v_C(0^-) = 0V.\ i_L(0^-) = 2A.$$

$$v_C(0^+) = 0V.\ i_L(0^+) = 2A.$$

At t>0

Equivalent circuit



Apply KVL

$$\frac{1}{C}\int i(t)dt + L\frac{di(t)}{dt} = 0$$

$$LT$$

$$\frac{1}{Cs}I(s) + \frac{v(0)}{s} + LsI(s) - Li(0) = 0$$

$$\frac{1}{Cs}I(s) + Ls\,I(s) - 2\text{L} = 0.$$

$$\frac{I(s)[1 + LCs^2]}{Cs} = 2L$$

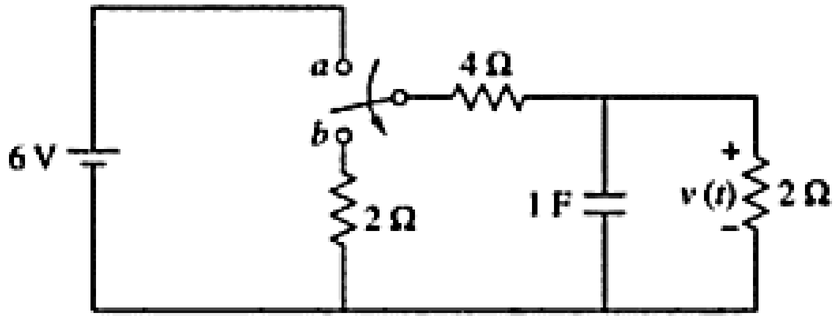
$$I(s) = \frac{2LCs}{1 + LCs^2}$$

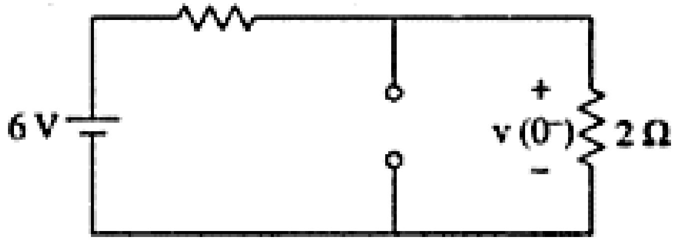
$$I(s) = \frac{2LCs}{1 + LCs^2} \Rightarrow \frac{2x200x10^{-6}x0.5s}{1 + 0.5x200x10^{-6}s^2} = \frac{800x10^{-6}s}{1 + 400x10^{-6}s^2} = \frac{2s}{2500 + s^2} \Rightarrow 2.\frac{s}{50^2 + s^2}$$

$$ILT$$

$$\boldsymbol{i(t) = 2.cos50t.}$$

Laplace Transforms- **Analysis of electrical circuits**





Examples

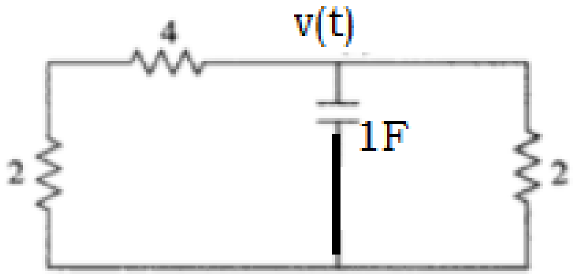
3. Find v(t) at t>0 for the electrical circuit shown in figure.

Solution:

1F capacitor is energy storage element and attained steady state at t=0-.

$$v_C(0^-) = v(0^+) = 2V = v(0^-) = v(0^+)$$

At t>0



$$Apply\ KCL$$

$$-\frac{v(t)}{6} = \frac{dv(t)}{dt} + \frac{v(t)}{2}$$

$$\frac{dv(t)}{dt} + \frac{2}{3}v(t) = 0 ---(1)$$

$$\frac{dv(t)}{dt} + \frac{2}{3}v(t) = 0 ---(1)$$

LT

$$sV(s) - v(0) + \frac{2}{3}v(s) = 0$$

$$sV(s) - 2 + \frac{2}{3}V(s) = 0$$

$$V(s)[s + \frac{2}{3}] = 2$$

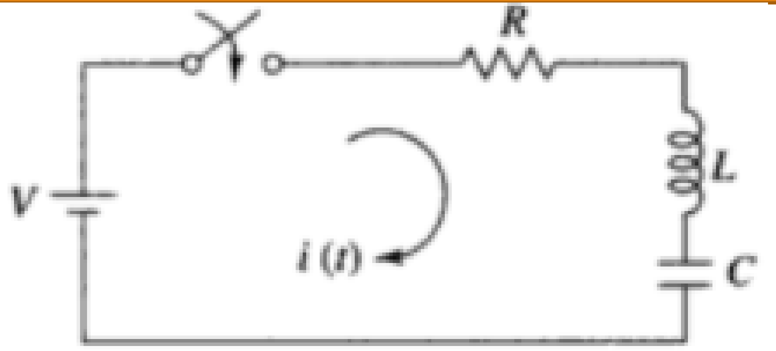
$$V(s) = \frac{2}{s + \frac{2}{3}} ---(2)$$

$$V(s) = \frac{2}{s + \frac{2}{3}}$$

ILT

$$v(t) = 2.e^{-\left(\frac{2}{3}t\right)}$$

Laplace Transforms- **Analysis of electrical circuits**



Examples

1. Find the voltage v(t) at t>0 for the electrical circuit shown in figure.

Solution:

Both Inductor and Capacitor are in uncharged condition, hence

$$i_L(0^-) = i_L(0^+) = 0A \text{ and } v_C(0^-) = v_C(0^+) = 0V$$

At t>0

KVL equation

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \;---(1)$$

Apply L.T

$$\frac{V}{s} = R\,I(s) + L\,[sI(s) - i(0)] + \frac{1}{Cs}I(s) + \frac{v_C(0)}{s} \;---(2)$$

$$\frac{V}{s} = \left(R + Ls + \frac{1}{Cs}\right)I(s)$$

$$\frac{V}{s}Cs = (RCs + LCs^2 + 1)I(s) \;---(3)$$

Divide LC on both sides

$$\frac{V}{s}Cs.\frac{1}{LC} = \left(\frac{RCs}{LC} + \frac{LCs^2}{Lc} + \frac{1}{LC}\right)I(s)$$

$$I(s) = \frac{\frac{V}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \;---(4)$$

$$I(s) = \frac{\frac{V}{L}}{(s+\alpha-\beta)(s+\alpha+\beta)}$$

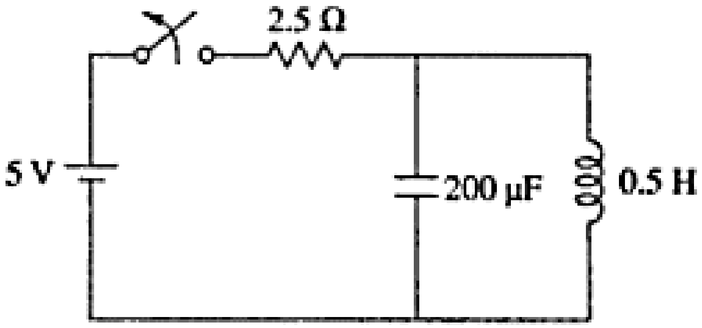
Where, $\alpha = -\frac{R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Laplace Transforms- **Analysis of electrical circuits**

$$I(s) = \frac{\frac{V}{L}}{(s+\alpha-\beta)(s+\alpha+\beta)}$$

Where, $\alpha = -\frac{R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Laplace Transforms- **Analysis of electrical circuits**



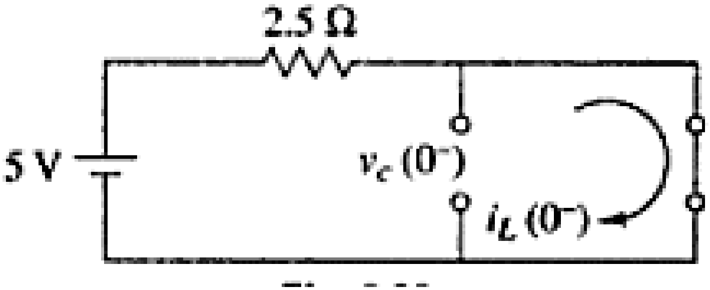
Examples

2. Find the current through the inductor at t>0 for the electrical circuit shown in figure.

Solution:

200uF and 0.5H are energy storage elements, and attained steady state at t=0-

At t=0-, the equivalent circuit.

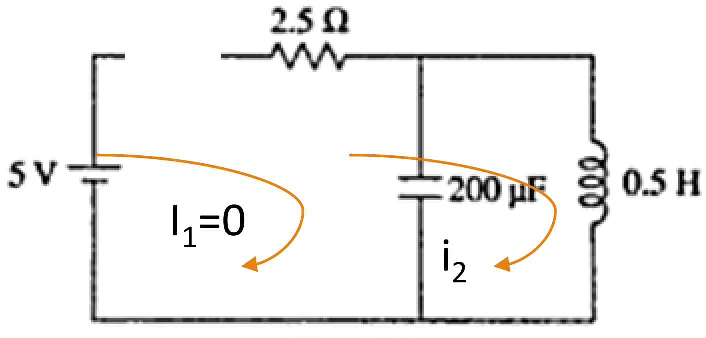


$$\boldsymbol{v_C(0^-) = 0V. i_L(0^-) = 2A.}$$

$$\boldsymbol{v_C(0^+) = 0V. i_L(0^+) = 2A.}$$

At t>0

Equivalent circuit



Apply KVL

$$\frac{1}{C}\int i(t)dt + L\frac{di(t)}{dt} = 0$$

$$LT$$

$$\frac{1}{Cs}I(s) + \frac{v(0)}{s} + LsI(s) - Li(0) = 0$$

$$\frac{1}{Cs}I(s) + Ls\,I(s) - 2\text{L} = 0.$$

$$\frac{I(s)[1 + LCs^2]}{Cs} = 2L$$

$$I(s) = \frac{2LCs}{1 + LCs^2}$$

$$I(s) = \frac{2LCs}{1 + LCs^2} \Rightarrow \frac{2x200x10^{-6}x0.5s}{1 + 0.5x200x10^{-6}s^2} = \frac{800x10^{-6}s}{1 + 400x10^{-6}s^2} = \frac{2s}{2500 + s^2} \Rightarrow 2.\frac{s}{50^2 + s^2}$$

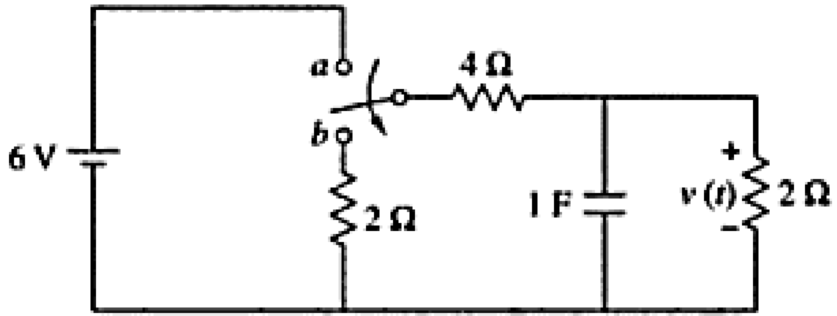
$$ILT$$

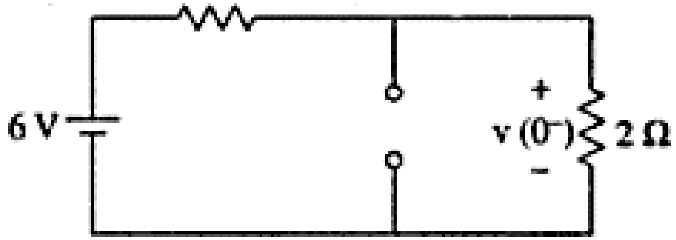
$$\boldsymbol{i(t) = 2.cos50t}.$$

Laplace Transforms- **Analysis of electrical circuits**

Examples

3. Find v(t) at t>0 for the electrical circuit shown in figure.



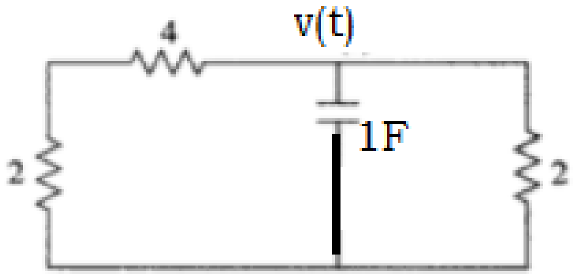


Solution:

1F capacitor is energy storage element and attained steady state at t=0-.

$$v_C(0^-) = v(0^+) = 2V = v(0^-) = v(0^+)$$

At t>0



$$Apply\ KCL$$

$$-\frac{v(t)}{6} = \frac{dv(t)}{dt} + \frac{v(t)}{2}$$

$$\frac{dv(t)}{dt} + \frac{2}{3}v(t) = 0 ---(1)$$

$$\frac{dv(t)}{dt} + \frac{2}{3}v(t) = 0 ---(1)$$

LT

$$sV(s) - v(0) + \frac{2}{3}v(s) = 0$$

$$sV(s) - 2 + \frac{2}{3}V(s) = 0$$

$$V(s)[s + \frac{2}{3}] = 2$$

$$V(s) = \frac{2}{s + \frac{2}{3}} ---(2)$$

$$V(s) = \frac{2}{s + \frac{2}{3}}$$

ILT

$$v(t) = 2.e^{-\left(\frac{2}{3}t\right)}$$

Laplace Transforms- **Waveform synthesis**

Introduction:

- Waveforms- signals
- Signals-Quantities which convey information (Current/Voltage)
- Two types

    1. Periodic and 2. Aperiodic Signals

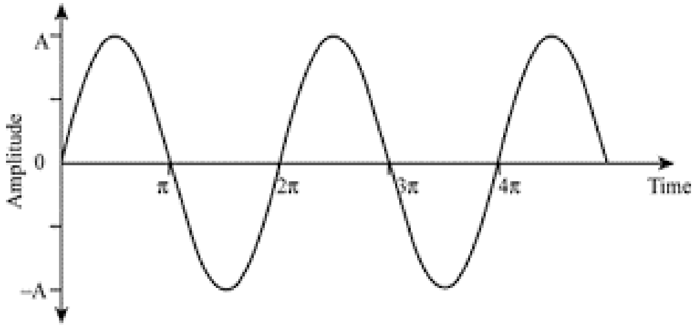
Periodic signal one which repeats the pattern exactly after a fixed time interval for all t.

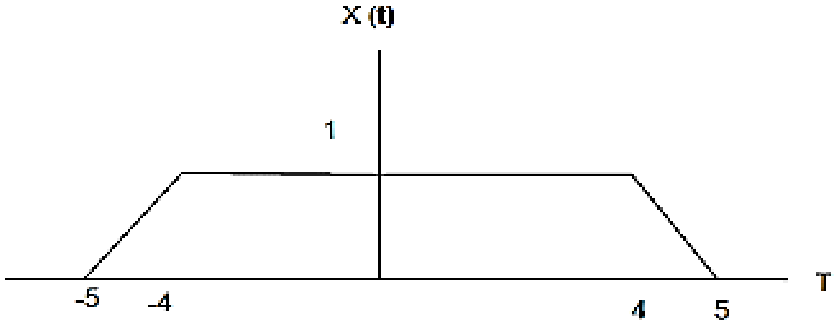
i.e., $\boldsymbol{f(t+T)=f(t)}, \textit{Where}, T = \textit{is the fixed time interval called Period.}$

A periodic signal is one for which no value of T satisfies the above equation

i.e.,$\boldsymbol{f(t+T)\neq f(t)}$

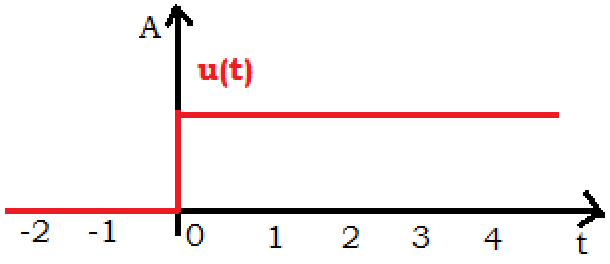
Examples:

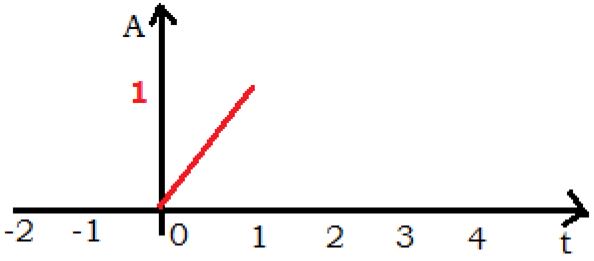


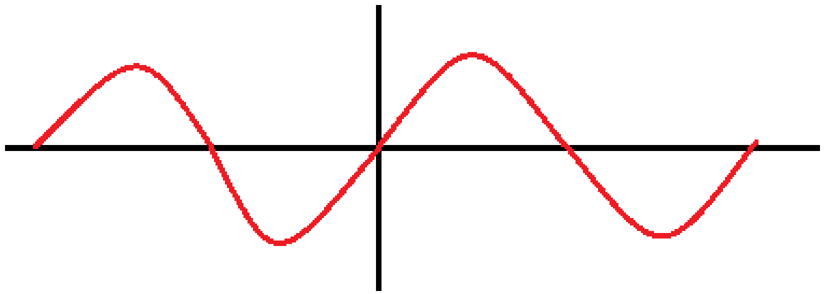


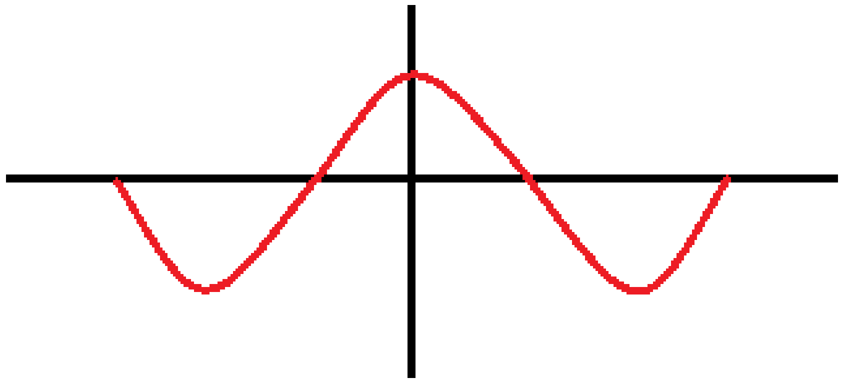
Laplace Transforms- **Waveform synthesis**

Basic Signals- step, ramp, exponential and sinusoidal signals



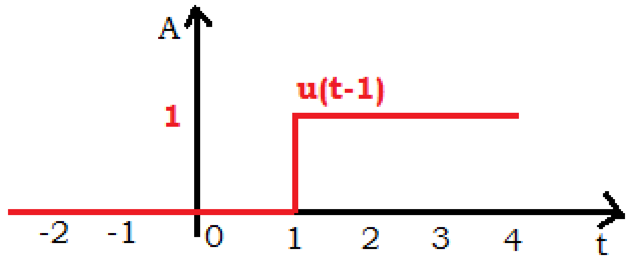


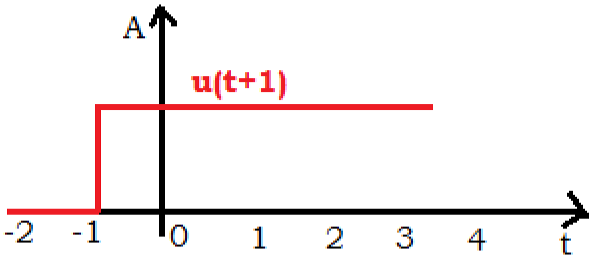


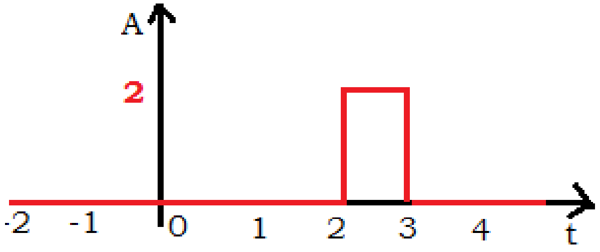


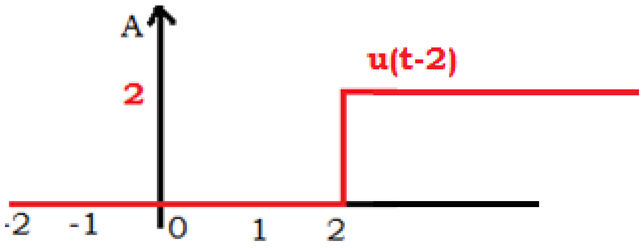
- No discontinuities – combination of only ramp signals
- Discontinuities and constant –only step signals

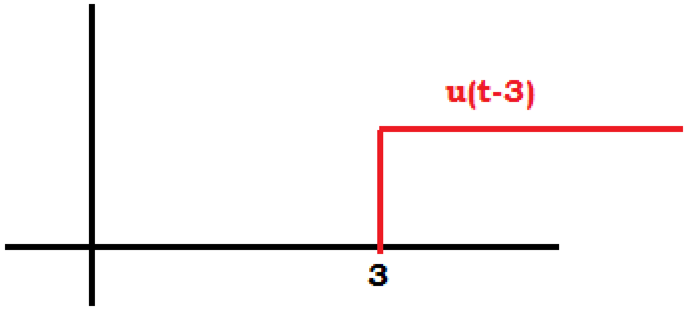
Laplace Transforms- **Waveform synthesis - Examples**











Time shifting property

$L\{f(t-T)\}=e^{-Ts}.F(s)$

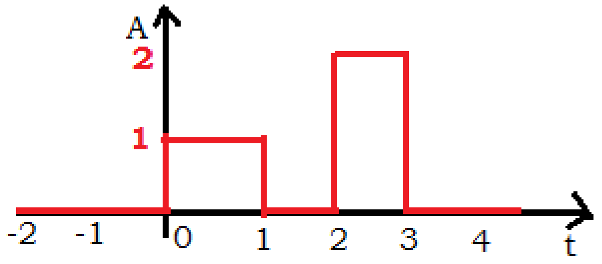
$$f(t) = 2u(t-2) - 2u(t-3) --- (1)$$

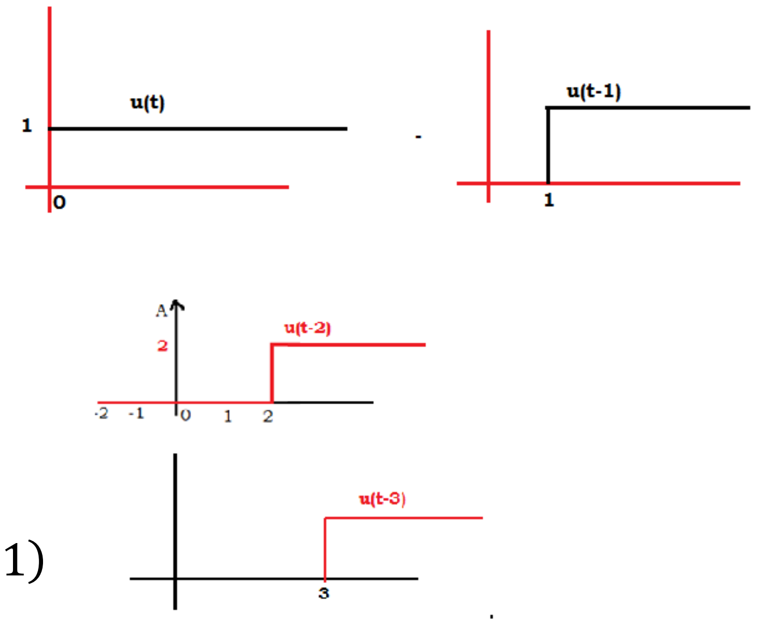
LT

$$F(s) = 2\frac{1}{s}e^{-2s} - 2.\frac{e^{-3s}}{s}$$

$$F(s) = \frac{2}{s}(e^{-2s} - e^{-3s}) --- (2)$$

Laplace Transforms- **Waveform synthesis - Examples**





$$f(t) = u(t) - u(t-1) + 2u(t-2) - 2u(t-3) --- (1)$$

LT

$$F(s) = \frac{1}{s} - \frac{e^{-s}}{s} + 2\frac{1}{s}e^{-2s} - 2\frac{e^{-3s}}{s}$$

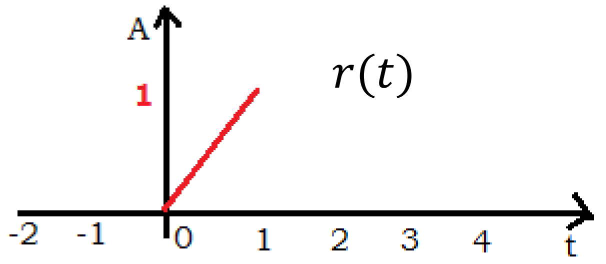
$$\boldsymbol{F(s) = \frac{1}{s}\left(1 - e^{-s} + 2e^{-2s} - 2e^{-3s}\right) --- (2)}$$

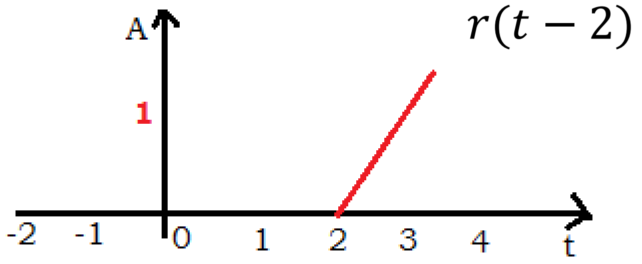
Method-II

$$\boldsymbol{f(t) = (1-0)u(t) + (0-1)u(t-1)) + (2-0)u(t-2) + (0-2)u(t-3) --- (1)}$$

$$\boldsymbol{f(t) = u(t) - u(t-1) + 2u(t-2) - 2u(t-3)}$$

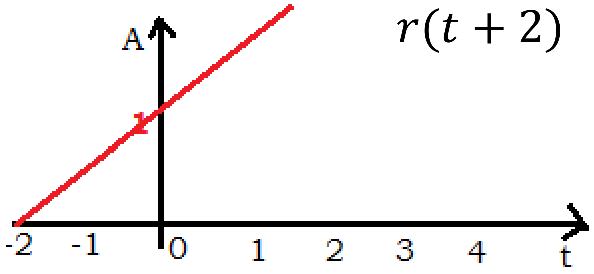
Laplace Transforms- Waveform synthesis - Examples



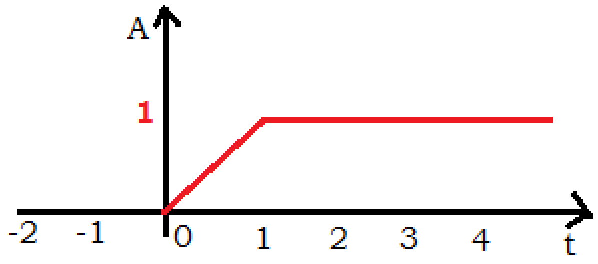


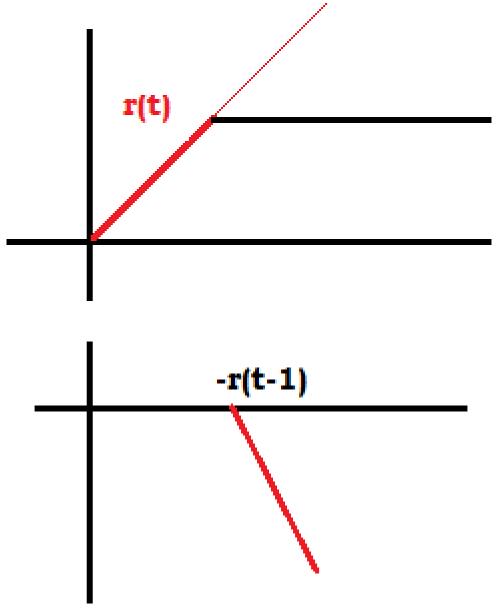
$$y = mx + c$$

$$m = \frac{y}{x} \Rightarrow \frac{y_2 - y_1}{x_2 - x_1}$$



Laplace Transforms- **Waveform synthesis - Examples**





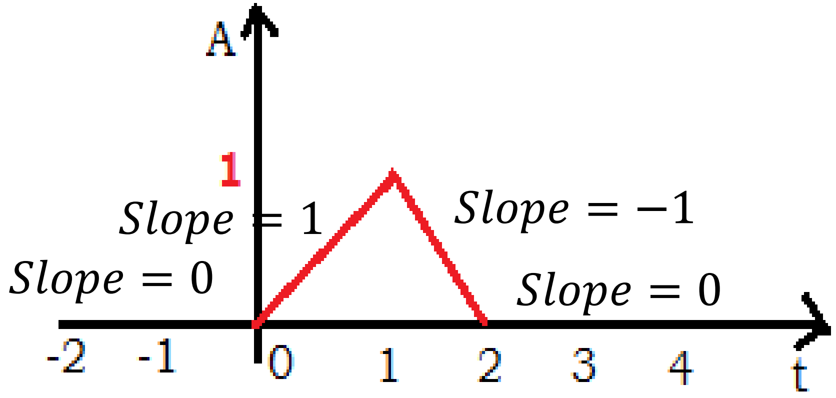
$$f(t) = r(t) - r(t-1) --- (1)$$

LT

$$F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2}$$

Method-II

$$f(t) = 1.r(t) - 1.r(t-1)$$
$$f(t) = r(t) - r(t-1)$$



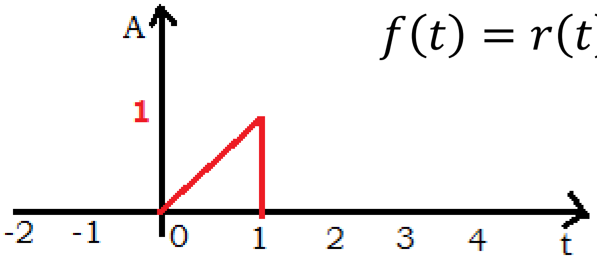
Method-II

$$f(t) = \mathrm{r(t)} + (-1-1)\mathrm{r(t-1)} + \big(0-(-1)\big)\mathrm{r(t-2)}$$
$$f(t) = r(t) - 2r(t-1) + r(t-2) --- (1)$$

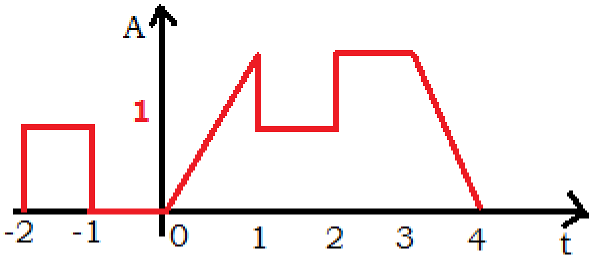
LT

$$F(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

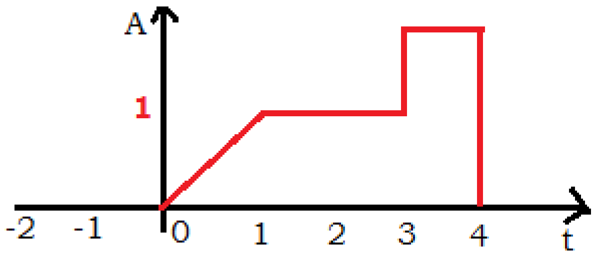
Laplace Transforms- **Waveform synthesis - Examples**



$$f(t) = r(t) - r(t-1) - u(t-1)$$



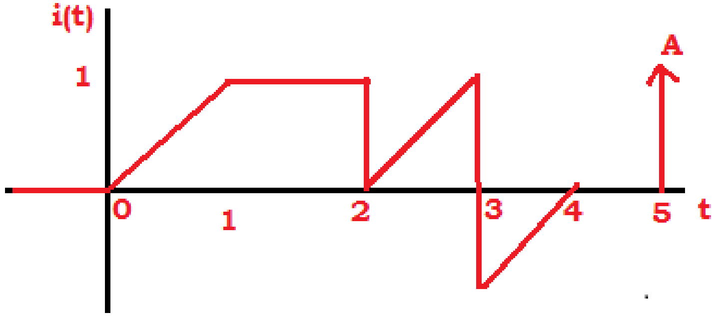
$$f(t) = u(t+2) - u(t+1) + 2.r(t) - 2r(t-1) - u(t-1) + u(t-2)$$
$$-2r(t-3) + 2r(t-4)$$

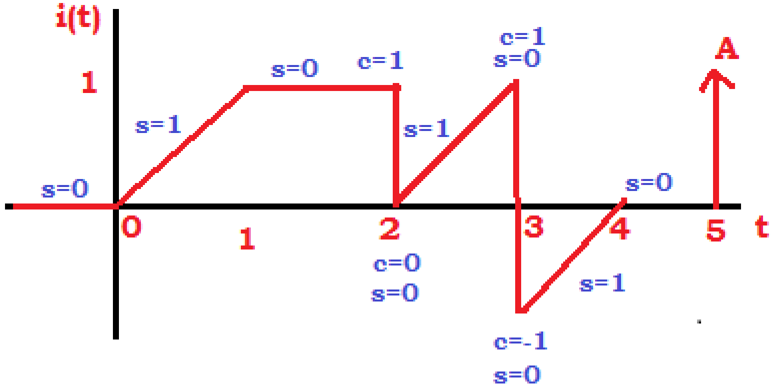


$$f(t) = 1r(t) - r(t-1) + 1.u(t-3) - 2u(t-4)$$

Laplace Transforms- **Waveform synthesis - Examples**

Find the Laplace Transform of the following signal.





$$i(t) = r(t) - \mathrm{r}(\mathrm{t}-1) - \mathrm{u}(\mathrm{t}-2) + \mathrm{r}(\mathrm{t}-2) - \mathrm{r}(\mathrm{t}-3) - 2\mathrm{u}(\mathrm{t}-3) + \mathrm{r}(\mathrm{t}-3) - \mathrm{r}(\mathrm{t}-4) + \mathrm{A}\delta(t-5)$$

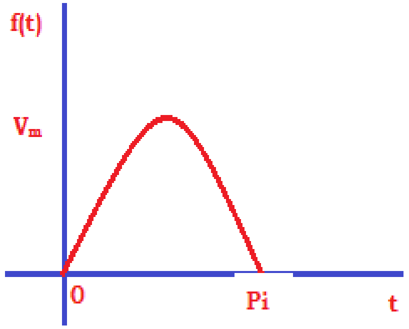
$$i(t) = r(t) - r(t-1) - u(t-2) + r(t-2) - 2u(t-3) - r(t-4) + A\delta(t-5) --- (1)$$

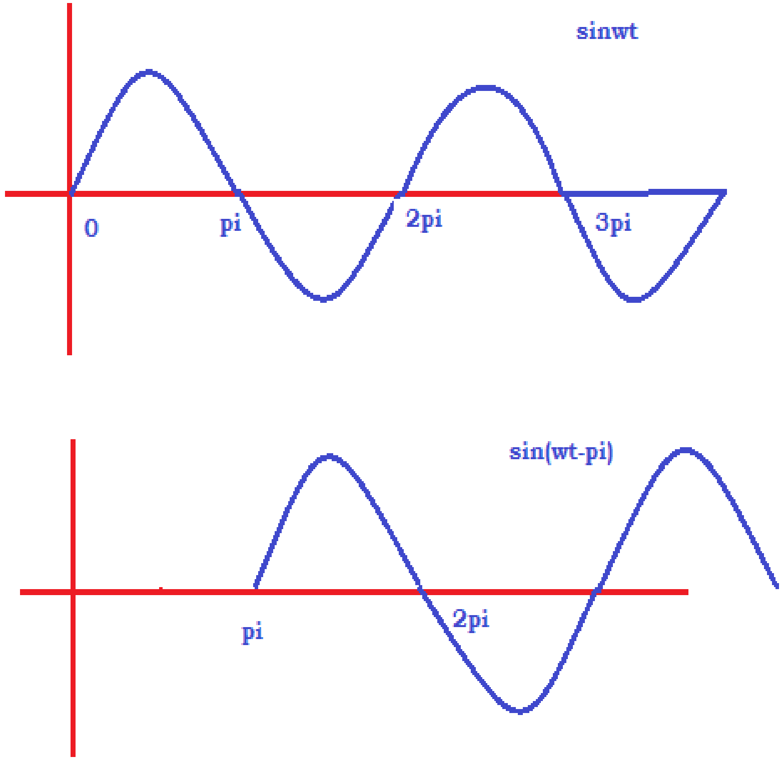
L.T.

$$\boldsymbol{I(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s} + \frac{e^{-2s}}{s^2} - \frac{2e^{-3s}}{s} - \frac{e^{-4s}}{s^2} + \frac{A}{e^{-5s}}}$$

Laplace Transforms- **Waveform synthesis - Examples**

Find the Laplace Transform of the following signal.





$$f(t) = sin\omega t + \sin(\omega t - pi) --- (1)$$

$$F(s) = \frac{w}{s^2 + w^2} + e^{-\pi s} . \frac{w}{s^2 + w^2}$$

Find the Laplace Transform for following periodic signal.

$L\{f(t)\} = \frac{1}{1-e^{-Ts}} F_1(s)$

Where, f(t) is a periodic signal/waveform and $F_1(s) is\ the\ laplace\ transform\ of\ single/first\ cycle\ signal$

Example: T=1.

$$F(s) = \frac{1}{1-e^{-s}} F_1(s)$$

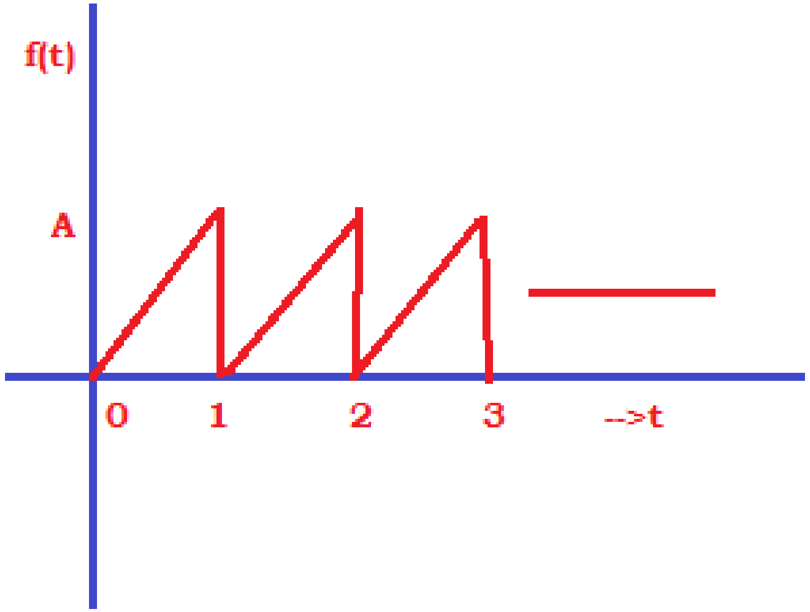
$$F_1(s) = L\{f_1(t)\}$$

$$f_1(t) = Ar(t) - Ar(t-1) - Au(t-1)$$

$$F_1(s) = \frac{A}{s^2} - \frac{Ae^{-s}}{s^2} - \frac{Ae^{-s}}{s}$$

$$F(s) = \frac{1}{1-e^{-s}} \left[\frac{A}{s^2} - \frac{Ae^{-s}}{s^2} - \frac{Ae^{-s}}{s}\right]$$

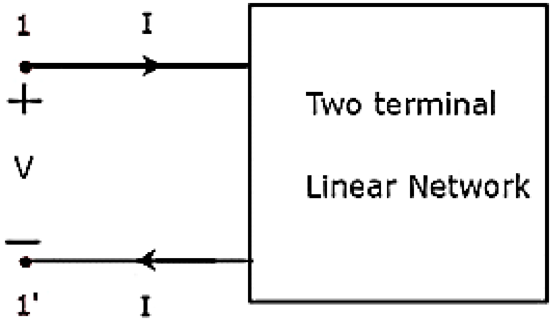
$$F(s) = \frac{A}{s^2(1-e^{-s})} [1 - \mathrm{e}^{-\mathrm{s}} - \mathrm{se}^{-\mathrm{s}})$$



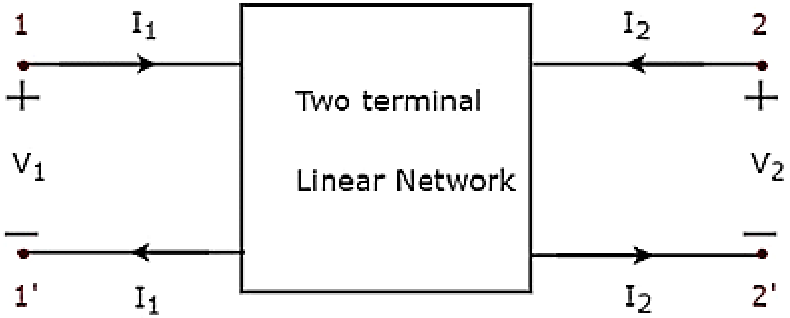
Two port Networks - Introduction

Port - A Pair of terminals, where electrical signal enters or leaves.

One port Network – Single pair of terminals, Current enters through one terminal and leaves through another terminal. Example: R, L, C



Two port Network – Two pairs of terminals, Current enters through one terminal and leaves through another terminal.



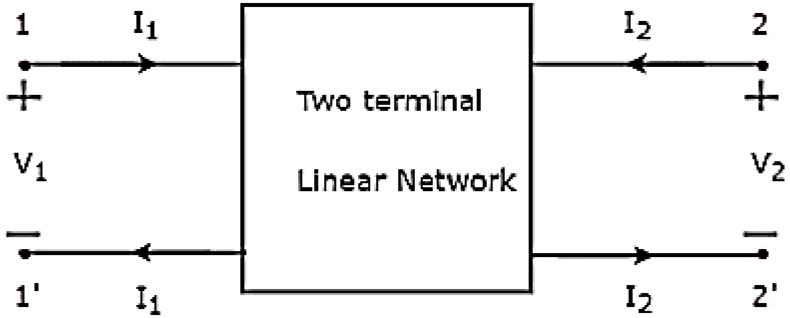
Two port Networks - Introduction

Three Port Network – Three pairs of terminal, Example: Co-axial circulars

Four Port Network – Four pairs of terminal, Example: Directional Couplers.

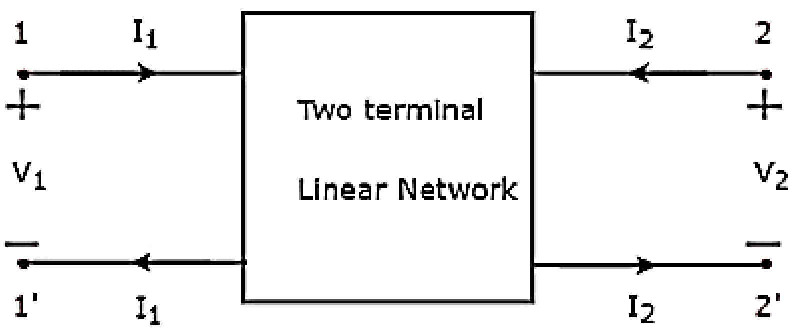
Two port Networks - Introduction

Two port Network – Two pairs of terminals, Current enters through one terminal and leaves through another terminal.



Two port Network – Two pairs of terminals, Current enters through one terminal and leaves through another terminal of each port.
Here, 1-1' and 2-2' are two ports, Port-1 and Port-2.
Four variables are associated with the network, they are I1, I2, V1 and V2.
Two variables are dependent variables and the remaining two variables are independent variables.
Six possible pairs of equations, these equations represents the dependent variables in terms of independent variables.
The co-efficients of independent variables are called Parameters.
Z, Y, h and T parameters are most important for the analysis of electrical and electronic circuits.

Two port Networks - Introduction



Z – Parameters

- V1 and V2 are dependent variables and I1 and I2 are independent variables

$$V_1 = f(I_1, I_2) ---(1)$$
$$V_2 = f(I_1, I_2) ---(2)$$

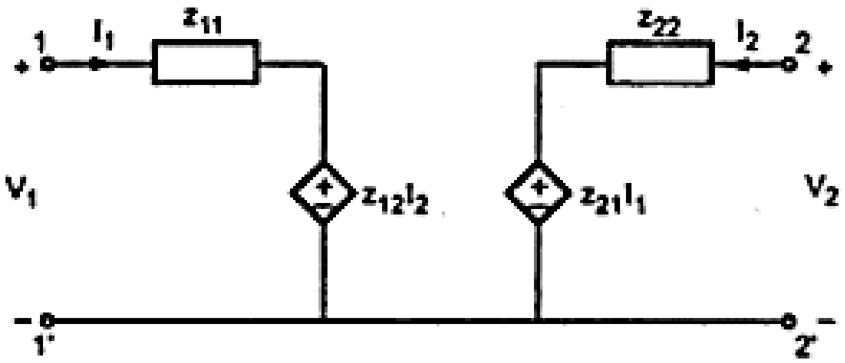
- Definition

$$V_1 = Z_{11}I_1 + Z_{12}I_2 ---(3)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 ---(4)$$

- In Matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} ---(5)$$

- Equivalent circuit



Two port Networks - Introduction

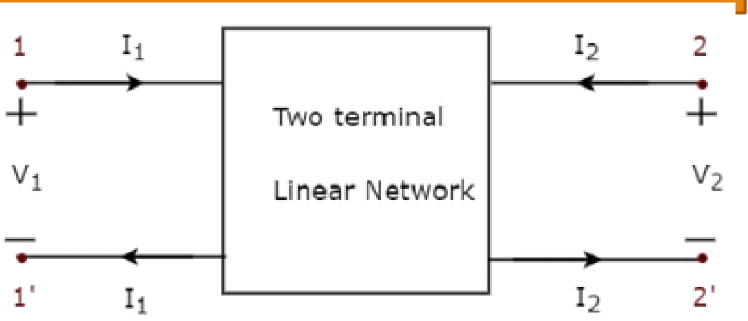
Z – Parameters

- Definition

$$V_1 = Z_{11}I_1 + Z_{12}I_2 ---(3)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 ---(4)$$

- Where,

$$Z_{11} = \frac{V_1}{I_1} \Rightarrow Open\ circuit\ input\ impedance$$
$$Z_{12} = \frac{V_1}{I_2} \Rightarrow Open\ Circuit\ reverse\ transfer\ impedance$$
$$Z_{21} = \frac{V_2}{I_1} \Rightarrow Open\ circuit\ forward\ tranfer\ impedance$$
$$Z_{22} = \frac{V_2}{I_2} \Rightarrow Open\ circuit\ output\ impedance.$$



Y – Parameters

- I1 and I2 are dependent variables, V1 and V2 are independent variables.

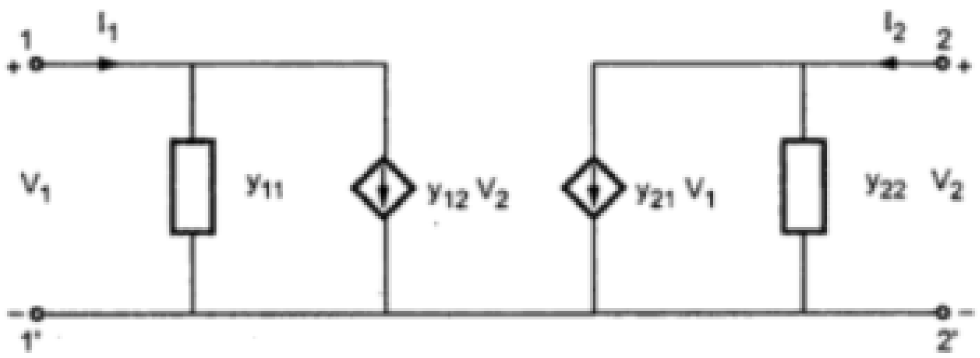
$$I_1 = f(V_1, V_2) ---(1)$$
$$I_2 = f(V_1, V_2) ---(2)$$

Definition

$$I_1 = Y_{11}V_1 + Y_{12}V_2 ---(3)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 ---(4)$$

Matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} ---(5)$$



Two port Networks - Introduction

$$I_1 = Y_{11}V_1 + Y_{12}V_2 --- (3)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 --- (4)$$

For V2=0, Short circuit the output port

$$Y_{11} = \frac{I_1}{V_1} \Rightarrow Short\ cicuit\ Input\ admittance$$

$$Y_{21} \Rightarrow \frac{I_2}{V_1} \Rightarrow short\ circuit\ forward\ tranfer\ admittance$$

For V1=0, sort circuit the input port

$$Y_{12} = \frac{I_1}{V_2} \Rightarrow short\ circuit\ revrese\ transfer\ admittance$$

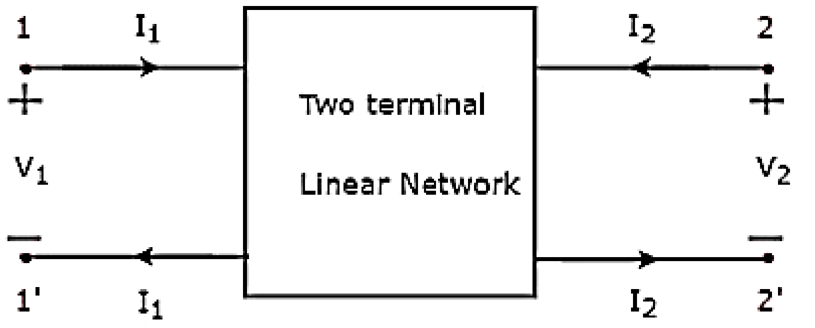
$$Y_{22} = \frac{I_2}{V_2} \Rightarrow short\ circuit\ output\ admittance$$

Y parameters are also called as Admittance parameters/short circuit admittance parameters.

Note: Reciprocal of Y parameters are not equal to the Z parameters.

Example: $Z_{11} \neq \frac{1}{Y_{11}}$

Two port Networks - Introduction



h – Parameters

- V1 and I2 are dependent variables and I1 and V2 are independent variables

$$V_1 = f(I_1, V_2) ---(1)$$
$$I_2 = f(I_1, V_2) ---(2)$$

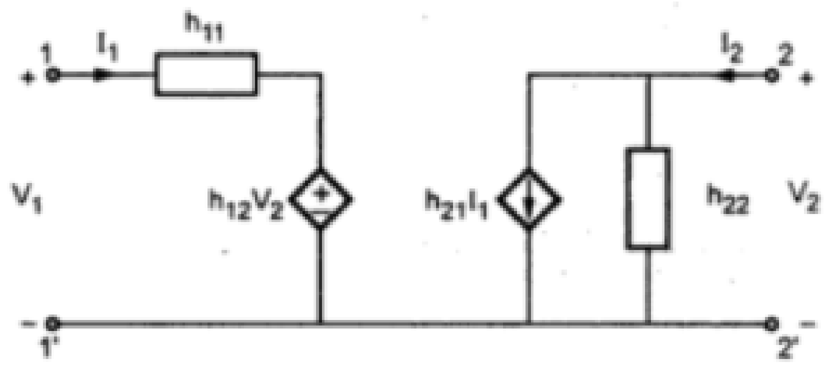
- Definition

$$V_1 = h_{11}I_1 + h_{12}V_2 ---(3)$$
$$I_2 = h_{21}I_1 + h_{22}V_2 ---(4)$$

- In Matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} ---(5)$$

- Equivalent circuit



Two port Networks - Introduction

$$V_1 = h_{11}I_1 + h_{12}V_2 ---(3)$$
$$I_2 = h_{21}I_1 + h_{22}V_2 ---(4)$$

V2=0, short circuit the output port

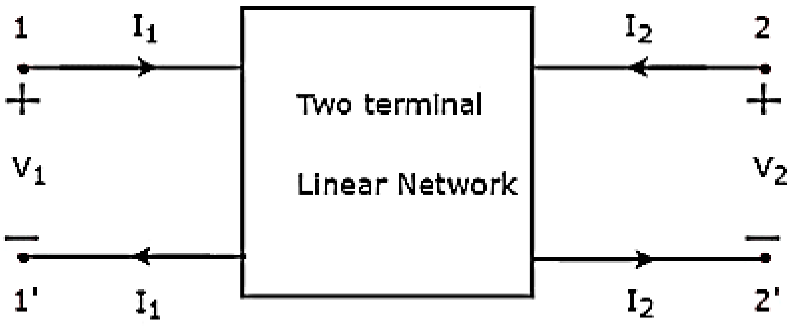
$$h_{11} = \frac{V_1}{I_1} \Rightarrow Short\ circuit\ input\ impedance$$
$$h_{21} = \frac{I_2}{I_1} \Rightarrow Short\ circuit\ forward\ current\ gain$$

I1=0, open circuit the input port

$$h_{12} = \frac{V_1}{V_2} \Rightarrow open\ circuit\ reverse\ voltage\ gain$$
$$h_{22} = \frac{I_2}{V_2} \Rightarrow open\ circuit\ output\ admittance.$$

Also called as hybrid parameters.

Two port Networks - Introduction



T – Parameters

- V1 and I2 are dependent variables and I1 and V2 are independent variables

$$V_1 = f(V_2, -I_2) ---(1)$$
$$I_1 = f(V_2, -I_2) ---(2)$$

- Definition

$$V_1 = AV_2 - BI_2 ---(3)$$
$$I_1 = CV_2 - DI_2 ---(4)$$

- In Matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ -I_2 \end{bmatrix} ---(5)$$

- Equivalent circuit

Two port Networks - Introduction

- Definition

$$V_1 = AV_2 - BI_2 ---(3)$$
$$I_1 = CV_2 - DI_2 ---(4)$$

I2 =0, output port is open circuit

$$A = \frac{V_1}{V_2} \Rightarrow Open\ circuit\ reverse\ voltage\ gain$$
$$C = \frac{I_1}{V_2} \Rightarrow Open\ circuit\ reverse\ admittance$$

V2=0, output port is short circuit

$$B = -\frac{V_1}{I_2} \Rightarrow Short\ circuit\ reverse\ impedance$$
$$D = -\frac{I_1}{I_2} \Rightarrow short\ circuit\ reverse\ current\ gain$$

Two port Networks – Relation Between Z and Y

Relation Between Z and Y, Z in terms of Y

Definition of Z- parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2 ---(1)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 ---(2)$$

Definition of Y – parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 ---(3)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 ---(4)$$

Fromm equation (3)

$$Y_{12}V_2 = I_1 - Y_{11}V_1$$
$$V_2 = \frac{1}{Y_{12}}I_1 - \frac{Y_{11}}{Y_{12}}V_1 ---(5)$$

Substitute equation (5) in (4)

$$I_2 = Y_{21}V_1 + Y_{22}\left[\frac{1}{Y_{12}}I_1 - \frac{Y_{11}}{Y_{12}}V_1\right]$$
$$I_2 = Y_{21}V_1 + \frac{Y_{22}}{Y_{12}}I_1 - \frac{Y_{11}Y_{22}}{Y_{12}}V_1$$
$$I_2 = \left[Y_{21} - \frac{Y_{11}Y_{22}}{Y_{12}}\right]V_1 + \frac{Y_{22}}{Y_{12}}I_1$$
$$I_2 = \left[\frac{Y_{21}Y_{12} - Y_{11}Y_{22}}{Y_{12}}\right]V_1 + \frac{Y_{22}}{Y_{12}}I_1$$
$$I_2 - \frac{Y_{22}}{Y_{12}}I_1 = \left[\frac{Y_{21}Y_{12} - Y_{11}Y_{22}}{Y_{12}}\right]V_1$$
$$\frac{Y_{12}I_2 - Y_{22}I_1}{Y_{12}} = \left[\frac{Y_{21}Y_{12} - Y_{11}Y_{22}}{Y_{12}}\right]V_1$$
$$V_1 = \frac{Y_{12}}{Y_{21}Y_{12} - Y_{11}Y_{22}}I_2 - \frac{Y_{22}}{Y_{21}Y_{12} - Y_{11}Y_{22}}I_1$$

Put $Y_{11}Y_{22} - Y_{12}Y_{21} = \Delta Y$

$$V_1 = \frac{Y_{22}}{\Delta Y}I_1 - \frac{Y_{12}}{\Delta Y}I_2 ----(6)$$

Compare equation (1) and (6)

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \text{ and } Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

Two port Networks – Relation Between Z and Y

$$\boldsymbol{I_1 = Y_{11}V_1 + Y_{12}V_2 --- (3)}$$
$$\boldsymbol{I_2 = Y_{21}V_1 + Y_{22}V_2 --- (4)}$$

From (3)

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 --- (7)$$

Substitute (7) in (4)

$$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22}V_2$$
$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right] V_2$$
$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{[Y_{11}Y_{22} - Y_{21}Y_{12}]}{Y_{11}} V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2$$

$$I_2 - \frac{Y_{21}}{Y_{11}} I_1 = \frac{\Delta Y}{Y_{11}} V_2$$

$$\frac{Y_{11}I_2 - Y_{21}I_1}{Y_{11}} = \frac{\Delta Y}{Y_{11}} V_2$$
$$\frac{Y_{11}I_2 - Y_{21}I_1}{\Delta Y} = V_2$$
$$V_2 = -\frac{Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 --- (8)$$

Compare equation (2) and (8)

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} \; and \; Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Two port Networks - Relation Between Z and Y

- **Learner Activity:**

Obtain the relation between,

i) Z and h
ii) Z and T
iii) Y and h
iv) Y and T
v) h and T
vi) Y in terms of Z

Two port Networks - Examples

- **Find Z parameters for the two port network shown in figure.**

SOLUTION:

Apply KVL to input circuit

$$V_1 = 1.I_1 + 6I_1 - 6I_3$$

$$V_1 = 7I_1 - 6I_3 ---(1)$$

Apply KVL to Output circuit

$$V_2 = 2I_2 + 2I_3 ---(2)$$

Apply KVL to loop 2

$$-6I_1 + 2I_2 + 12I_3 = 0 ---(3)$$

w. k. T.

$$V_1 = Z_{11}I_1 + Z_{12}I_2 ---(4)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 ---(5)$$

$$I_3 = \frac{1}{2}I_1 - \frac{1}{6}I_2 ---(6)$$

Substitute (6) in (1)

$$V_1 = 7I_1 - 6\left(\frac{1}{2}I_1 - \frac{1}{6}I_2\right)$$

$$V_1 = 4I_1 + I_2 ---(7)$$

Compare (4) and (7)

$$Z_{11} = 4\ Ohms$$

$$Z_{12} = 1\ Ohm.$$

Subsitute (6)*in* (2)

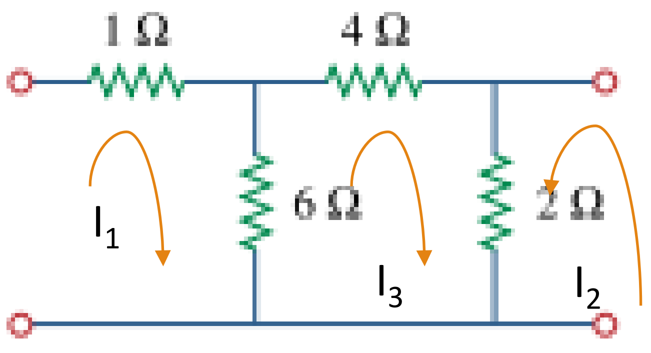
$$V_2 = 2I_2 + 2\left(\frac{1}{2}I_1 - \frac{1}{6}I_2\right)$$

$$V_2 = I_1 + \frac{5}{6}I_2 ---(8)$$

Compare (5) and (8)

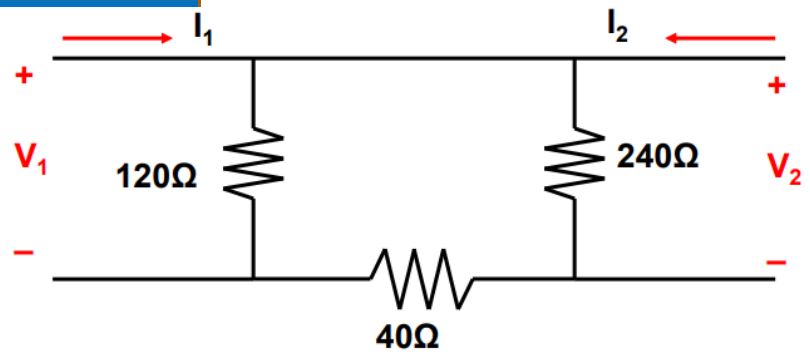
$$Z_{21} = 1\ Ohm$$

$$Z_{22} = \frac{5}{6}Ohms.$$



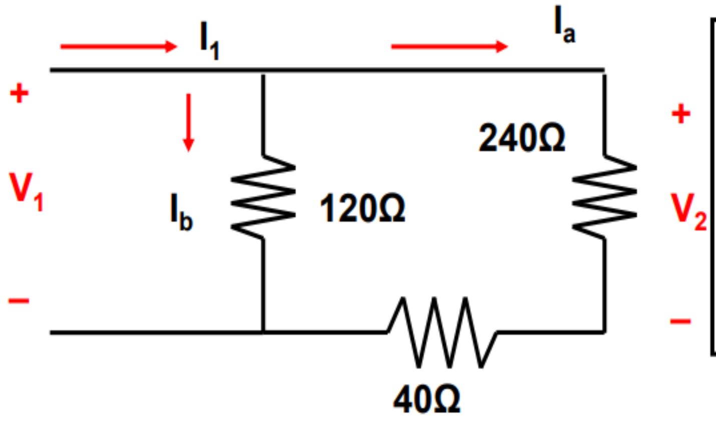
Two port Networks – Examples

1. Find Z – Parameters for the two port network shown in figure.



SOLUTION

i) $I_2 = 0$(open circuit port 2). Redraw the circuit.



$$V_1 = 120 I_b \ldots\ldots(1)$$

$$I_b = \frac{280}{400} I_1 \ldots\ldots(2)$$

sub(1) → (2)

$$\therefore Z_{11} = \frac{V_1}{I_1} = 84\Omega$$

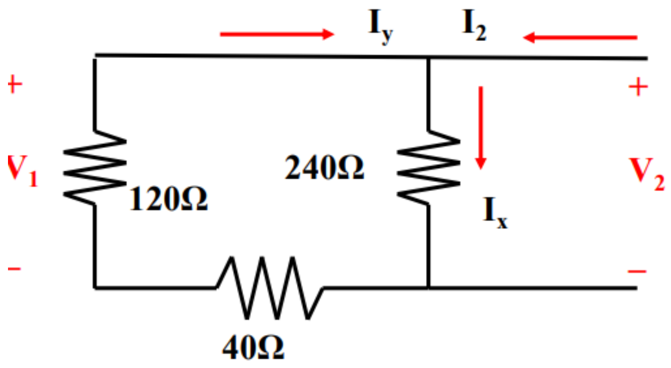
$$V_2 = 240 I_a \ldots\ldots(3)$$

$$I_a = \frac{120}{400} I_1 \ldots\ldots(4)$$

sub(4) → (3)

$$\therefore Z_{21} = \frac{V_2}{I_1} = 72\Omega$$

ii) $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_2 = 240 I_x \ldots\ldots(1)$$

$$I_x = \frac{160}{400} I_2 \ldots\ldots(2)$$

sub(1) → (2)

$$\therefore Z_{22} = \frac{V_2}{I_2} = 96\Omega$$

$$V_1 = 120 I_y \ldots\ldots(3)$$

$$I_y = \frac{240}{400} I_2 \ldots\ldots(4)$$

sub(4) → (3)

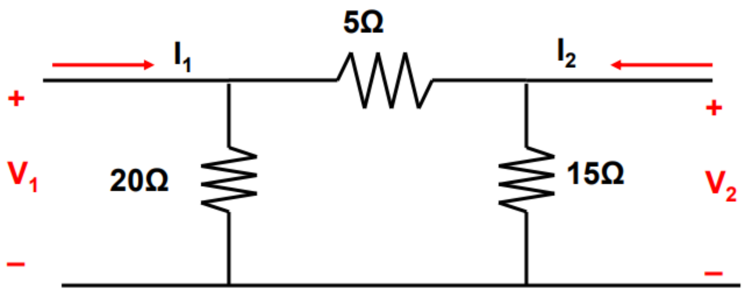
$$\therefore Z_{12} = \frac{V_1}{I_2} = 72\Omega$$

In matrix form:

$$[Z] = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix}$$

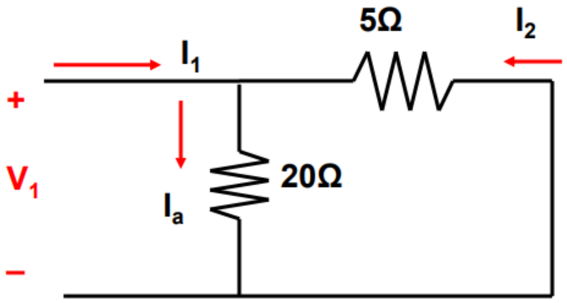
Two port Networks – Examples

2. Find Y – Parameters for the two port network shown in figure.



SOLUTION

i) $V_2 = 0$



$$V_1 = 20 I_a \ldots\ldots (1)$$

$$I_a = \frac{5}{25} I_1 \ldots\ldots (2)$$

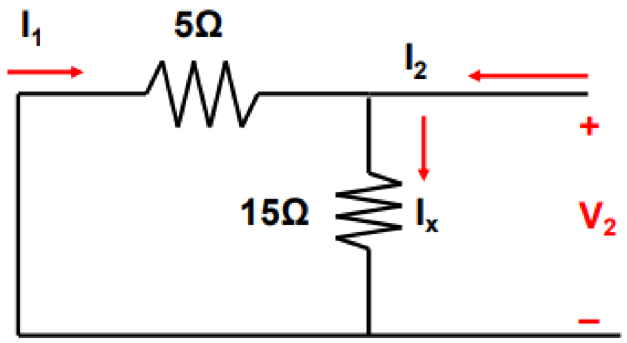
$$\text{sub}(1) \rightarrow (2)$$

$$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{4} S$$

$$V_1 = -5 I_2$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = -\frac{1}{5} S$$

ii) $V_1 = 0$



$$V_2 = 15 I_x \ldots\ldots (3)$$

$$I_x = \frac{5}{25} I_2 \ldots\ldots (4)$$

$$\text{sub}(3) \rightarrow (4)$$

$$\therefore Y_{22} = \frac{I_2}{V_2} = \frac{4}{15} S$$

$$V_2 = -5 I_1$$

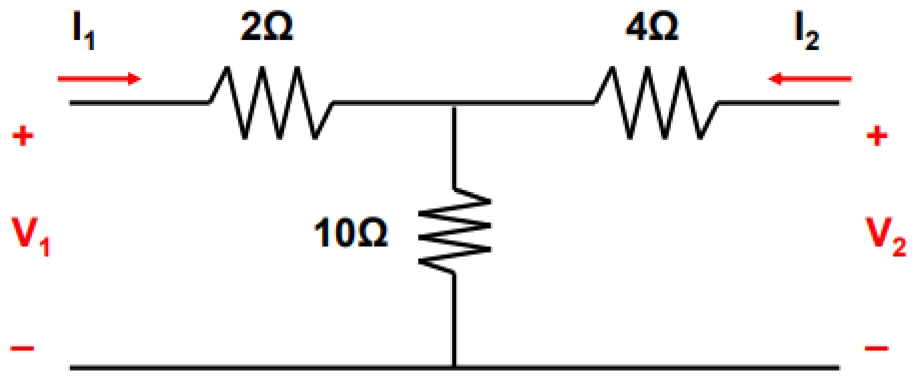
$$\therefore Y_{12} = \frac{I_1}{V_2} = -\frac{1}{5} S$$

In matrix form;

$$[Y] = \begin{bmatrix} \frac{1}{4} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix} S$$

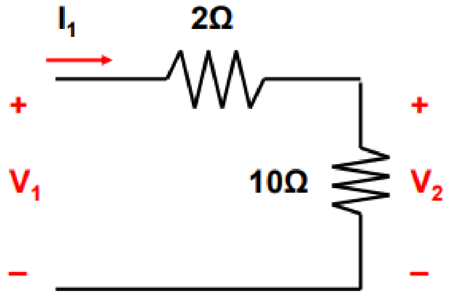
Two port Networks – Examples

3. Find T – Parameters for the two port network shown in figure.



SOLUTION

i) $I_2 = 0$,



$$V_2 = 10I_1$$

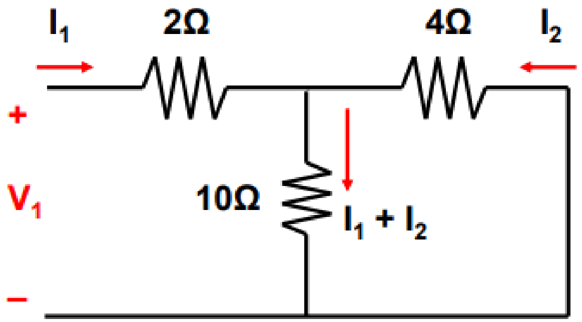
$$\therefore C = \frac{I_1}{V_2} = 0.1S$$

$$V_1 = 2I_1 + V_2$$

$$V_1 = 2\left(\frac{V_2}{10}\right) + V_2 = \frac{6}{5}V_2$$

$$\therefore A = \frac{V_1}{V_2} = 1.2$$

ii) $V_2 = 0$,



$$I_2 = -\frac{10}{14}I_1$$

$$\therefore D = -\frac{I_1}{I_2} = 1.4$$

$$V_1 = 2I_1 + 10(I_1 + I_2)$$

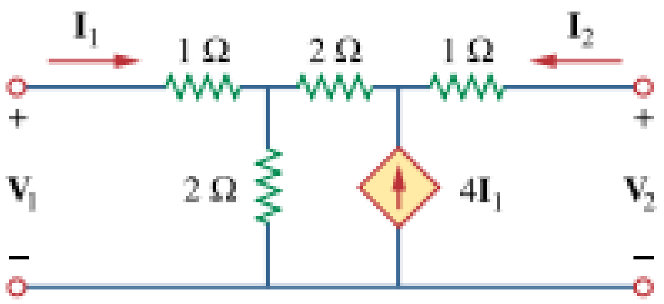
$$V_1 = 12I_1 + 10I_2$$

$$V_1 = 12\left(-\frac{14}{10}I_2\right) + 10I_2$$

$$\therefore B = -\frac{V_1}{I_2} = 6.8\Omega$$

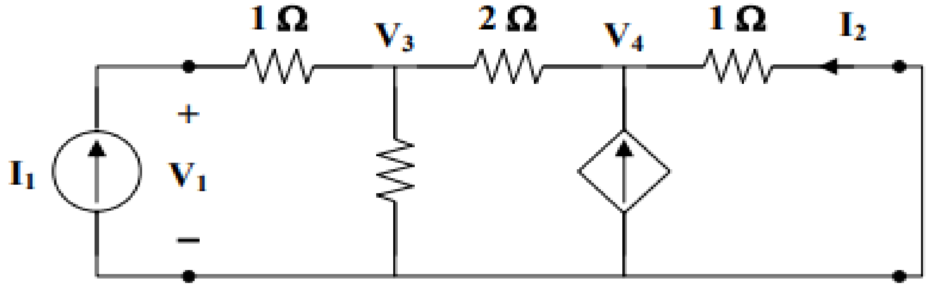
$$[T] = \begin{bmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{bmatrix}$$

Two port Networks – Examples



4. Find h – Parameters for the two port network shown in figure.

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = -\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = -2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

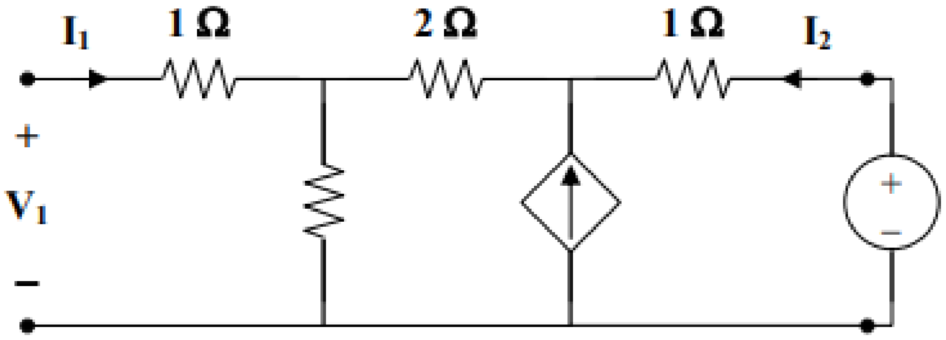
$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\,\Omega$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_4}{1} = -3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -3.6$$

Two port Networks – Examples

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\ \mathrm{S}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} 38\ \Omega & 0.4 \\ -3.6 & 0.2\ \mathrm{S} \end{bmatrix}$$

Two port Networks – Examples

Two port Networks – Examples

Two port Networks – Examples

Disclaimer

Some Contents and Images showed in this PPT have been taken from the various internet sources for educational purpose only.

Thank You

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